

... how
man
thinks

by
Richard Hope

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...To L.K.H.

PREFACE . . .

Man's chief glory is to think. Reflections of astronomers revolve around the stars; of workers, around their jobs; of citizens, around government; of parents, around their children; of playwrights, around dramas; of botanists, around plants; of geologists, around rocks; and of logicians, around reflective thought itself.

In logic our first thinking becomes the subject matter of our second thinking. W. Stanley Jevons in the Preface to his *Elementary Lessons in Logic* said of this science that: "It should take its place as an indispensable study for every well-informed person." For this vital branch of learning deals, as he had remarked, with "those simple principles and forms of reasoning which will enter into the thoughts of every hour." Not that logic presumes to analyze all the impulses, habits, and reflections which may characterize our actual thinking. But logic is concerned with an analysis of thought operations so ordered as to justify their outcome.

What logic undertakes, by way of a central task, is to formulate rules of inference. To this end it distinguishes necessary from probable inference and both of these from fallacies. By means of such distinctions and rules it provides guidance for discourse and inquiry.

From this point of view we may as logicians examine the results of thought in discourse as well as trace the processes of inquiry which lead to those results. As to discourse, logic considers especially how we may clarify our terms, relate our affirmations and denials, and support our conclusions. As to inquiry, our science asks how we may attempt to solve our

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problems; how, in particular, we may verify our hypotheses and demonstrate our theorems; and how we may apply the methods of reflective inquiry and of reasoned discourse to the weighing of opinions and values.

Hence, in the arrangement of this book, a general consideration of inference and discourse introduces the topics of terms, propositions, and syllogisms; and a general analysis of reflective and scientific inquiry introduces accounts of factual, formal, and evaluative thought. Numerous exercises, possessing varying degrees of difficulty and supplementing the brief expositions given, are added to encourage the student to enlarge and discipline his own thinking.

Reasoning is indeed best studied in extended examples. A very practical difficulty arises because illustrations must appear in manuals of logic in highly condensed form. But the application of logic to his own life and thought will remain each learner's constant problem. Nor will he expect from his introductory study exact prescriptions to fit every specific case which may arise in his experience. He will get something very much better in the way of perspective and background. Above all, his attention will be directed to questions applicable to the formation of his own judgments.

As an introductory treatment, the present book presupposes no previous familiarity with its subject. The point of view here taken stresses the fundamental contribution of the science of logic to a general or liberal education. If the uninitiated suspect logic of being a dry affair, let them bear in mind: how dry you find a crust of bread, may depend upon how much you chew on it.

More than enough material is offered in these pages for two semesters of study, each (if so desired) fairly independent of the other. Appropriate selections may be made as circumstances suggest for a single semester's work.

HOW MAN THINKS

Writers on logic to whom I owe most include Aristotle, W. Stanley Jevons, W. E. Johnson, John Dewey, and M. R. Cohen and E. Nagel. Responsibility for mistakes and failures in the following pages is my own.

The presentation here offered combines elements of traditional, symbolic, and experimental logic. In particular I regard forms of inference as generalizations from our actual inferences. Also continuities in discourse seem to me more basic in logic than distinctions which a highly developed symbolism suggests and which I find significant rather in appropriate contexts. Methods, too, besides varying with one's purposes and from one type of inquiry to another, appear to have common features amenable to logical analysis. However, an introductory manual hardly affords suitable opportunity for elaborate discussions of attitudes to the various "schools" of logicians.

RICHARD HOPE

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Inference and Discourse

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Inference

1. INFERENCE AND KNOWLEDGE

Logic may be defined briefly as the science of right thinking. Moreover, in order to learn how man ought to think, we must study how man does think. For just as experience in farming discloses distinctions between good and bad farming, so man's exercise of thought in his life reveals that some ways of thinking lead to failure, whereas others lead to success. Then, too, just as we may in the science of agriculture formulate the conditions under which farming may be done more economically and efficiently, so we may in the science of logic formulate the features distinguishing right from wrong thinking.

A good approach to logic is through the topic of inference. From this standpoint, logic may also be provisionally defined as the science of types of inference. This definition of logic has an advantage in that it stresses inference as central in the discrimination of good from poor thinking. For some of our inferences are more reliable than others. Thus, if a figure is rectangular, we may reliably infer that its diagonals will bisect each other. Or, if it is raining, we may more reliably than not infer that the scheduled baseball game will be postponed. But a stranger's supposedly honest face would afford little or no warrant for an inference to his honesty.

This approach to logic, as the science of types of inference, suggests at least three questions: What is inference? What are its main types? And in what sense is logic the science of them?

With a view to determining what an inference is, we may note how it differs from such processes as apprehension and comprehension. For we may on the one hand apprehend an object directly (like a figure or a face) or an event (like rain). To "apprehend" is to "seize" or to "grasp," either literally in some physical way, or figuratively by a mental act. Again, we may comprehend a meaning directly (like the meaning of "rectangular figure"). To "comprehend" is to "grasp together." Inference on the other hand is indirect.

In an inference, we pass from directly given facts or statements (e.g., that it is raining) to other facts or statements connected with them (e.g., that therefore the scheduled baseball game will be postponed). Obviously, we cannot observe an event directly in the future, or in the past, or going on at some inaccessible place in the present; but we may infer it, and invite another to infer it, on the basis of something that does engage our attention directly. So the burned child, having learned from experience how a lighted candle behaves, leaps in thought at the sight of a flame to what may happen to a finger.

Because our thinking, then, is not limited to direct apprehension and comprehension, but may proceed by way of inference, we may extend our knowledge indefinitely. Thus Aristotle declared: "All instruction given or received by way of argument proceeds from pre-existent knowledge."¹ For, as scientists build on past discoveries and achievements, so learners and teachers, in acquiring and

¹ Aristotle *Posterior Analytics* i.1.

imparting new knowledge, begin with knowledge already possessed: with a fact apprehended, with a meaning comprehended, or both. From what is thus known, or better known, we advance to what is as yet unknown or less known to us. Thus our questions concerning nature's ways gradually become answered; though sometimes what has been obscure to us becomes clear to us "in a flash."

Surely an act like inference, so pervasive in our progress from ignorance to knowledge, needs to be examined in any endeavor to understand and improve our thinking. We shall, accordingly, for our present purposes regard inference as a passage from given facts or statements to other facts or statements connected with them.

However, our attempts to learn by exploring the world in which we live are by no means free from erring excursions, so that we must also face the question into what main types our inferences may be divided. There are at least three kinds of inference to be distinguished at the beginning of our study of logic: necessary inferences, probable inferences, and fallacies.

First, as an example of necessary inference, take the argument: "If all mammals breathe through lungs, and if some mammals are water animals, then some water animals breathe through lungs." This is an inference in the sense that we have passed from a given fact or statement ("if," etc.) to another fact or statement connected with it ("then," etc.); and this is a necessary inference in the sense that the evidence ("if," etc.) for the conclusion ("then," etc.) is complete or conclusive.

It is important in this and in other inferences to distinguish premises from conclusions. Premises state the evidence for a conclusion; and a premise, in cases like that exemplified, may be introduced by such words as "if,"

"since," "because." A conclusion is said to "follow from" the premises—not in time, but as part of their meaning; and a conclusion may be introduced by such words as "then," "hence," "therefore." The reader will with a little practice be able to identify readily, in our examples, premises and conclusion.

Additional examples of necessary inference, or of inference based upon complete or conclusive evidence, may be cited as follows: "If speech requires a larynx, and fishes lack a larynx, then fishes are voiceless." "If B is greater than C, and A is equal to B, then A is greater than C." "If all free citizens vote, and no convicts vote, then no convicts are free citizens."

To be sure, the instances given illustrate many points and raise many problems. Yet for the present, we may content ourselves with seeing in them instances of inference (represented by "if—then"), of premises (represented by "if") and conclusions (represented by "then"), and of necessary inference (so that the conclusion may be said to "follow necessarily" from the premises).

Secondly, an inference may be not necessary, but probable; viz., when it is based upon evidence that is not conclusive, but partial. Suppose a weather bureau, with a record of eight true forecasts out of ten, predicts rain. We may then reasonably maintain that the rain foretold is more likely than not, because it is "probable" on the basis of evidence.

Similarly, rigorous proof is unavailable for the following: your friend is glad to see you; a # 87 trolley will take you into town; this marriage will be successful; Socrates walked on the streets of Athens; you will recover from your illness. We must, in such situations, rely upon assertions and circumstances that point to the conclusion

without altogether excluding some alternative to it. The conclusion would not "follow necessarily" from such assertions and circumstances, but may be more or less probable.

Thirdly, we must, from both necessary and probable inferences, distinguish fallacies. Inferences of this type are based upon premises which, even if true, constitute no evidence. Fallacies are most likely to deceive when they look like, without being, genuinely necessary or probable inferences. Thus some have argued incorrectly that by improving education for all we would thereby abolish individual differences.

As to fallacies, there is a sense in which logic is not obligated to deal with them—any more than mathematical science is required to give a systematic account of mistakes in arithmetic. Yet early attention to some of the more common fallacies may facilitate for the beginner an orientation concerning the province of logic.

The kinds of inference mentioned are distinguished by the weight of the evidence on which they rest: necessary inference, by complete or conclusive evidence; probable inference, by incomplete or partial evidence; and fallacies, by alleged evidence which is no genuine evidence at all. Logic accordingly may also be defined as the science of the weight of evidence.

In what sense, then, is logic a science? In what sense, if any, may we also speak of logic as an art? In general, a science teaches us to know, whereas an art teaches us to do. Logic does not indeed make us rational; for, as the art of navigation finds men navigating, so logic finds men making inferences and even testing them in simple cases. But navigation cannot be carried far without theory. So the science of logic aims at an understanding of our thinking, as the art of logic aims at its improvement.

Logic as a science examines our reasoning with a view to teaching us to know what we are doing when we reason; to know not so much whether, as rather why, our reasoning is correct or incorrect; to know wherein our rationality consists. Knowledge of types of inference will react favorably upon our practice, provided we apply the rules according to which logic, as an art or technique, teaches us to do our thinking. Assuredly, whatever progress man has made in life and letters, in the arts and sciences, is due to the application of his intellect, whose procedures it is the office of logic to help us understand.

DRILL

State briefly the distinction between: apprehension, comprehension, and inference; old and new knowledge; premises and conclusion; necessary and probable inference and fallacies; logic as a science and as an art. (See Exercises I, 1-16.)

2. NECESSARY INFERENCE

What is the test of a necessary inference? Not any single truth by itself, as stated in a premise or in a conclusion. For, as we have seen, we pass in an inference from one fact or statement to another. Rather must we therefore get our clue for the test we are seeking from the ways in which premises and conclusion may be related.

After Jephthah had vowed to sacrifice as a burnt offering whatever would come forth to meet him, and his own daughter came forth, he made the inference to the need of her sacrifice with great consternation; but he recognized what the given premises implied by way of conclusion. Later, after Saul had on a certain day cursed anyone eating food before evening, and his own son Jonathan,

ignorant of the vow, had tasted a little honey, the people prevented Saul from putting Jonathan to death; but failure to carry out the oath did not hinder anyone from acknowledging a relation of implication between the premises and the conclusion they implied. For, in contrast to an inference, which is a procedure, an implication is an objective relation between statements; and, whereas it is we who do or do not infer a conclusion, it is premises which do or do not imply a conclusion. Whether or not we are then justified in drawing an inference depends upon whether or not given premises genuinely imply a conclusion.

Let us therefore briefly state, and then examine, those relations between premises and conclusion which will help us in formulating a test for a necessary inference. On the one hand, (*a*) true premises cannot imply (or warrant) a false conclusion; but (*b*) true premises may imply a true conclusion; and (*c*) true premises may fail to imply a true conclusion. On the other hand, (*d*) false premises may imply a false conclusion; and (*e*) false premises may even imply a true conclusion.

Reflection on a simple example may suffice for the assurance that true premises cannot imply a false conclusion. Could anyone who says that genuine friends are likeminded, and that the ancient Greeks and Persians were not likeminded, consistently maintain that the ancient Greeks and Persians were genuine friends? In general, an inference from true premises to a false conclusion is invalid.

True premises can imply such conclusions only as are likewise true. Given the assumption that a body heavier than water will not float when put into water, and the assumption that iron is heavier than water, we must deduce that iron will not float in water; because the premises are so connected with the conclusion that, if they are true, it is

bound to be true also. Anyone who accepts the premises cannot rightly reject a conclusion drawn from them by way of a necessary inference. By the same token, if the conclusion of a necessary inference is false, a premise is false. In general, then, a valid inference from true premises leads to a true conclusion.

Yet true premises may fail to imply a true conclusion. It may be true that many mathematical treatises are difficult; that this is not a mathematical treatise; and that this book is not difficult. But the third of these statements does not necessarily follow from the other two; the premises setting forth the supposed evidence are not so connected with the conclusion in question that, if they be true, the latter is bound to be true also. Even on the premises stated, the conclusion may be false and would therefore have to be proved on other grounds. Accordingly, truth of premises and of conclusions—their agreement with facts—cannot in general guarantee the validity—the correctness—with which an inference as such is to be made. An inference, or passage from “if” to “then,” is valid only when our premises imply or warrant a statement drawn from them as part of their meaning.

On the other hand, false premises may yield a false conclusion. Were all Americans Christians, and Zoroastrians Americans, then Zoroastrians would be Christians. Though each of these statements be false, there is a necessary connection between the premises and the conclusion such that anyone who accepts as evidence the false premises cannot consistently reject as false the conclusion which would necessarily follow, even if the latter be in fact false. Moreover, it is important for inference that premises contrary to fact have logical consequences. That is why we can argue: “If there were no friction, no one could start an

automobile." "Did man not wear clothing, he could not survive in cold climates." "If lines a and b were not perpendicular to each other, the equal lines c and d would be unequal." Having committed ourselves to a false premise, we have thereby committed ourselves to its implications, which belong to its meaning. In general, then, the validity of an inference as such, the fact that given premises imply a conclusion, does not guarantee that we have achieved truth.

What is more: false premises may yield a true conclusion. That every triangle has three sides is a truth which would follow from the false premises that every square has three sides and that every triangle is a square. Given that no dog is carnivorous, as are lions, we could, would, and must conclude that no dog is a lion. Here again, if anyone accepts as true the premises given, he cannot then justify rejection of the implied conclusion. Admittedly, the conclusion, even if it be true, is not proved true by the false premises in which it is implied. But is it not important to know that a conclusion is not necessarily false or proved to be false when it happens to be supported by a weak or false argument? Hence also, in general, if the conclusion of a valid inference is true, the premises are not therefore necessarily true.

However, as previously noted, no true premises imply a false conclusion. A conviction that all students learn would of necessity hold also of logic students. If our premises are true and imply a given conclusion, that conclusion is necessarily true and cannot be false.

In any inference, therefore, we must distinguish, at the same time we consider together: What is true or false—i.e., correct or incorrect as a description of facts? And what is valid or invalid—i.e., correct or incorrect as an inference?

What then is the test of an implication or of a necessary inference? An inference is necessary if and only if it belongs to a type of inference whose conclusions are never false when the respective premises are true, or whose conclusions can never justifiably be supposed to be false when the respective premises are supposed to be true. Briefly, an implication exists when the premises cannot be true without the conclusion being true. Or, in other words, the test of an implication is the impossibility of the premise being true and the conclusion being at the same time false.

DRILL

State carefully the test of a necessary inference. Show, by various combinations of premises and conclusions, true or false, that no single truth of premise or conclusion can by itself guarantee a necessary inference. (See Exercises I, 17-30.)

3. INFERENCE AS FORMAL

The test of a necessary inference is closely connected with what we call the "formal" nature of logic. For, as we have seen, premises yield a conclusion whether they happen to be true or not; and we test a necessary inference by the *impossibility* of the premises being true and their consequences false.

Now what do we mean by "formal"? Take a "formal" wedding invitation. If we substitute lines or other symbols for those items which vary from one case to another, we no longer have an invitation, but the form, pattern, structure, or fixed order of an invitation. So, although inferences deal with an endless variety of subject matters, they also contain relations exhibited in their form.

Consider: "If minors are ineligible to vote, and Tom is a minor, then Tom is ineligible to vote." This reasoning meets the test of a necessary inference. Moreover, the validity of this reasoning is not affected if we replace Tom by Dick; Dick, by Harry; Harry, by X; minors, by aliens, or by Y; ineligible to vote, by ineligible for office, or by Z; or X, Y, and Z, by A, B, and C.

Consider also: "If minors are ineligible to vote, and Tom is ineligible to vote, then Tom is a minor." This argument does not meet the test of a necessary inference. Substitute "an alien" for "Tom," and it will become clearer that, though the premises be true, the conclusion would or could be false. The validity of an inference may thus be tested by its structure just as a check, to be valid, may have to conform to a pattern as exhibited in its blank form.

A form of inference filled with true premises and possibly false conclusions is thereby shown to be invalid. Inferences may be validated by forms filled with true premises such that conclusions cannot then be false. Compare and contrast the two skeleton forms here given.

Valid: If all Y is Z and all X is Y then all X is Z.

Invalid: If all Y is Z and all X is Z then all X is Y.

In the first of these diagrams, the premises imply the conclusion; not so in the second diagram. To be "implied in" is to be "folded up in," to be "entwined in." An inference to be valid must unfold what is folded up, untwine what is entwined, in the premises. But a detailed study of validating forms of inference must be reserved for a later stage in our study.

However, we may here ask: How do these formulas come to be constructed? Obviously, as in the case of formal invitations, by generalization from specific instances. Were there no marriages, there would be no formal wed-

ding invitations. Yet it is equally true that absence of forms would multiply confusions and misunderstandings.

So a logic which would illuminate actual reasoning must develop its formalism at the hand of content. Without the latter, the forms would be meaningless marks. Surely, it is not because X-is-Z follows from X-is-Y and Y-is-Z, that Tom's ineligibility to vote follows from his status as a minor. Hence we must keep in mind what our symbols are intended to symbolize. At the same time it is optional whether we employ the symbols X, Y, and Z, or A, B, and C, or S, M, and P; provided we do not fall into an inconsistency which promotes confusion.

Form, then, is in no case separate from content, but is the form "of" an invitation or of some other document or act; and, what interests us here, there are forms "of" inference. Hence truth and falsity are not matters of complete indifference to logic. For knowledge is concerned with the truth of our premises and conclusions. Apart from such truth, implication loses its significance. And logic is chiefly concerned with what premises imply.

But have we not shown that false premises have implications? Indeed. Yet it is to be noted that to validate our inferences from false premises we have to suppose (at least "for argument's sake") that those false premises are true and that they do imply a conclusion. We are dealing either with actual, or with supposedly actual, truths and implications. Our suppositions get their significance from the actual. So the laws and the forms and the symbols of logic get their significance from the concrete content which they generalize.

For this very reason, however, it would be inadvisable to try to do away in logic with forms and symbols. Shall we examine separately every possible specific inference? That

were an impossible task. To discuss at length Tom and voters and aliens and whatnot, would carry us far afield. What interests us as logicians about the objects of thought and speech is the part they play in inference. Our attention is centered on formal properties they have in common, as disclosed in structures, or patterns, or skeleton forms of logic. Inferences must then, in systematic study of them, be reduced to a limited number of types and be analyzed as to their forms.

DRILL

Show: that an argument may be tested by its form; that we obtain forms of inference by generalizing from specific instances; that actual or supposed truth enters into the testing of a valid inference; that formal logic illuminates and facilitates our actual inferences. (See Exercises I, 31-40.)

4. PROBABLE INFERENCE

Not all the evidence on which we reason is complete or conclusive; much of it is incomplete or partial. Hence not all our inferences are necessary; many of them are probable. And we may regard as probable an inference belonging to a type of inference which leads to true conclusions more often than not.

Among important kinds of probable inference are generalizations and presumptions of fact. Thus, one who has observed numerous crows, none other than black, may generalize with a high degree of probability: "All crows are black." Again, I may say, by way of presumption of fact: "The train on which I am travelling will probably arrive at my destination on time, for it usually does." Such arguments, though not conclusive, have partial evidence in their favor.

So it is that an inference which is fallacious when taken as a necessary inference, may have validity as a probable inference when based on something that takes place, not "always" indeed, but at least "usually" or "for the most part"; e.g., "As we become permanent drunkards by so many separate drinks, so we become saints in the moral, and authorities and experts in the practical and scientific spheres, by so many separate acts and hours of work."²

Then, too, where evidence is incomplete, additional findings may increase or decrease the probability of an inference. As a case in point, the more books of an author I find to my liking, the stronger the likelihood that I shall welcome his next literary production. Thus probability, being relative to the evidence, may vary in degree, as partial evidence may itself have varying degrees of relevance to the truth to be established.

However, despite the differences between them, necessary inference may enter into the determination of probable inference. For I am never justified in rejecting a conclusion which necessarily follows from true premises. Also, if the table on which I am writing has satisfactorily supported my papers, I am not usually justified in refusing to conclude that it will probably continue to do so for some time to come.

Yet the interrelation of necessary and probable inference is best illustrated from the application of fundamental theorems in the calculus of probability. We shall here consider, for this purpose, a few of the simplest formulas only.

If a coin can fall head or tail only, and if the two faces are equiprobable, the probability that a coin will fall head is $1/2$. In general, the probability of a favorable event is a fraction whose numerator is the number of possible favorable events and whose denominator is the total number of

² William James *The Principles of Psychology* (New York: Henry Holt & Co. 1890) I.127.

possible events favorable and unfavorable— f/t , or $f/(f+u)$ —provided that all the possible events are equiprobable. This proviso would be disregarded if we argued that, in throwing a die, we may get either a six or other than six, so that the probability of getting a six is $1/2$. Not so. For, the total number of possible alternatives being six, the probability of getting a six, with the proviso stated, is $1/6$. Similarly, the probability of drawing a spade from a shuffled pack of fifty-two cards is $13/52$ or $1/4$; and the probability of the complementary drawing (of a heart or a diamond or a club) is $39/52$ or $3/4$.

In tossing a coin twice, we may get: HH, or HT, or TT, or TH. Hence the probability of getting a head twice is $1/4$, or $1/2 \times 1/2$. In general, the probability of a joint occurrence of two independent events a and b may be expressed in the formula: $P(ab) = P(a) \times P(b)$. Here too, all the possible alternatives must be enumerated. May we legitimately argue that, in trying to get at least one head in two throws, one of the alternatives is getting a head on the first throw? On this basis (H, TT, TH), the probability would be $2/3$. But the first alternative includes two that are equiprobable with the others: HT, HH. Accordingly, the probability of getting at least one head is $3/4$.

The formula for the probability of a joint occurrence may sometimes be used when the events are not independent. Let a bag contain four white and three black balls. What is the probability of drawing two white balls in succession if the white ball drawn first is not put back into the bag? The probability of getting the first white ball is $4/7$; but, since after the first drawing there remain six balls of which three are white, the probability of getting a white ball a second time is $3/6$. Therefore the probability of getting two white balls in succession is $4/7 \times 3/6 = 2/7$.

What is the probability of getting either two heads or two tails in tossing a coin twice? Of the four possible events—HH, or HT, or TT, or TH—two are favorable. The probability in question is the sum of the probability of two heads, or $1/4$, and of two tails, or $1/4$, and hence is $1/2$. In general, the probability of either of two disjunctive events is the sum of the probabilities of each: $P(a + b) = P(a) + P(b)$.

In order to calculate the probability of the disjunction of two occurrences which are not exhaustive, we proceed by an indirect method. Take the probability of getting six on either the first or the second throw of a die, where six may occur on both throws. The complementary probability (getting six on neither throw) is $5/6 \times 5/6 = 25/36$. Hence the probability we are seeking is $1 - 25/36 = 11/36$.

Like the forms of inference considered in the previous section, so the basic formulas in the highly developed calculus of probability are generalizations from specific cases. The calculus explores the necessary consequences of our premises concerning matters of fact. Further application of its formulas is subject to such assumptions as the equiprobability and independence of possible events. The more extensive and the more highly organized our knowledge, the more reliable the inferences which a systematic science supports. From the simple examples given, however, it is clear enough that necessary inference enters into the determination of probable inference.

DRILL

Distinguish: the tests of necessary and of probable inference; generalization and the presumption of fact; degrees of probability; the probability of a favorable event, of a joint occurrence, of disjunctive events. (See Exercises I, 41-55.)

Some Common Fallacies

5. FALLACIES OF AMBIGUITY

To discriminate the genuine from the spurious is not always easy, especially for the inexpert; be it a question of physical vigor and beauty, of gold and silver, or of reasoning and refutation. In particular, words, like counters, are readily manipulated so as to mislead the unwary: words are symbols used instead of the things for which they stand; and care is required in such use if we are not to be tricked, by ourselves or by others, into mis-reasoning.¹

A fallacy, in the strict sense, is an invalid inference which resembles a valid one. Yet fallacies hardly lend themselves to strict classification. Not only is their number legion, but one frequently involves another. A student does well, therefore, by considering in each case the reason why he selects a particular label for a given fallacy.

Such mistakes in reasoning as turn upon a double meaning in language may be called fallacies of ambiguity. Not that every ambiguity is necessarily fallacious; it is that when, because of the ambiguity, we commit an error in reasoning.

In particular, the fallacy of equivocation may be traced to the use of the same term in two distinct senses;

¹ Aristotle *On Sophistic Refutations*, chap. i.

the fallacy of amphibology (or amphiboly), to an ambiguous grammatical structure of a sentence. It would be an equivocation to argue that "partisans" are not worthy of a public trust and that all Republicans and Democrats are "partisans"; an amphibology, to suppose that newly-born infants can speak because: "Job cursed the day he was born." Often indeed it is difficult to determine an intended meaning. Should the term "general welfare" in the Preamble of the Constitution of the United States, be interpreted strictly or broadly? When we speak of someone's "first important poem," do we have reference to a poem written after others of no importance or to the first he ever wrote?

Again, one who reasons as if properties of elements or individuals are always properties of the whole they constitute, commits the fallacy of composition. Not under all circumstances will the best soprano, alto, tenor, and bass form the best quartet; for what is true of each and any taken separately is not therefore necessarily true of the whole taken together. The converse error of reasoning is the fallacy of division. Does the bravery of a regiment render unquestionable the bravery of an individual soldier in the regiment; or the wisdom of a law, its freedom from unwise provisions; or the existence of ninety-odd elements, the occurrence of all of them in living creatures? What is true of a whole taken together is not therefore necessarily true of the parts taken separately.

The fallacy of accent consists of an ambiguity arising from a misplaced accent or emphasis. Different interpretations result according as we read in the Sermon on the Mount: "Seek, and ye shall *find*," or "*Seek*, and ye shall find."² To observe differences in meaning when stress is laid upon one or another portion of discourse affords a

² Matt. 7:7.

profitable logical exercise. Often strong prejudices have their roots in a fallacy of selective emphasis. Allied to the fallacy of accent is the misquoting of an author's or speaker's words by separating them unfairly from their context or by italicizing them unfairly. How little Francis Bacon's *Advancement of Learning* has in common with psychological "laws of learning" in current parlance! Faulty accentuation or use of verbal forms underlies the fallacy of figure of speech. Thus John Stuart Mill argued:

"The only proof capable of being given that an object is visible, is that people actually see it. The only proof that a sound is audible, is that people hear it; and so of the other sources of our experience. In like manner, I apprehend, the sole evidence it is possible to produce that anything is desirable, is that people desire it."^a

Is what I desire always worth desiring?

Failure, in applying a general or abstract rule to a specific concrete case, to take into account complicating circumstances and counterbalancing considerations constitutes the fallacy of accident: an inference from a statement without qualifications to a statement subject to some special condition (*a dicto simpliciter ad dictum secundum quid*). Because men of good will desire peace, they do not therefore necessarily in war refrain from violence. The same error in reverse bears the name of the converse fallacy of accident (*a dicto secundum quid ad dictum simpliciter*). Abuse of property rights does not establish their alleged or inherent incompatibility with human rights. Akin to this sort of confusion is invalid reasoning from one special case to another. As a common proverb reminds us: "Circumstances alter cases." To draw the moral of these considerations: Look well to your speech, but look also to concrete situations!

^a John Stuart Mill *Utilitarianism, on Liberty, and Representative Government* (Everyman's Library; New York: E. P. Dutton & Co., Inc. 1910), chap. iv.

DRILL

List, explain, and illustrate common fallacies of ambiguity. (See Exercises II, 1-35.)

6. FALLACIES OF BEGGING THE QUESTION

As distinct from fallacies due to some ambiguity of speech, there are false claims or illusions of proof, among them, various ways of begging the question (*petitio principii*). Anyone who assumes proved what he sets out to prove may be said to beg the question.

A typical form which this fallacy takes is arguing in a circle (*circulus in probando*), as in the contention that a student reading the classics wastes his time instead of fitting himself for the serious duties of life, since he wastes his time on the classics. Can we prove that the Koran is to be followed because Mohammed is to be followed, if the latter pretension depends in turn upon the former; or does not this amount to an attempt to prove a proposition by itself?

* K—F

M—F

(M—K)

(K—M)

∴ M—F

*. ∴ K—F

Names used as if they proved the truth of what they suggest are question-begging epithets. Usually they express praise or blame without evidence that they have been deserved. "Protective" tariffs, we are given to understand, "protect" home industry. Ours is "the party of the people." Candidates for political office have promised us a "full dinner pail." Before this or that leader came upon the scene, no one in government had thought of the "common welfare." How many acquiesce in confident assertions that "pagans are materialistic"; that "orthodoxy is my doxy";

that fire from heaven ought to strike one that followeth not us!

When stated in interrogative form, question-begging goes by the name of the false question or of many questions. Here the question-begger smuggles into a question a false or dubious proposition by means of which he seeks to establish some other proposition. Or he rolls two or more questions into one as if they had a single answer. After a visiting teacher, keeping his hat on, had spoken to a class, the regular teacher asked the suggestive question: "In what hand did he hold his hat?" Seventeen answered, "right"; seven, "left"; only three responded correctly.

An argument which, instead of addressing itself to the merits of the issue in hand (*ad rem*), bases its appeal upon prejudice (pre-judgment), exemplifies special pleading. What partisan does not plead, consciously or unconsciously, sincerely or otherwise, on behalf of his own interest (*pro domo*)? On the other hand, popular psychoanalysis denies the logical force of an argument, thus seeming to prove the opposite, by an unfavorable account of the personal motives which may (or may not) have led some spokesman to the view he holds. Such an account would, at best, have some bearing on the credibility of a witness testifying to what he has himself observed. Since when does alleged overstatement of a point logically justify rebuttal by understatement of it? Despite the popular game of debunking, there are people genuinely interested in good causes and in sound arguments.

The special pleader may rest his case on an appeal to someone's character or position (*argumentum ad hominem*); to popular feeling (*ad populum*); to authority (*ad verecundiam*); to force (*ad baculum*); to sympathy (*ad misericordiam*); to impossibility of disproof (*ad ignoran-*

tiam); and the like. However, the numerous forms of special pleading may also be interpreted as examples of an irrelevant conclusion.

DRILL

List, explain, and illustrate common ways of begging the question. (See Exercises II, 36-66.)

7. FALLACIES OF IRRELEVANCE

Every fallacious inference contains a factor in some sense irrelevant. The fallacy of irrelevant conclusion, popularly known as arguing to the wrong point, consists in trying to prove something in a manner which suggests that something else is proved. The technical term, *ignoratio elenchi* or ignorance of the refutation, has special reference to a conclusion which does not genuinely contradict an opponent's position. Take a claim to the right of private judgment: insistence that private judgment is not always right cannot refute a claim which has made no such pretension. Nor may a dissenter properly impugn the value of a tradition because of the lapses of some who have upheld that tradition. So the principle of judicial review cannot validly be overthrown by the true but irrelevant observation that judges are fallible. And what does the circumstance that the framers of the Constitution of the United States were propertied men prove about the merits of that political instrument? Also a typical case of an irrelevant conclusion is the fallacy of objections, in which a critic of a proposal states its disadvantages, but fails to compare them with its advantages.

Again, premises may be so loosely connected with a conclusion as to warrant the charge of irrelevant premises, customarily called the fallacy of the consequent (or *non*

sequitur). Thus Machiavelli wrote in *The Prince*: "The chief foundation of all states, new as well as old, are good laws. As there cannot be good laws where the State is not well armed, it follows that where they are well armed they have good laws." Similarly, someone has said that since a rule restricting the employment of teachers to members of particular societies or unions would be void, "it follows logically" that a rule restricting employment to nonmembers is also void. In neither of these examples does the conclusion follow.

Then too fallacies of irrelevance include, besides sophistical refutations, abuses of logic and of scientific method. Originally designating a savant, the word "sophist" has come to mean one who makes the worse appear the better cause. It is sophistical in the latter sense to oppose a valid argument by misrepresenting, confusing, depreciating, or exaggerating its claims.

An abuse of logic occurs especially in the pretense to logical rigor and in unreasonable demands for definitions and proofs. Hence we must cultivate an insight into the scope of the principles of logic, along with right habits in their use.

Similar cautions hold for just estimates of scientific procedure. If bodies are masses, they are not therefore "nothing but" masses, any more than sentences are "nothing but" words. Either of these misconceptions would exemplify a fallacy of reduction. Science, properly understood, does not identify objects with their elements, but analyzes its objects into elements related in ascertainable ways. By the same token, the totality of things cannot, without gross oversimplification, be reduced to nothing but matter or to nothing but ideas, just as day and night do not cease to differ because together they constitute a "day."

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Many people, moreover, tend to conceive alternatives in too simple a fashion, as if a hypothesis which cannot be absolutely proved is therefore without value, or as if people are "either" wholly attractive "or" wholly unattractive rather than "both" attractive in some ways "and" unattractive in others.

Among other widespread varieties of pseudosimplicity may be reckoned the confusion of origin with value or of temporal with logical order. Consider as an illustration of the first indiscrimination the question: "Can there any good thing come out of Nazareth?" The second distortion underlies excessive reliance on the law of development from the simple to the complex—examples to the contrary notwithstanding—or the failure to realize that the order in time according to which our knowledge has grown differs from the ways in which the items in our knowledge may be arranged in a system.

DRILL

List, explain, and illustrate common fallacies of irrelevance. (See Exercises II, 67-90.)

8. FALLACIES OF NEGLECTED ASPECT

Reasoning which seems relevant as far as it goes may nevertheless contain a mistake due to neglect of some aspect. So we may heedlessly take necessary conditions for sufficient conditions. Whatever, as a case in point, a combination of labor and management might do, it would not guarantee economic stability. And bodily or social ills are necessary, but not in themselves sufficient, to prove that some given reform or remedy is desirable. In particular, temporal succession does not suffice to prove causal connection. Yet, like Chanticleer, who attributed the sun's rising

to his own crowing, men have blamed economic depressions on preceding bank failures, public calamities on meteorological phenomena, etc. Those who offer superficial explanations along such lines commit the fallacy of false cause, or of saying, "after this therefore because of this" (*post hoc ergo propter hoc*)—as if deaths occurring after medical treatment were therefore necessarily caused by the treatment.

Mankind suffers also from hasty generalization, a fallacy rooted in neglect of negative instances. To suppose that our ancestors built more strongly than we do, is to overlook the crumbling away of their more fragile structures. Inference from limited observation that redheaded people have a bad temper ignores redheads not irritable, as well as non-redheads who do or who do not have an ill-humored disposition. Further illustrations, like race prejudice, will readily suggest themselves to the reader.

It is also appropriate in this context to take notice of our reasonings from one case to another by resemblance or analogy. The term "analogy" originally designated a mathematical proportion ($2:4::6:12$). But the meaning of the term has been extended to similarity in any kind of relationship between two sets of terms and to similarity generally. Analogies in this wider sense have been condensed in such expressions as a "sweet" smile, the "wings" of butterflies, "airships," "*pomme de terre*." In the history of science, analogies have been a fruitful source of hypotheses: witness Newton's law of gravitation (suggested, according to a well-known anecdote, by the fall of an apple), and Darwin's theory of natural selection (suggested by the artificial selection practiced by breeders).

It is not uncommon to argue that if a certain serum has cured monkeys of some disease, the same serum will

probably cure men of the same disease. But careful experimenters refrain from acting upon such reasonings until they have applied every available test. On what does the reliability of an analogy depend? Clearly, on the weight of the similarities between the supposedly analogous cases, in contrast to the weight of the dissimilarities. But to weigh adequately the relevant likenesses and differences requires no little experience and knowledge. It is a far cry from Plato's analogy between watchdogs discriminating friend from foe and rulers discerning what is good or bad for the State, to Darwin's analogy between artificial and natural selection.

Neglect of dissimilarities leads to false analogies. Because in some sciences the latest information has the highest value, it does not follow that past writers in other fields may not have much more to teach us than current best sellers. Though electrons revolve around the nucleus of an atom much as planets revolve around the sun, and though the distance of an electron from its nucleus, relatively to the size of the former, is comparable to the distance of the earth from the sun, the path of an electron about the nucleus is not therefore a special case of the law governing the motions of the planets. Obviously, false analogies have much kinship with hasty generalizations.

Moreover, according to an ancient proverb, examples or comparisons illustrate but do not prove. Even an illustration false in itself, like a fable, may have considerable value in making clear some truth we wish to express. But a comparison must not extend beyond the point of comparison. It is only in certain particulars that the godly man "shall be like a tree planted by the rivers of water, that bringeth forth his fruit in his season"; and the ungodly,

"like the chaff which the wind driveth away." No analogy can be made to go on all fours.

DRILL

List, explain, and illustrate common fallacies of neglected aspect. (See Exercises II, 91-120.)

9. FALSE PREMISES

Does acceptance of false premises have a place in a catalogue of fallacies? Not if we define the latter as mistakes in reasoning from "given" premises to a conclusion. Yet, as we have seen (sec. 2), one who bases his reasoning on false premises fails to "prove" the conclusion he infers. This failure we may, if we so desire, call a fallacy in an extended meaning of the term.

Perhaps a logician does not go beyond his province by laying down the principle that false claims to knowledge may be pointed out when that is relevant. Apparently, not enough time is spent by popularizers generally in trying to discover whether the ideas they present are true. To refute falsity in affirmation or denial, one must establish the truth or greater probability of some other position. A specialist who asserted that ethical philosophers separate ethics from such sciences as biology and psychology plainly ignored the first systematic ethical treatise in our tradition. Sometimes indeed errors have a limited relevance. When Thomas Aquinas said that the heavenly bodies change only in position, his mistake did not invalidate his account of things accessible to sense perception as composed of matter and form. At other times, uncritical adoption of false premises has far-reaching disastrous effects. Supposing that life has no meaning when interpreted in its own terms, many have fallen victims of misleading "authorities." So too has preva-

lent indifference to truth opened the floodgates to shrieking demagogues and fanatics and even to outspoken faith in violence.

It cannot fall within the province of logic, however, to guarantee the truth of a premise or conclusion taken by itself. Logic, strictly regarded, confines its critical examination to the comparative truth of premises and conclusions. On what conditions are we entitled to say that when one proposition is true, another is true also? This is the question at the heart of logic, at least on its formal side. As the study of types of inference, logic deals with relations in truth-value among affirmations and denials. Logic gives an account of the ways in which the truth or falsity of one statement may determine the truth or falsity of another.

"It guarantees the truth—not of any proposition that is consistent with the premises—but only of the proposition whose contradictory is inconsistent with the premises. And even this statement goes too far; for logic does not allow any arbitrarily chosen premises to be taken as true; and thus the only conclusions that it can be said in any sense to guarantee are those which have been correctly inferred from premises that are themselves true."⁴

Formal fallacies in the relating of one statement to another remain to be treated as we go along in our exposition. Most of them may be classified under fallacies of inadequacy; i.e., of neglected aspect. Meanwhile, an introductory account of the subject matter of logic must include a consideration of discourse. For, in analyzing types of inference, a student of logic must be conscious of the factor of language.

DRILL

Review the main types of inference, including common fallacies. (See Exercises II, 121-130; 131-163.)

⁴ W. E. Johnson *Logic* (Cambridge: The University Press 1921) I.222.

Discourse

10. ORDERED SPEECH

Most of us are familiar with the stock example of a syllogism: "All men are mortal; Socrates is a man; therefore Socrates is mortal." Let us treat it here as a convenient peg on which to hang some general comments. Does logic deal with words, or thoughts, or objects? What main problems, moreover, does logic try to solve?

That thinking and the science of it have much to do with language and linguistics there can be no doubt. Broadly considered, our utterances develop through the stages of expression, direction, and communication. Whether in soliloquy or in the presence of hearers, we give expression to things and to our own reactions: "What a piece of work is man!" We also make requests and frame questions of various sorts: "Please keep off the grass!" "What did he say?" "Did he say that?" And we give information: "This little boy picked up a green apple and immediately ate it."

The grammarian tells us about words as general expressions, their formation (pick-ed, immediate-ly, ate), and their combination. He distinguishes parts of speech: substantives (boy, apple), adjectives (little, green), pronouns (this, it), verbs (picked, ate), and particles (up, and, immediately). He shows how words, in a junction (the very little boy) and in a nexus (the boy was very little), fall

into ranks: primary (boy), secondary (little), and tertiary (very). In his analyses, the grammarian employs such categories as number, case, tense, mood, voice, person, gender. He deals with the general, whereas the lexicographer deals with the special, facts of language.¹

To the extent that grammar becomes philosophical, there is a close connection between it and logic. Nevertheless grammar concerns itself primarily with questions of linguistic usage which, to the logician, are incidental. What verbal symbols mean on a common-sense level depends upon group habits and expectations. "Scientific" language, in contrast, forms a "system" in which each meaning is determined in relation to other meanings and in which the symbols are chosen and used so as to indicate the relation. As a consequence, the logician is enabled to consider speech as ordered by rules of inference. He formulates rules of inference by which the development of discourse may be guided to a certain purpose—namely, the drawing of conclusions from premises and the ordering of knowledge. Such rules, then, contribute to making our ordinary use of language more determined and precise; and they aid us in attaining the goals of our inquiries.

Although words or symbols are necessary for logic, reasoning is not therefore "nothing but" an affair of language. For the names of things vary from one vernacular to another and even within the same vernacular. In order to reason correctly, we must use our linguistic medium consistently and with faithfulness to the order and connection of things. Besides, a systematic study of words itself depends for its intelligibility upon logical discrimination.

¹ O. Jespersen *The Philosophy of Grammar* (New York: Henry Holt & Co. 1924), pp. 41, 92, 96, 53, 34.

DRILL

Point out the relation of logic to discourse and to linguistics. (See Exercises III, 1-12.)

11. LAWS OF THOUGHT

In one traditional definition, logic is the science of the laws of thought. By a "law of thought" is not meant a human decree which prescribes or proscribes a course of action and which may from time to time be altered. A formula stating a principle of reasoning describes a uniformity or agreement in the ways in which all persons think when they think. Like the law of gravitation in physical astronomy, or of equivalent proportions in chemistry, so a canon in logic has the status of a natural law.

"Things identical with the same third thing are identical with each other." This is a law of thought in the sense that all people think in accordance with it and that they do so regardless of the subject matter about which they think. No matter who the thinker, he would reason that if Scott wrote *Waverley*, and the author of *Waverley* was the author of *Marmion*, then Scott wrote *Marmion*. Whatever X, Y, and Z stand for, if X is Y, and Y is Z, then X is Z.

It is in the same sense of a natural law that we phrase the formula: "Part of a part is part of the whole." Geniuses, being totally included in men, who in turn are totally included in mortals, are themselves totally included in mortals. From the fact that iron is one of the metals, all of which are elements, we necessarily conclude that iron is one of the elements. If all X's are Y's, and all Y's are Z's, then all X's are Z's.

Admittedly, men do not always reason validly. But this may mean that at some point they fail in the exercise

of their mental powers; or that they may be doing something else, perhaps "emoting," instead of thinking. *Omnis qui fallitur, id in quo fallitur, non intelligit.*² Because man has gained knowledge, he may analyze his thought processes as psychological events, in order to determine the factors which have gone into their actual occurrence and with which they may have become entangled. Psychology studies the activities of the individual organism on a scale intermediate between that of physiology and sociology. Logic examines reflective practices, with emphasis on the methods by which they result in knowledge and on the relations of truth-value in the results. To regard logic as a branch of psychology because thought is an intellectual process would be like viewing logic as a branch of biology because thought is a vital performance, or like identifying transportation and all the branches of technology with psychology because they require intelligence.

The major interests of logic differ also from those of rhetoric, the art of persuasion. A speech has persuasive power to the extent that an audience has confidence in the character of the speaker, has been brought into an emotional state favorable to his objective, and considers his reasoning sound. Since the rhetorician aims to produce conviction, he abbreviates his arguments and adapts them to the capacities of his audience and to the exigencies of the occasion; he chooses words which through association have strong emotional influences; he seeks to establish and maintain rapport with his hearers. But the feeling of certainty, or unquestioning assurance, does not constitute demonstrative evidence for logical certainty or even demonstrable evidence for logical probability.

Like grammar, so psychology and rhetoric co-operate with logic at certain points. But words and thoughts alike

² Augustine, quoted in A. Gratry *Logic*, tr. H. and M. Singer (La Salle, Ill.; The Open Court Publishing Co. 1944), p. 138.

concern the logician from the standpoint of their role in inference and in systematic discourse. Most of our words and thoughts refer directly not to words and thoughts, but to objects and their relations in the natural and social world. Hence we cannot reduce all branches of knowledge to linguistics or to psychology, which must themselves be branches of knowledge, and which therefore, like every other special science, can establish their findings only by conformity to the rules of logical inference.

DRILL

Show: the relation of logic to thought; to psychology; to rhetoric. (See Exercises III, 13-20.)

12. THE UNIVERSE OF DISCOURSE

Careful discrimination discloses that what we call laws of thought have reference to things. It is things equal to the same third thing that are said to be equal to each other. Such equality does not depend upon a fiat of ours to speak or to think in a certain way. Logical relations obtain among entities recognized in any art or science and even among entities said not to exist. Were objects not interconnected in their own right, how could speech or thought have reference to them or sustain ordered relations in the articulation of them?

Inasmuch as we can set no antecedent limits to what we can think or speak about, the universe of things and processes is at the same time an intelligible universe and a universe of discourse. It is one universe; just as Socrates—who drank the hemlock, about whom we think, and who gets into a syllogism about himself—is one, not three. Objects, thoughts, and words, with all the differences between

them, do not fall into separate parts of logic any more than do spoken and written words.

By the universe of discourse is sometimes meant some context or subject matter in terms of which a hearer or reader is expected to supply the restrictions intended by a speaker or writer. Thus, there was a time when vegetables could be said to be motionless and animals alone endowed with the power of locomotion; but we can now see, with the aid of a microscope, minute plants more active than minute animals. So it is a physical whole which may be said to be greater than any of its parts; the principle would not apply to an infinite series of numbers in relation to a part such as the infinite series of odd or of even numbers. So too Americans may be said to function under a two-party system; viz., under certain spatial and temporal conditions and with other supposedly well-known qualifications. A limited universe of discourse, taken in this sense, has the relation to the total universe of a part to the whole.

However, to speak as if the universe of discourse were, along with the world of the imagination and other worlds, outside "the" universe is at best misleading. The statement that "the gods on Mount Olympus drink nectar and ambrosia" is distinct from the statement that "some people have believed in Olympian gods drinking nectar and ambrosia." Truth or falsity of such propositions may be determined in ways not unlike those in which we seek to find out, respectively, whether it is true or false that "our neighbor drinks goat's milk" or that "a mutual friend has declared that our neighbor drinks goat's milk." In order to talk intelligently about things in doubt or in dispute, it is not necessary to invent separate world systems for them. How many separate universes would we require in which to put

events, narratives, and narrators; or chemical elements, motion, and discourse?

Similarly, to distinguish objects and relations is not tantamount to separating them. If one body is equal to, greater than, or less than another, then the relations of equality, greater than, or less than hold just as objectively as the bodies exist between which they hold. Of course, qualities and relations are not bodies or things additional to the bodies or things they qualify or connect.

Moreover, particulars, like "Socrates," differ in many ways from universals, like "man" or "mortality." But there can be no logic without the one or without the other. No less than plant and animal, fish and bird, does man interact with a distinctively qualified environment. But because man can think, he can respond not only to particulars: this family, this government, this school, this church; but also to universals: the family, the government, the school, the church. Man responds not only to immediate excitations, like noise, but also to possibilities, as those contained in the statesman's plans. No less one-sided than love of humanity and hatred of people would be the love of people and hatred of humanity.

Have universals then the status of names, concepts, or realities? According as they answered this question, medieval and other thinkers have been classed respectively as nominalists, conceptualists, or realists. When we believe in democracy, oppose totalitarianism, make sacrifices for our country, devote ourselves to education, do our duty, are we responding to words, to ideas, or to something real (not physically, yet somehow real)? A complete answer, were one possible, would carry us too far afield. Perhaps it would be wise to combine at least some of the advantages of the different points of view about universals.

At any rate, the logician need not divorce words, thoughts, and things. Between his science and every other, there is a two-way connection. That is why we speak of cosmology, biology, sociology, and the rest.

DRILL

Discuss what is meant: by the universe of discourse; by a universal; by a two-way connection between logic and other sciences. (See Exercises III, 21-34.)

13. PROBLEMS OF LOGIC

Like taking three steps towards a goal, so syllogistic reasoning from premises to a conclusion forms a single and undivided act. To know or to suppose the truth that Socrates is a man, as dependent upon the known or supposed truth that all men are mortal, is at the same time to know or to suppose the truth that Socrates is mortal. The premises are brought together in the conclusion. Yet the syllogistic reasoning is not therefore, as an act, simple and indivisible; on the contrary, it is complex and may be divided into three distinct acts in an orderly sequence. Each of these is a judgment, expressed in a proposition—i.e., in an affirmation or in a denial.

To affirm or deny is, again, a single and undivided act, like that of taking a step. As a movement between one point and another, it is in strictness also simple and indivisible. The proposition is a unit for logic, never wholly isolated from other propositions, but not itself made up of three separable parts (A, is, B).

As taking a step requires lifting the foot, so affirming or denying presupposes that about which something else is affirmed or denied and that which is affirmed or denied about it. In thinking "Socrates," "men," "mortal," we ap-

prehend the objects of our thought and comprehend the meanings of the terms referring to the objects. Apprehension, an act single and undivided as well as simple and indivisible (though it may have a complex object), thus constitutes our first intellectual operation. We express no truth or falsity, however, until we combine one term with another in significant affirmation or denial. When someone says, "Socrates," we look for a completion of the utterance.

At least three problems have here been posed for logic: clarifying our terms; affirming and denying; and supporting our conclusions. But terms and propositions will occupy our attention chiefly as factors in the reasoning process.

Other problems of logic are often grouped under the general heading of method or methodology. The syllogism, as a simple kind of demonstration, leads naturally to a more comprehensive consideration of the latter. Demonstration in turn raises questions about the nature of proof in a wider sense. What warrant do we have, e.g., for the premises that "all men are mortal" and that "Socrates is a man"? Then there is the general issue of what the varied sorts of inquiry may have in common and wherein they may differ.

As we have begun our exposition of logic with a broad consideration of inference and discourse, so we shall conclude our exposition with some suggestions logic may offer about thinking in realms which extend beyond the requirements of proof and in which we are restricted to the discrimination of opinions as better or worse, informed or ill-informed.

The order of topics in our treatment is influenced in large part by factors of pedagogical convenience.

In sum, the controlling questions here taken up include: How may we distinguish the main types of inference; clarify our terms; relate our affirmations and denials; sup-

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port our conclusions; solve our problems; verify our hypotheses; demonstrate our theorems; and reflectively weigh our values?

DRILL

Outline the principal problems of logic. (See Exercises III, 35-45.)

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Clarifying Our Terms

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Terms

14. THE AMBIGUITY OF WORDS

A child is ever asking "What is this?" His demand may not be for information so much as for a name, for words in which to give voice to what he perceives, for an increasingly effective vocabulary. Soon he discovers one of the chief imperfections of language: its ambiguities. Comparatively few words have a single clear signification. And the uncertainty and variety in the import of our expressions leads to fallacious reasoning when one meaning is confounded with another.

Sufficiently clear indeed are the names of some individual persons, objects, buildings, or events: William Shakespeare, Schubert's *Unfinished Symphony*, the Cathedral of Learning, the World War of 1914 to 1918. Definite enough are also such common names as anticlimax, exploration, parasol, yokel. Often we invent technical terms or formulate technical definitions of ordinary words, especially in the sciences, in order to keep our meanings precise and dependable: mass, work, power, efficiency, oxygen, valence, protoplasm, chromosome, mammal, neurosis. In general, things are said to be named "univocally" when they have in common not only the same name, but also, in explanation of that name, the same statement of what they are. Thus, if we ask in what sense of the word both man

and beast are said to be "animal," the answer would be the same for both.

On the other hand, things are said to be named "equivocally" when the name they share calls for different definitions in its different applications. By the word "church" are designated certain buildings or people or organizations or acts devoted to religion, and even different sorts of each of these in the usages of different churchmen themselves. Among words equivocal in sound we may note: to, too, and two; leaf and lief; pain and pane; passed and past; some and sum; son and sun. Words equivocal in spelling include: tear (a drop) and tear (pronounced tare); bow (from *būggan*) and bow (from *boga*); o'verthrow and overthrow'. The numerous class of words equivocal in both sound and spelling has been subdivided into three groups according as the equivocation arises: (1) from accidental confusion of entirely different words (e.g., "sound"); (2) from change of meaning by habitual association of ideas (e.g., "church"); and (3) from logical transfer of meaning to analogous objects.

Closer examination of the last of these causes reveals two opposite processes in the growth of language: generalization, by which a name comes to be applied to a wider set of objects; and specialization, by which a name comes to be restricted in its application to a smaller range of objects. Whereas "paper" originally referred to the papyrus used in the Roman empire, it was next employed to designate the writing material of cotton or linen rags; now it symbolizes as well the product of chemically treated wood pulp. The word "number," once embracing integers only, gradually included fractions, irrationals, transcendentals, and determinants. Contrariwise, a "publication," or a communication to others, has since the invention of printing

come to be restricted (except in legal parlance) to printed communications. The name "surgeon," originally referring to all who worked with their hands, has long been restricted to those with specialized medical training. The term "statistics," first applied to collections of data concerning matters of importance to the state, has been both narrowed to numerical data and broadened beyond the state to cover any social group or any other branch of knowledge or investigation.

Like other "conventional" features of life, a vernacular presents much that is arbitrary, but also affords considerable opportunity for fascinating study of historical developments. Characteristic doublets in the English language, as in the expression "ways and means," are accounted for in part by the onetime official use of both English and French. The multitude of words derived from Greek or Latin roots, and nearly synonymous with other words of Saxon or French origin, has made possible extensive substitution of one word for another, which easily creates the illusion of advance in knowledge. Have we not been told that our "consciousness" is "nothing but awareness"?

A fertile source of change in the meaning of words arises when their application is widened by what is called a "metaphorical extension" of their meaning from an ordinary to a peculiar purpose. It is interesting to trace the similarities condensed in such words as "governor," which originally meant a steersman; "spirit," which meant breath; "palaver," from "parable"; etc. A "metaphor" is itself a "carrying over." What one thing is, is carried over by means of something else, especially by means of speech, and from one word to another. And what words convey—that is what we call their "meaning" or an "idea." If a word is then to bear a definite meaning, we must remove its ambi-

guities and in some way indicate how we propose to regard the word.

DRILL

Differentiate the "univocal" and the "equivocal" use of words; the various sources of ambiguity. (See Exercises IV, 1-15.)

15. KINDS OF TERMS

There are words of which we may say little beyond describing their role as words: "of" is a preposition, etc. Inasmuch as such words must be combined with others to form parts of names or terms (as in "the Father *of* his Country"), they are called syncategorematic (Greek: *syn*, with; *kategorein*, to predicate). Categorematic words may stand alone as names or terms with a reference beyond themselves (father, country, etc.). In the interest of facilitating an understanding of terms we may, moreover, assign them to appropriate kinds.

Popularly, the familiar is lauded as "concrete"; the unfamiliar, disparaged as "abstract." Careful thinkers, however, work with more subtle distinctions. Let us take for our guiding clue the observation that certain terms point in the direction of what we perceive by means of the sense organs, whereas other terms direct us to that which we conceive intellectually. We may then safely call "concrete" any names directly applied to "things" and their kinds: the Mississippi River, tree, house, ax; this, that, now, then, here, there; red, loud, sweet, solid. "Abstract" words would then stand for qualities, attributes, or circumstances of things, apart from direct application to the things themselves: solidity, redness, widowhood, gratitude, equality, presence, absence, location.

Whether, in specific instances, we do or do not possess pairs of corresponding concrete and abstract terms is largely a matter of historical accident. Soap has soapiness. But does color have colority? At any rate, words are concrete or abstract not because of their verbal form but according to the way in which they are used in some context. As ordinarily experienced, color, sound, the circular, the solid, being traits of actual objects, are certainly concrete; in technical scientific contexts where they stand for possibilities, they are abstract.

Nevertheless, it is highly significant that intellectual terms can be traced etymologically to an origin in sense terms: lucid, articulate, sagacious, sapient, comprehensive—a brief list in which the traditional five senses are represented. According to an old maxim, there is nothing in the intellect which is not first in sense. And Kant declared, "Perceptions without conceptions are blind, and conceptions without perceptions are empty."

Another obvious but important distinction is that of singular and general terms. As long as a singular term is employed with the same meaning, it has exclusive reference to some one determinate unit: Thomas Jefferson, New York City, Mars, the last Puritan, the author of *The Last Puritan*, this book, this world, George Washington's birthday, the death of Queen Victoria, England, Mont Blanc, Uncle Sam, the cube root of 8, the smallest country in Europe. Each of these terms is understood to signify something individual as such, unique, limited to a single instance only. A general term is one which may be applied in the same sense equally to each of an indefinite number of units: president, sergeant, American, planet, book, animal, ghost, a Croesus, "a Daniel come to judgment."

In this connection, however, a frequent confusion is to be avoided. Terms designating a number of things may be used "distributively" or "collectively." A general term, such as soldier, is the name of all the members of a collection taken distributively (*omnes*); i.e., of each taken separately. But how about a regiment? Here we have the name of a collection as a whole, of all the members of the collection taken together (*cuncti*), but not of each taken separately; it therefore exemplifies not a general but a collective term. Confusion of the two underlies the fallacies of composition and of division. With respect to soldiers, then, regiment is collective. Yet there are numerous regiments; so that, with respect to these, regiment is indeed general. So too, as a kind, "word-list" is general; with respect to the words it contains, it is collective. To distinguish general and collective terms, therefore, bare inspection will not suffice; we must, in construing them, look closely to the manner in which they are used.

Still another division claiming attention is that of terms positive and negative. We may indicate the presence or possession of a quality: honest, blue, excellent. On the other hand, we may stress the absence of a quality: sick, alien, unmarried, imperfect, disagreeable, unformed. Not many examples are needed to show that a language does not always provide a ready-made pair of positive and negative terms; that terms positive in form may be negative in meaning, and vice versa; and that positive or negative character is not inherent in a term, but depends upon the meaning in which a term is used in a specific context. Insofar as negative terms are not further expressly restricted, they are infinite or indefinite: "non-men" includes everything in the universe save men. There is, moreover, a special class of negative terms, called privative, which express

that a thing has been deprived of a quality normally possessed or expected; as when we speak of the lame, the halt, and the blind.

Is evil then the simple negative of good; or does not rather the good exclude also everything that is neither good nor evil? When two terms are both mutually exclusive and together exhaustive, they are contradictory: white and non-white; rich and non-rich; possible and impossible; and, for integers, even and odd. In the case of these pairs, no third alternative is possible: *tertium non datur*. Each term in a pair of contradictory terms is the negative of the other. When two terms, though mutually exclusive, are not together exhaustive, they are contrary: white and black; rich and poor; young and old; first and last; virtue and vice. Each of these pairs allows of things neither the one nor the other. Unlike contradictories, contraries admit of a mean. In particular, extremes in a scale of degrees of more or less are called opposites, like "sweltering" and "icy," between which we may list: hot, warm, tepid, lukewarm, mild, fresh, cool, chilly, cold, frosty.

Finally, terms may be divided according as they are absolute or relative. Although everything is indeed related to something else, certain terms may be singled out as relative in the sense that their signification cannot be explained without reference to a correlative: twice, half; leader, follower; partner, partner. Terms not relative in this manner may be called non-relative or absolute.

Consider the term "progress" in the statement: Only a changed heart and mind makes progress possible. As here employed in the meaning of a steady advance to the goal of full realization of human powers, the term "progress" is categorematic, abstract, general, positive, and relative.

DRILL

Distinguish terms concrete and abstract; singular, general, and collective; positive, negative, and privative; contradictory and contrary; absolute, relative, and correlative. (See Exercises IV, 16-33.)

16. DENOTATION AND CONNOTATION

To "denote" is to "mark down"; whereas to "connote" is to "mark along with." Terms accordingly have denotative force when they "point," so to speak, like gestures, and when they name objects, somewhat as do labels pasted on bottles. Plainly among denotative terms are demonstrative words (this, these), proper names (Peter, James, John), common nouns (man, fisherman, apostle), verbs of change or action (grows, barks, thinks); perhaps also descriptive terms lacking demonstrative reference (next year's model, future civilization, the ether, atoms, mountains of gold, and sea-serpent).

Generic terms, which designate kinds and indirectly denote individual objects, may also be said to possess extension, in the sense of having reference to subordinate kinds. Thus Copernicus, Galileo, and Darwin form part of the denotation of "scientist"; part of the extension would be astronomer, physicist, biologist.

How is a kind (like "scientist") marked out and identified as such? By a set of traits which may be put to use in description. Such traits may be said to comprise the intension of a term, as distinguished from its extension. But, for effective communication, there must be a measure of agreement as to the traits both necessary and sufficient to mark out a kind.

Reflection shows that a term declares more than itself in its naming capacity. We cannot utilize a term as a mark without marking, along with it, much besides. The denoting of objects is attended by the connoting of ideas. Certain organisms are (de)noted as human beings; at the same time are (con)noted their animality and rationality. Especially when we are challenged to justify our denotative application of a term, we appeal to connotative meanings. Does a given literary passage possess "lyrical quality"? Yes, if it exhibits a union of such characters as unity of theme, intense emotionality, and concentrated richness of language. In some such manner we prescribe conditions under which we describe a literary passage as lyrical. Abstract terms thus turned to account give warrant together for the claim that a term, whose pertinence someone has questioned, is adequate to perform its denotative function in a given instance.

As a concrete term designating a kind has an extension, so an abstract term may be said to have a logical scope or comprehension. Corresponding to the various kinds of triangles, triangularity is necessarily right-angled, or scalene, or isosceles. The triangularity (con)noted is thus rendered determinate.

The traits which are necessary for something to be a triangle or money or whatever it is, are brought together in a definition. "Papa! What's money?" asked Paul. Hesitating to reply in the technical terms that came to mind, Mr. Dombey answered: "Gold, silver, and copper. Guineas, shillings, halfpence." But Paul persisted in his questionings: "I don't mean that, Papa; I mean what is money after all." And explained: "I mean, Papa, what can it do?"¹

Irrelevant to logic, of course, is the total intension, which may be in large part unknown. The logician must rule out also an arbitrarily selected intension varying from

¹ Charles Dickens *Dombey and Son*, chap. viii.

one group or person or occasion to another and sometimes called "connotation," by literary people, in the sense of overtones or nuances of association. Logical importance attaches to what may be called the conventional intension, or connotation, of a term: the meaning agreed upon in discourse and perhaps formulated in a definition.

Denotation and connotation go together in the progressive development of our knowledge. We designate by name, objects, their kinds, their traits. From superficial resemblances and differences, our attentive interest proceeds to more reliable abstract features, their scope, their interrelations. So our use of terms comes to be modified from time to time. Whatever the historical causes of the selection of a name, we must, from the point of view of developed knowledge, understand the connotation of a term in order to assign to the term its proper denotation.

Consider the terms: squares, rectangles, parallelograms, and quadrilateral figures. As the extension or denotation increases, the intension or connotation decreases; and vice versa. However, the proportion is not numerically exact: joining the adjective "blind" to "man," narrows the extension of "man" much more than would joining the adjective "white" to "man." Moreover, the extension of "metal" is unaffected by the apparent increase of intension in the addition of "elementary."

No one would condemn every loose use of words. Yet, as a common practice, it does hinder the precise fixing of denotations and connotations. So we debate: Is this society "democratic"? Is that artist a "genius"? Are Ibsen's plays "sordid"? For, to the extent that the denotation of one word shades off into that of another, our word is vague. Again, we easily slip into confusion of "cavities" with "passages," "nonpartisan" with "bipartisan," "barbarian" with "bar-

barous," "affect" with "effect." To the extent that the connotation of a word in some context is replaced by a different one, that word is ambiguous and its use confusing. In general, the more irresponsible the treatment accorded to words in everyday conversation, the more serious becomes the need of a technical vocabulary. As the gap widens between the public and the expert, and between one specialist and another, occasions are multiplied for misunderstanding and for intellectual tension. Standards of denotation and connotation are therefore pertinent to careful handling of words in accordance with the large variety of their settings.

DRILL

Explain and illustrate: denotation, extension, intension, connotation, logical scope or comprehension, definition; the inverse ratio of extension and intension; vagueness, ambiguity. (See Exercises IV, 34-47.)

17. COMPARISON AND CONTRAST

"A rose by any other name would smell as sweet." For names refer to objects and events, but do not convey them to us. The freight words carry consists of ideas: that about things which is known and is said. There is, moreover, a fourfold challenge in ideas; namely, that they be clear, distinct, true, and adequate.

Take a common flower. Our idea of it is clear if we can distinguish it from other flowers; obscure, if we cannot. So too we may have a clear notion of a friend, of a household pet, of a tree, of the boundaries of a state. But it is difficult to decide whether a certain micro-organism is a plant or an animal; whether a certain plant is herbaceous or woody; or of what materials certain rocks are composed.

Clearness is attended by confusion when, though we recognize a thing as a whole, we are unable to discriminate its parts or the qualities by which we know it. The notion we have of a sunflower is distinct as well as clear when we not only can tell it from others, but also specify and describe its parts: sepals, petals, stamens, pistils, and receptacle. The difference between confused and distinct ideas is apparent in the crude drawings made by amateurs. Neither clear nor distinct are many ideas commonly expressed about reality, sensation, mind, personality, goodness, beauty.

We find it possible, within limits, to speak about flowers and their parts from hearsay only; to know them by name alone, or symbolically. How impressive, too, are large numbers: the billions and trillions of dollars in government finance; the thousand sides of a chiliagon; the four years and two-hundred million miles our nearest star is declared to be away from the time and place in which we see it! Clearness, distinctness, and truth are more likely if we do not forget the significance of the simplest monetary transaction; the three sides of a triangle; the distance of our neighbor's house, or of the tip of our own nose, from where we see it. To be sure, much knowledge is directly occupied with symbols. So we expand a binomial: $(x+y)^2 = x^2 + 2xy + y^2$. But manipulation of symbols is not therefore a satisfactory substitute for intimate familiarity with things, even if we are able to repeat parrot fashion a sound argument: "Every labiate is square-stalked; the dead-nettle is a labiate; therefore the dead-nettle is square-stalked." For clear and distinct ideas may be false: witness the mythologies. Still, progressive approximation to the ideal of truth is promoted, not by suitably ordered observation and experiment alone, but by suitably ordered discourse as well.

This would include open acknowledgment of the limits within which our findings are true.

Knowledge that is clear, distinct, and true as far as it goes, may yet be inadequate. How far must analysis of objects, of their parts, and of the parts of their parts in turn, be carried? Certainly, to the extent that such analysis is adequate for the purpose in hand. A knowledge of flowers which would suffice for the amateur gardener would be far from adequate for the purposes of a botanist. Moreover, analysis is correlative to synthesis. In analyzing a whole into its components, we exhibit it as a synthesis of such component parts. A flower, a sentence, or a psychosis is determined in its essential nature by the elements into which it may be analyzed; but also by the manner in which these factors are combined.

At each step in clarifying our terms we evidently institute comparisons. A comparison is any activity we perform with a view to establishing identities and incompatibilities or likenesses and differences. Since every selection involves its negative counterpart of rejection, to compare is at the same time to contrast. Comparison-contrast is extraordinarily fundamental in inference and discourse and in every phase of the thinking process. *Qui bene distinguit, bene docet.*

Aristotle's monumental *Historia Animalium* described and explained in detail the similarities and diversities of animals accessible to observation: in their modes of living, their activities or functions, their characters or dispositions, and their parts.² In the opening passages, Aristotle distinguished simple bodily parts (like flesh), divisible into parts like themselves, from complex parts (like the hand), divisible into parts unlike themselves (such as flesh, sinew, bones).

² H. D. Hantz *The Biological Motivation in Aristotle* (Ph. D. thesis; New York: Columbia University 1939), chap. i.

HOW MAN THINKS

Thereupon he considered the ways in which parts are alike or different, so as to determine the individual or racial look of animals. Some parts may be alike in form (nose, eye, flesh). Similar parts may vary in degree with respect to their characteristics (color, shape); may vary in size (the beak); may vary in number (feathers). Parts differing in form may have the same function (bone and fish-bone, nail and hoof, scale and feather). And parts may be alike or different in position (teats). An extensive survey of such facts, especially in conjunction with the criterion of modes of generation, rendered possible for Aristotle a grouping of animals far in advance of what had previously been known.

DRILL

Explain and illustrate: clear, distinct, true, adequate ideas; analysis and synthesis; comparison and contrast. (See Exercises IV, 48-66.)

Categories

18. WAYS OF BEING

If we were to bore carefully upward through a person's foot and leg, would we eventually (say in the chest) come upon his soul? This, a noted thinker has informed us, is the first "philosophic" question he could remember asking. While his mother patiently tried to satisfy his eager youthful curiosity, his father earnestly warned against thinking of the soul in that way: "it was not at all the kind of thing that had a place inside the body from which it could be fished out."¹

Quite differently did Aristotle conceive the soul. Taking as a starting point the difference between a living and a dead body, he noted that the latter has ceased to act as the former acts. The *psyche*, he accordingly declared, is an activity: The characteristic behavior of a living body; not anything like what the body is, but rather what the body does; neither identical with the body nor separate from it—therefore to be distinguished from, yet at the same time related to, the body.²

Shall we speak of the soul as substance or as activity? Certainly, whatever construction we put upon it, will color whatever else we may say about it. Should we now take the time and trouble to write out such divergent accounts, we would sense in a vivid manner the importance of a doctrine

¹ *Contemporary American Philosophy* eds. George P. Adams and W. P. Montague (New York: The Macmillan Co. 1930 [Copyright 1930 by The Macmillan Company used by permission of The Macmillan Company, publishers]), Vol. II.

² Aristotle *De Anima* 413a21; 412a, b.

of categories. Because of the failure to distinguish clearly one category from another, there have been centuries of confused and confusing controversy about the so-called "mind-body" problem.

As derived from the Greek, to "categorize" is to "predicate." Categories are the widest significant predicates. To follow one of Aristotle's lists: man and horse exemplify substance; two and three cubits long, quantity; white and grammatical, quality; twice, half, greater, and less, relation; the market and the Lyceum, place; yesterday and last year, time; lying down and sitting, posture or position; shod and armed, state or condition; cutting and burning, activity; being cut and being burned, passivity.^a

To be sure, there are still wider predicates. Everything may be said somehow to "be." But such all-comprehensive predication is not too significant. "Being is being" affords as little illumination as, at the other extreme, "Socrates is Socrates." If the various categories are discriminated ways of being, then "being" is not itself a category (or is the category of all categories). Etymology, moreover, suggests the comparison of a "category" with an "accusation": to place anything under its most general significant predicate, is to bring it to trial, so to speak, before the highest court in the realm of discourse, where the judgments of the lower courts are reviewed.

The chief distinction among categories is between substance and the categories dependent upon substance. Quantities, whether equal or unequal, are so much "of" something; just as qualities are "of" things like and unlike. Again, that is relative which cannot, without explicit reference to something else, be said to be what it is; and primary to places and times are objects and events. So too posture and condition, along with activity and passivity, presup-

^a Aristotle *Categories*, chap. iv.

pose that to which they are ascribed. Even in Wonderland, the unattached grin is the grin "of" the Cheshire cat.

Whatever stability the various kinds of being have is due, in last analysis, to some substance maintaining itself through the changes it undergoes. Substance is primarily a concrete individually existing thing ("this man"), the first object of utterance, that about which something is said. In a secondary sense, substance is that which can both be said about something else and about which in turn something else is said; especially a species ("man") and a genus ("animal").

Given a subject matter of discourse, emphasis falls upon "what" it is or is said to be; interest is focused upon predication. How men become entangled in their speech when turning a predicate around into a subject! In Plato's dialogues, we learn of those who, not content with characterizing a certain act as just or unjust, carried the questioning habit out of bounds by asking: Is justice then also itself just or unjust? Confusion of this sort may be avoided with the aid of a logic of discourse which is based on a doctrine of categories.

DRILL

Give Aristotle's list of "categories," with comments on each. (See Exercises V, 1-16.)

19. CATEGORIAL EXPLANATION

Our age is keenly aware of scientific progress and of social problems. So we find it congenial to turn, for further light upon the categories, to our natural sciences and social studies. A recent author has enumerated typical modern categories, from the simple to the complex: "Order, relation, class, implication, meaning, validity, number, figure,

quantity, chance, time, velocity, mass, pressure, color, sentience, organism, life, death, consciousness, sign, plan, mind, behavior, speech, good, bad, freedom, tragic, ugly, value, labor-time, rent, class-struggle, revolution.”⁴ Who can fail to note that any such inventory would reflect a point of view? Yet the selection quoted is synoptic enough, having been made from such varied fields as “logic, mathematics, physics, biology, psychology, ethics, aesthetics, economics, sociology.”

What purpose is served by a catalogue like the one here drawn up? The terms so arranged, we are given to understand, are effective in answering questions, in organizing segments of experience, in solving problems, in communicating interests. How the words singled out as categories meet the demand for explanation, is shown in a few illustrations: “The series is well-ordered”; “The number 2 is the only member of the *class* of even primes”; “Light exerts *pressure*”; “An idea is a *plan* of action.”

Within the list itself, terms become clarified as we work backward: a play is “tragic” when it portrays “behavior” which ends in “death.” Not that the complex has been rendered inherently more simple; but its conditions have been pointed out. Thereby it has been made more intelligible. For explanation and control lie in the direction of simplicity. The categories of a “lower” level may be applied to the “higher” level dependent upon them; although the reverse is not equally true. Nevertheless, what is “unique” in the “higher” levels is not therefore to be denied. On the contrary, it is the complex which we try to explain, which is the starting point of our reflection, and to which our analysis is relevant.

Categories are controlling concepts in any investigation that is far-reaching or systematic. They are so inter-

⁴ Sidney Hook *The Metaphysics of Pragmatism* (Chicago, London: The Open Court Publishing Co. 1927), p. 107.

related as to furnish us with tests for types of valid inference. One thing implies another because of what categories imply. Thus man is mortal because of an organic constitution with traits which indicate that his life-functions will eventually cease. Of what use were seeing and hearing in a dark and silent world; or a multiplication table where nothing could be counted or separated; or geometry where one thing could not be superimposed upon or displaced by another? Just so the making of inferences, the tracing of implications, presupposes a world that is logically ordered or that is at least amenable to logical ordering.

Composing a schedule of categories is a difficult but incisive way of summing up one's knowledge and beliefs. Hence Charles Renouvier is reported⁵ to have said towards the end of his life:

"I should have wished to write a small book . . . Its point of departure would have been a study of the categories. The problem of the categories is the most arduous one which can present itself to a philosopher. It is the key to everything I have studied, so to speak, all my life; yet I have not studied it sufficiently."

DRILL

Show the uses of a schedule of categories in stating and answering questions, in explanation and control, in inference and discourse, in organizing and summing up knowledge and belief. (See Exercises V, 17-20.)

20. POLAR CONCEPTS

There is a striking feature none of us ought to overlook in his use of basic concepts; namely, the principle of polarity.⁶ Every magnet has a positive and negative pole and operates in a magnetic field. We cut with scissors which

⁵ *Ibid.*, p. 105.

⁶ Morris R. Cohen *Reason and Nature* (New York: Harcourt, Brace & Co. 1931), p. 165.

have two blades moving in opposite directions. Breathing with our lungs consists of intake and outgo, each a draft upon an environment itself polarized as organ and atmosphere. Necessary and probable inference presuppose natural necessity and chance. In short, existence is pervaded by opposites involving one another.

Among those fascinated by this theme was the Greek sage Heraclitus. "Strife is the father of all things," he insisted, comparing this strife at the heart of things with the harmony effected through the tension of opposites in the string of the bow or of the lyre. Being and not-being, the one and the many, up and down, hot and cold, dry and moist, light and darkness, sleep and waking, youth and age, life and death, good and evil—these and other antitheses like them he cited to show how each moment of change must contain the old and the new.

The principle of polarity is violated when one opposite is avowed to the exclusion of the other. Who would argue that all cups have an outside but no inside? That would be no more misleading, however, than to contend that everything changes and nothing is permanent. What would become of a game of bridge if not only the course of the game but also the players, cards, rules, and everything else mentionable were to change while the game is going on, irrespectively of any fixed or definite reference? Change and constancy, like other polar opposites, constitute cross-grained patterns in nature as in thought and in life. Not that everyone always correctly identifies which is which. It is rather to the point to remember that a distinction is not a separation; whether we are dealing with matter and form, action and reaction, giving and taking, substance and function, body and mind, means and ends, pleasures and

pains, supply and demand, perfect and imperfect, faith and doubt, or the sacred and the secular.

Time and again we find it necessary to adjust opposite considerations as best we may. Skilled artists strive to achieve a balance between extremes in color or sound, space or time, shape or motion; as artistry in living requires moderation in responses to stimuli generally. That is why thorough thinkers search out suitable distinctions and qualifications. Although more is not less, it is quite possible to live more than five miles from one part of a city and less from another part. There are, of course, impossibilities with which we must reckon. You cannot eat your cake and have it too. But some difficulties, at least, may be successfully overcome. Given six matches, four equilateral triangles may be formed with them, provided we recall that there are more than two dimensions.

Sometimes indeed our knowledge and insight are not adequate to do justice to seemingly diametrical opposites. Is light composed of corpuscles or waves? We need not therefore, however, deliberately confuse the rational and the irrational, the temporal and the non-temporal, the real and the ideal, and the like. Perhaps it is not irrelevant to remark that history has repeatedly enforced the seriousness of the prophetic warning: "Woe to those who call good evil, and call evil good, who make out darkness to be light, light to be darkness, who make out bitter to be sweet, sweet to be bitter!"⁷

DRILL

Indicate the pervasiveness of opposites and the contrasts between their recognition and separation, their distinction and confusion, their insistence and their mutual adjustment. (See Exercises V, 21-30.)

⁷ Isa. 5:20 (Moffatt tr.).

21. POSSIBLE OPERATIONS

In popular language, a "category" is any term that represents a possible mode of operation. When a clock is said to fall within the category of mechanisms, the clock is declared to be a mechanism, to exemplify the principles by which being mechanical is defined, and to call for an orientation and a method of treatment in accordance with the kind of being that is specified. So an act subject to a lawsuit may be assigned to the category of civil or criminal law; or a form of conduct, to some pertinent moral principle or category.⁸

This usage does not restrict categories to ultimate terms. Neither does it have regard to the widest possible frames of reference; but rather to captions or attitudes with a more immediate application. Yet the very recognition of categories aids in ordering our speech, in rendering it less chaotic, more coherent. Not only the theoretical but also the practical value of categories is brought home to us: categories suggest points of view from which actions and processes may be divided into kinds and may be explored and regulated.

Moreover, as knowledge increases and science progresses, one formulation of operations leads to another. Metals were at one time distinguished as such by directly observed traits, such as lustre and opacity, and by traits manifested in use, such as malleability and tenacity. More recently emphasized traits, including affinity and electric capacity, are consequences of experimentally instituted interactions of things with one another. Hence the recognition of numerous additional metals, the more accurate determination of kinds of metals, and the linking of metals with

⁸ John Dewey *Logic: The Theory of Inquiry* (New York: Henry Holt & Co. 1938), p. 273.

non-metals in an extensive system of inferences in chemical science.⁹

Within any range of inference and discourse, however, there is need of concerted effort to avoid an unseemly mingling of categories. Understanding of art, for example, is easily hindered by the intrusion of historic, biographical, religious, political, and other extraneous preoccupations. Again, in the usual debates about democracy, socialism, communism, capitalism, and fascism, it is not always clear whether these are considered as human ideals or as systems of administration. And what shall we say of the widely publicized pictures of the universe, so often taken as necessarily calling for radical transformations of our modes of living in the world?

Equally faulty is the Procrustean disposal of varied and complex affairs with categories of one sort. Not every problem is individual; or social; or scientific; or moral; or religious. Issues of war and peace concern not mapmaking alone, but reach, in the end, to the hearts and homes of men, to their labor and their leisure, their education and their aspirations. More than a matter of the laboratory, life is also a drama suggesting the vital terms in which it is, after all, to be construed: birth, food, sleep, work, joy, sorrow, love, victory, defeat, and death; above all, the ever-renewed balancing of our circumstances and our ideals.

DRILL

Discuss categories as possible operations; their range in discourse and inference; the fallacies of their confusion and reduction. (See Exercises V, 31-34.)

⁹ *Ibid.*, pp. 275 f.

Classification and Definition

22. THE PREDICABLES

Socrates is said to be a man. What then is a man? A rational animal. And what is an animal? A sensible living body. And a body? It is a corporeal substance or being.

This is the famous "tree of Porphyry." As we climb this "tree," we obtain an increasingly inclusive "look." Our speech too becomes more "general." Upon descending, we find that our understanding of the less general has been enhanced. Evidently, too, the particular and the general go hand in hand in the development of our knowledge.

Summum Genus	Substance	
Differentia	Corporeal	Incorporeal
Subaltern Genus	Body	
Differentia	Animate	Inanimate
Subaltern Genus	Living Thing	
Differentia	Sensitive	Insensitive
Subaltern Genus	Animal	
Differentia	Rational	Irrational
Infima Species	Man	
	Socrates, Plato, and others	

The relation of Socrates, Plato, and others to man differs from the relation of man to animal. Individuals as startingpoints of our classificatory scheme form members of a class; whereas a sub-class is included in a superordinate class. Technically, a less inclusive class is known as a *species*; a more inclusive class, as a *genus* (both "species" and "genus" being understood in a logical, not in a biological, sense). Between the *infima species*, or least inclusive class, and the *summum genus*, or most inclusive class, are intermediate or *subaltern genera*, each a species or a genus according to the direction in which it is read. But so far as any related species and genus are concerned, the former is a subject; the latter, a predicate. Also the upper and lower limits of a classificatory scheme are relative to our knowledge and purpose.

Of the species, we may also predicate a *differentia*, a trait which distinguishes the species from other species in the same genus. Thus a highly developed rationality differentiates man from the cat and other animals. Genus and differentia, taken together, constitute a *definition*, telling what the species defined is in distinction from other species. From the definition in turn, we may (on sufficient systematic knowledge) derive another kind of "predicable"; namely, a *property*. If man, for example, is rational, then he is capable of learning grammar. Unlike the definition, a property in the same classificatory scheme does not endeavor to tell what the species is. Nevertheless a property, like the definition, belongs to all of the species and to the species exclusively. In other words, definition and property are the two predicables which are "commensurate" or "convertible" with the subject: if anything is a man, it is a rational animal, and if anything is a rational animal, it is a man; likewise, if anything is a man, it is capable of learn-

ing grammar, and if anything is capable of learning grammar, it is a man. A trait singled out as a property differs, in this important respect, from some other trait which may be incidentally predicated of part of the species. The latter sort of trait is, for the purpose of the classification in hand, to be viewed as an *accident*. The sitting posture is therefore an accident which may or may not belong to Socrates as a man (whatever his posture as an individual at a given time).

Predicate		
Commensurate with the Subject	Not Commensurate with the Subject	
Definition, Property	An Element in the Definition	Not an Element in the Definition
	Genus, Differentia	Accident

There are then, according to Aristotle's analysis, five possible relations between a predicate and its subject: definition, property, genus, differentia, and accident. Porphyry obtained his own list of five predicables by omitting definition, which combines genus and differentia, and adding the species, which is indeed predicable of a concrete individual. Aristotle's analysis, however, applies to the relation of one universal to another; for scientific or systematic knowledge is not of a concrete individual as such but of a form, a type, a universal (therefore of Socrates not as Socrates but as man).

DRILL

Explain: "the tree of Porphyry"; class-membership and class-inclusion; Aristotle's list of predicables; Por-

phyry's modification of Aristotle's list of predicables. (See Exercises VI, 1-11.)

23. DIVISION AND CLASSIFICATION

To group individuals into classes, and classes into wider ones, is a practice known as "classification." In the opposite direction, the process of breaking up a class into its constituent sub-classes, or a genus into its species, is known as "division." Logical division is, of course, to be carefully distinguished from physical and from metaphysical division: physically, a tree may be divided into such parts as roots, trunk, branches, leaves, etc.; metaphysically, into material, structural, developmental, and functional features; logically, by an appropriate enumeration of its species.

A classification or division may be said to be both natural as well as artificial: natural, inasmuch as it is based upon some trait actually possessed by the members of a species or of a genus; artificial, inasmuch as the trait on which the classification or division is founded is selected by us. For there is not any one classification which is exclusively true. We employ in our thinking the classification or division best suited to our purpose and most fruitful in organizing our knowledge. Although whales live, not like horses on land, nor like birds in the air, but like fish in water, yet, for the understanding of the life-cycle, it is more important to classify whales with mammals, which suckle their young, whereas fish deposit their eggs to be fertilized.

At the same time, a classification or division, to be sound as well as useful, must conform to certain requirements. Thus, the constituent species must be mutually exclusive; they must together exhaust the genus; and the division must, at least at any one stage, proceed upon one principle or basis, upon a single *fundamentum divisionis*.

Note the faults which would be committed if rectilinear figures were divided into triangles, parallelograms, rectangles, and polygons of more than four sides: parallelograms and rectangles do not exclude each other; there is an omission of irregular four-sided figures which are not parallelograms; and no less than three principles of division are employed, the number and the directions of the sides in addition to the angles contained. We may, however, divide rectilinear figures, according to the number of their sides, into triangles, quadrilaterals, and polygons; quadrilaterals, according to the directions of their sides, into parallelograms and trapeziums; and parallelograms, according to the equality of their sides and the size of the angles, into squares, rhombuses, oblongs, and rhomboids.

As has been noted, a classification may extend from an infima species through a number of intermediate or subaltern genera to a summum genus. But the species enumerated should be those of the proximate (or next higher) genus; the division should not make a leap (*divisio non faciat saltum*) by proceeding from a wide genus (such as geometrical figures) at once to a narrow species (such as figures with or without parallel sides). Apart from knowledge of subject matter, however, there is no guarantee that a classification or division is exclusive, exhaustive, or based on a significant principle. Are all forms of business organization divisible into partnerships, corporations, individually owned businesses, and business trusts? Often we have to content ourselves with groping for a satisfactory method of organizing our knowledge. No less than a hundred language-groups have not yet been connected in larger units. Then again, knowledge of some subject matter may be so highly developed that we cannot content ourselves with the simple rules of classification and division.

We may indeed assure ourselves that the species we exhibit are mutually exclusive and together exhaustive if we adopt the method of binary classification or dichotomous division: parallelograms may have their sides equal or not equal; they may be right-angled or not right-angled; etc. It is by means of bifurcate division that we distinguish four sub-classes of vertebrate animals: *mammifera* (having lungs and *mammae* but not wings), birds (having lungs and wings but not *mammae*), reptiles (having lungs but neither *mammae* nor wings), and fish (having neither lungs, *mammae*, nor wings). When such dichotomy is unnecessary (as in numerical arrangements), it has, as a rule, little more than formal value. It is, moreover, frequently misleading. Plato satirized the Greek division of mankind into Greeks and barbarians by comparing it with an intelligent crane's division of animals into cranes and non-cranes! Clearly, the negative terms in point are "indefinite" or "infinite" terms, including as they do everything in the universe save what is designated by the corresponding positive term.

Among the purposes of classification we may mention: to distinguish different types of data; to facilitate comparison; to simplify a multiplicity of data; to aid in the process of inference. Since all and only ruminant animals have the foot cleft, the print of a cleft foot yields to the observer information about the teeth, jaws, vertebrae, and the bones of legs, thighs, shoulders, and pelvis of the animal which has left the impression.

DRILL

Explain and illustrate: classification or division as natural and artificial; its rules; its dependence upon knowledge; its value in dichotomy; its purposes. (See Exercises VI, 12-27.)

24. DEFINITION

An important device for the clarification of terms is definition. Although no one can reasonably be expected to define every term he uses, there are occasions when the need for definitions is acutely felt.

Notoriously, many words carry emotional associations and overtones which obscure thought. Out of conflicting passionate attitudes arise interminable disputes about moral obligation, social justice, civic rights, public welfare, private property, legal principles, religious sentiment, and the like. Could the disputants be prevailed upon to give adequate explications of their terms, many of their strifes would be reduced, if not eliminated altogether. Still, not all disagreements over meanings are quarrels about words only. Often more than one approach is needful in locating fundamental or central features of complex phenomena. Thus, recent theories have stressed as clues for the study of religion: personification of natural objects, worship of ancestors, belief in superior beings, recognition of mysterious forces, practice of magic, social rites, ceremonies attending life-crises, symbolism, and experience of the supernatural, or the sense of the "numinous," moralized and intellectualized.

Definition accordingly plays a vital role in inference. To the extent that a defining formula is adequate, it may be substituted for the expression it defines, provided our discourse is coherent. In any case, what our ideas mean, depends less upon what we say they mean than upon the ways they operate in our thought, speech, and action. When Euclid defined a point as "that which has no parts," he did not have in mind everything that has no parts. In the procedure to which Euclid was committed, his reference was to the ends and intersections of lines as over against lines

and what is made up of them or enclosed by them; for lines can be divided as the ends of lines cannot. Though designed to clarify, definitions themselves stand in need of clarification by the procedures in which they find their place and function.

We may accordingly avail ourselves of definitions as explicit or implicit. Nevertheless careful writers recognize the obligation, when they introduce a new symbol, to state clearly the sense in which they propose to make use of that symbol. When speaking of "monetary circulation," they inform us whether they exclude or include bank deposits. Such definitions are called "verbal" or "nominal." However, there is no sharp dividing-line between verbal or nominal and "real" definitions: any defining formula worthy of the name endeavors, in one way or another, to lay bare the structure of some subject matter.

Since a definition aims to tell "what" something "is," it must obey the rule (applicable to substantive terms): State the proximate genus and the differentia. Clearly, the proximate genus (that next in order) is more appropriate than one more remote: a fish is more informatively and properly said to be a "vertebrate" breathing by means of gills than an "organism" differing from others in this respect. Statement of "essential attributes" requires also that we select a differentia rather than an accident or a property: though man be the only animal that cooks his own food, such a description would yield an exposition too restricted in application; and though man be a rational animal having powers of articulate speech and of the intelligent use of his hand as a tool, such an interpretation would contain superfluous marks which do not render our formula more precise. A definition is, in short, a brief statement of such qualities as constitute necessary and sufficient

conditions for one class to be distinguished from other classes and to be properly placed in a classification of types.

In order that a definition may denote the species, the whole species, and nothing but the species, the *definiendum* (or the expression to be defined) and the *definiens* (or the defining expression) must be exactly equivalent, in the sense of being coextensive. "Science" would be too narrowly defined as "experimental inquiry" if mathematics is to be recognized as a "science"; and anyone mindful of a plucked hen would find the formula "featherless biped" too broad as a definitive description of man.

Other rules remind us that a definition must not be circular, negative, or expressed in obscure or figurative language. We make no logical advance when we define a term by itself or by some synonym; as when we say that "life" is the sum-total of "vital" processes. Then, to state what a thing is not, instead of what it is, does not, as a rule, sufficiently delimit it from other things: a watch is indeed a timepiece which is not a clock, but needs to be differentiated from other timepieces as well. So too, since the less known is to be explained by the better known, many would derive little clarification from Samuel Johnson's famous definition of a "net" as "a reticulated fabric, decussated at regular intervals, with interstices at the intersections." As for metaphors, they are likely to mislead when treated as definitions, as in the saying that "architecture is frozen music."

Despite such rules, we do, for ordinary purposes, substitute synonyms: "fatigue," we may say, means "weariness." Many terms, though negative in form, have positive meaning; so that a "bachelor" may be satisfactorily defined as an "unmarried man." Whether or not defining expressions contain unfamiliar terms, often varies from person to

person; and scientists necessarily replace the loose language of everyday conversation by technical terms which, though unintelligible to the untrained, may be well understood by the specialist.

It is also advisable to remember that no single definition of a given term may claim exclusive correctness. On the contrary, a statement of meanings may assume various forms relative to the purpose it is intended to serve. Why should not the chemist and the physiologist select different genera and differentiae for their respective definitions of oxygen? Furthermore, the contents of a definition depend upon the state of our knowledge. Only in recent times has it been possible to define malaria as the presence, in the red blood corpuscles, of animal parasites of the genus *plasmodium*. Every system, moreover, contains undefined terms whose meanings can be made plain only by an exhibition of what they denote or by the relations they bear to other terms in the same system. Sooner or later we come, in any classification, upon individuals which are not classes at all and upon ultimate abstractions, such as "entity" and "relation," for which there is no genus.

A definition depends for its utility upon the scope of the deductions it makes possible in the systematic exploration of a given subject matter. That is why gold is defined as a yellow, incorrodible, dense metal of atomic weight 197. According to Aristotle, "the basic premises of demonstrations are definitions."¹ We therefore choose definitions stating those features of a thing from which its other features follow.

DRILL

Point out the role of definition in inference and discourse; some of its main kinds; its rules; its limitations and values. (See Exercises VI, 28-44.)

¹ Aristotle *Posterior Analytics* 90b24.

25. PROPERTY AND ACCIDENT

A discerning Oriental, asked to comment on a religious assembly he had visited, made an acute observation: the speakers, he gently suggested, appeared to assume that God were necessarily a member of the white race, of a major American political party, and of a leading Protestant denomination. So blithely do some identify that to which they have grown accustomed with the nature of things!

Much discourse confuses the necessary or essential and the accidental or incidental. Perhaps everyone would recognize this confusion if it were said that white "happens to be" a color, or justice "happens to be" a virtue. Yet who, bringing some logical sensitivity to bear upon recurrent controversies, can fail to be impressed with the frequent lack of discrimination between predicables? It would not be difficult to multiply examples like that of the man to whom all church-bells sounded discordant because his betrothed had jilted him to marry a clergyman; the argument that a political candidate has a good radio voice or is a good father; the war-cry in the psychology of the 1860's, "*Ohne Phosphor, kein Gedanke*"; etc.

As the necessary enters into the determination of the probable and the contingent, so we need not deny the existence of doubtful or borderline cases as regards property and accident. But we have not thereby obliterated the distinction. "Properties" holding only in restricted spatio-temporal relations, or only relatively to some definite contrasted species, are more suitably treated as "accidents"; as when a given individual is the only one of a company who has appeared in full dress, or when man is declared to differ from the horse in being two footed. A "property" in the strictest sense is a trait peculiar to some subject at all

times and in contradistinction to everything else: triangles, e.g., have their internal angles equal to two right angles.

An accident, on the other hand, may or may not belong to a class without in either case affecting other qualities of the class. As regards the geometrical properties of a triangle, its size, however small or large, is a pure accident; and the form of a crystal is not affected by its size or by the manner in which it is grouped with other crystals. Sometimes a distinction is made between separable accidents, such as the clothes a man wears, and inseparable accidents, such as the place of his birth.

It may be objected that present day science is concerned with "accidents," in the sense of apparent exceptions to alleged laws, inasmuch as science inquires into the correlated conditions of their occurrence. True, the fixing of definite essences, properties, and accidents of things antecedently to inquiry about them would be an indefensible procedure; and the doctrine of the predicables is more readily applicable to the ideal entities of geometry than to empirical objects. But the relativity of the distinctions made confirms rather than negates those distinctions, which are abundantly utilized in our predications. Latterday repudiations of the doctrine of the predicables, on the ground that the "essential" and the "accidental" differ as the indispensable and the superfluous in inquiry, appear to overlook that, also in its original formulation, the doctrine has a "functional" significance.

DRILL

Show the need of distinguishing property and accident. Review methods of clarifying terms. (See Exercises VI, 45-60.)

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Affirming and Denying

Chapter VII. Propositions page 79

Chapter VIII. The Opposition of
..... Propositions.. page 94

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..... Propositions.. page 104

Propositions

26. SENTENCES AND PROPOSITIONS

Like many another death, that of Socrates stimulated a variety of estimates concerning the deceased. Pronounced wise by some, philosophy's famous martyr was contemned by others as not wise but quite unwise.

Easily the mistaken notion gained ground that affirmations or denials generally could be contradicted in the same way. But, although the statement that "all men are wise" is incompatible with the statement that "no men are wise," or that "all men are unwise," such incompatibility is not tantamount to contradiction. Here¹ we have, on a small scale, an indication of the reasons why logic must undertake to answer systematically the questions: What are the main types of affirmation and denial? How are these types interrelated?

Terms taken by themselves do not ordinarily affirm or deny anything; to do this, they must be combined in certain ways. There is indeed a natural development of combinations and separations effected in nature and in life to those made in experience and thought and in spoken and written discourse. Traits and tricks of speech, however, afford abundant occasion to ask exactly what our utterances express, call for, or communicate. Even "simple" sentences are not always as simple as they seem, whether exclamatory

¹ Aristotle *De Interpretatione* 20a23-30.

("O tempora! O mores!"), interrogative ("Is life worth living?"), or declaratory ("To be is to be perceived").

A "sentence" interests a logician to the extent that it states a "proposition." Whereas the former includes every utterance which is by itself relatively complete and independent, the latter may be defined as anything that can be said to be true or false. Thus two or more sentences may express one proposition: "*Gallia est omnis divisa in partes tres*"; "All Gaul is divided into three parts." On the other hand, wishes, questions, or commands, though not as such true or false, implicitly contain propositions in the form of assumptions; as in the prayer, "Give peace in our time, O Lord!" Symbols must, moreover, be arranged in appropriate ways if they are to convey a proposition; only in a special code may this be accomplished through nonsense syllables. Then too, like symbols and propositions, so judgments and propositions can be distinguished, even though mental acts called "judgments" are necessary to entertain, doubt, accept, or reject propositions; as in Othello's attitudes to the proposition: "Desdemona loves me." Concrete events or objects (like "Socrates") do not, of course, enter physically or literally into our sentences, wishes, symbols, judgments, or propositions (like "Socrates is overrated"); but only as to their form, to their relations, to their abstract aspects.

Must we then, in order to identify a "proposition," know whether it is in fact true or is in fact false? In statements of the type: "One hundred cents make a dollar," we must distinguish a resolution to use a word in a certain way from the factual question whether a given word is actually used in the manner specified. Again, we have no assurance today whether "there will" or "there will not be a naval battle tomorrow"; yet affirmations and denials con-

cerning a possible or probable future may be classed as propositions, though we cannot tell which of the alternatives will as a matter of fact be true, which false. May a proposition then be sometimes true, sometimes false? Clearly, the expression: "this is war-time," is incomplete; if the intention is to assert a limited state of affairs universally, the assertion is evidently false.

Issues like those presented reveal how widely a logical treatment of propositions may differ from a grammatical treatment of sentences. Not that grammar has no bearing on logical distinctions; on the contrary, grammatical and logical analysis are often mutually illuminating. We should be quite at a loss in logic if we did not learn from the grammarian what forms a subject, or predicate, or object, or modifier may take, or how simple sentences may be combined into compound or complex sentences.

DRILL

Distinguish propositions from terms, sentences, wishes, questions, commands, symbols, judgments, events, objects, and the knowledge whether a proposition is true or false. (See Exercises VII, 1-18.)

27. CATEGORICAL PROPOSITIONS

So numerous are the types of propositions that we cannot arrange them once for all in a perfectly satisfactory scheme. Yet even a rudimentary classification of them is desirable to facilitate an introductory study of syllogistic and other inferences. We may begin with subject-predicate propositions, in which a subject and a predicate are said to agree or to disagree in some respect: "Napoleon was superstitious," where an attribute is ascribed to an individual;

"the pole-star is the slowest-moving star," where a simple identity-relation is asserted; etc. In general, a "proposition" lays or places before us (*proponere*) the (tentative or definitive) result of a judgment or comparison, involving an "underlying" subject matter (*subjectum*) and something "asserted" about it (*praedicatum*).

The logical subject is that about which an affirmation or denial is (or is to be) made: "*Talk* is cheap"; "No *artificially manufactured musical wind instrument* possesses an adjustable reed"; "Blessed are *the merciful*"; "*He* jests at scars *who never felt a wound*." The logical predicate is that which is affirmed or denied of the logical subject: "No planets are *self-luminous*"; "Muscles are *the motor members of sensory-motor arcs*." Putting a proposition into the logical form "S is P" articulates connection of subject and predicate by means of the copula, or some form of the verb "to be." In the interest of logical manipulation, we therefore temporarily sacrifice a certain vitality and spontaneity of speech by saying, not: "Isadora Duncan danced expressively," but: "Isadora Duncan *was* an expressive dancer"; not: "It is raining," but: "The present case *is* a case of rain"; not: "Two is-greater-than one," but: "Two *is* greater-than-one." The standard form "S is P" also indicates why a "term" is so called: in a proposition stated in logical form, the words or groups of words serving as logical subject and predicate are placed at the extremes, or in the "terminal" position. The term is the last element into which every argumentation is necessarily resolved.

Propositions about singular terms may be subdivided into "singular-transient" and "singular-representative" propositions. The former apply to an immediate change, to something taking place at a limited here-and-now: "This is wet" (a proposition of sense-perception). The latter,

however, determine "this" or "that" to be one of a kind: "This is a chrysanthemum" (a class-membership proposition). How then shall we classify the statement: "You are thoughtful"? Apparently, it is the context alone which will justify us in saying of a singular proposition that it qualifies a relatively isolated occurrence at a specified time or place ("You are thoughtful here and now"), or that it implies more permanent traits which mark out a kind ("You are a thoughtful person").

Wider opportunities for inference are opened to us in generic propositions, which relate one kind to another. From the generic proposition: "Iron is a metal," more can be inferred than from the singular proposition: "This is iron." Indeed, there is a sense in which the latter is logically grounded upon the former. For kinds may be ordered in series from the least to the most extensive. By means of such a system, the logical basis is shown for characterizing any one kind as the kind it is. Moreover, to include a kind in another is at the same time to exclude certain others: "Wood is not a metal." Besides propositions of total inclusion and exclusion, there are propositions of partial inclusion and exclusion: "Some books are worth repeated careful study"; "Some mushrooms are not poisonous."

Leaving aside for a moment singular propositions, we thus obtain a fourfold schedule of propositions called "categorical" in that they are straightforward affirmations or denials. This schedule is basic to an understanding of syllogistic and other inferences. Consider first the universal affirmative proposition: "All men are fallible." It is said to be universal in quantity, as pertaining to a definite class, to a class as such, as a whole, to all of a class; and affirmative in quality, as asserting a certain agreement between subject and predicate. Of the form: "All S is P," it is briefly desig-

nated as an "A" proposition, from the first vowel in *affirmo* (I affirm); which has led to the symbolic notation SaP. Consider next the universal negative proposition: "No reptiles are vertebrates." Since a certain disagreement or discrepancy is here noted as obtaining between a subject and a predicate, this proposition is said to be negative in quality. Propositions of the form: "No S is P," are known as "E" propositions, from the first vowel in *nego* (I deny); hence the symbols SeP. In the third place, we speak of particular affirmative propositions, such as: "Some metals are corrodible substances." Here the predication applies to a class not as a whole but in part, partially, to an indefinite part of a class; the proposition is in this sense said to be "particular" in quantity. Hence also the symbolic notations: "Some S is P"; "I" propositions, from the second vowel in *affirmo*; SiP. Take, finally, the particular negative proposition: "Some Hindus are not inhabitants of India." It is of the form: "Some S is not P"; is called an "O" proposition, from the second vowel in *nego*; and is symbolized by SoP. The negative particle "not," as here used, characterizes not the subject or predicate but the copula.

As to singular propositions ("this S is P," "this S is not P"), we may, for some purposes of elementary logic, treat them as A or E propositions, according as they are affirmative or negative. Indefinite or indesignate propositions are those which give us no clue as to whether the predicate is to be applied to the whole or only part of the subject: "Clannish people entertain contracted views of general character"; "Rules are made to be broken"; "Gentlemen prefer blondes." Such propositions must be rendered definite before they can be used as A, E, I, or O propositions.

Unfortunately, the signs of quantity, "all" and "some," are not without ambiguities which affect inference. Under

what circumstances may we announce that "all the committee members are present"? When each has been counted; when a set limit has been attained; when the group is complete. In other collective propositions, there is greater stress on qualitative wholeness and none on counting of items: "All's right with the world." However, no entirety as an inherent objective limitation is suggested by "all the people at the county fair," any more than by "all the kernels of sand in a pile"; propositions about them are aggregative rather than collective. Note too that geometers refer to ideal rather than to actual figures: "All circles have their radii equal"; this is therefore a necessary proposition. But a generalization, exemplified by: "All men are mortal," is intended to apply to each and every one of the kind specified, even though complete enumeration is impossible. Expressions like "not all," or "not every," or "not everything," easily mislead beginning students of logic, who ought therefore to avoid them; as in the proverb: "All is not gold that glitters," which is not an E but an O proposition meaning that "some of the things that glitter are not gold."

Does "some" mean "some only (but not all)" or "at least some (and perhaps all)"? Usage varies. But the latter may be accepted as the usual meaning in logic, unless otherwise specified. On the one hand, it is a part but not the whole of the class of brilliant men who would ordinarily be said to be eccentric. On the other hand, in calling a proportion of the great artists geniuses, we do not exclude the possibility that all may be so honored. Again, as information comes to us that some candidates have been successful, this may mean at one stage "at least some"; at another stage, "some only." In strictness, a particular proposition, SoP (or SiP), has the purpose of overthrowing a universal, SaP (or SeP, respectively). Particular propositions will readily

be seen to include such as the following: "Many doubted"; "Most drunkards are shortlived"; "A few recognized him" (SiP); "Few recognized him" (SoP); "Not a few of his friends are influential" (SiP); "Certain members of the organization did not hesitate to take a definite stand on the issue"; "Ten per cent of the manuscripts submitted to us are accepted for publication."

DRILL

Explain and illustrate: logical subject, logical predicate, and copula; A, E, I, O propositions; singular propositions; ambiguities of "all" and "some." (See Exercises VII, 19-28.)

28. THE DISTRIBUTION OF TERMS

A familiar specimen, in text-books, of a statement to be reduced to logical form (A, E, I, or O) is: "None but the brave deserve the fair." Why such an illustration? By way of reply to this complaint, a critic was asked whether he construed the proverb as saying: "All the brave deserve the fair." Only after some discussion did he come to see the proper interpretation: "All who deserve the fair are brave." Grateful for the lesson learned, he continued his interest in logic with a fresh sense of its significance and value.

When we are told that man alone is a maker of tools, the intention is by no means to credit all men with tool-making activities; rather is it to underline that every maker of tools is a human being. A proposition of the form: "Only S is P," called an "exclusive" proposition, is to be reduced, not to: "All S is P" (SaP), but to: "All P is S" (PaS). For the effect of the restrictive adjunct "only" is to universalize not the subject term but the predicate term. To become convinced of this principle, we need but apply to it

a few appropriate cases: "Only undergraduates are members of this class." "Only those adequately trained will be hired." "Human beings alone contract leprosy." "None but fools are utterly unafraid." "The cost is only a dollar." "Only the courageous will face the truth." "Only logically relevant evidence is admissible in court."

"Exceptive" propositions may, where feasible, be restated in "exclusive" form. Take as a sample: "He is to be found everywhere except where he is most needed." That is to say, "only places where he is not most needed are places where he is to be found." Or, "all places where he is to be found are places where he is not most needed." Try similarly: "All persons except the color-blind respond differently to red and to green light of equal intensity."

Exclusive propositions help to introduce us to a topic of supreme importance in elementary logic: the distribution of terms. A term is said to be "distributed" when it is covered in the whole of its extent; "undistributed," when covered in part of its extent. Obviously, universal (A or E) propositions distribute their subject term: "All *sellers* are buyers"; "No *street-cars* are noiseless." Particular (I or O) propositions leave their subject term undistributed: "Some *dances* are ludicrous"; "Some *labor* is not pleasant."

As to the distribution of predicate terms, close attention makes evident that when we exclude something, in whole or in part, from something else, we exclude it from the whole extent of the latter. Negative propositions, accordingly, distribute their predicate terms: "No birds are (any) *quadrupeds*"; "Some pleasures are not (any of the things that are) *good*." When, on the other hand, we include something, in whole or in part, in something else, the latter is covered in part, but not necessarily in the whole, of its extent. Hence affirmative propositions leave

their predicate terms undistributed: "All animals with more than four feet are (some of the things that are) *bloodless*"; "Some languages are (some of the things) *difficult to learn*." SeP means "no S is any P"; SoP, "some S is not any P"; SaP, "every S is some P"; SiP, "some S is some P."

The rules given for the distribution of terms hold for propositions of A, E, I, and O form. Instead of complicating the rules, it would seem more convenient to accommodate certain other forms to them. Consider: "All equiangular triangles are all equilateral triangles." But "all S is all P" may be handled as two A propositions: SaP and PaS. Again, "some of those on business are all those admitted." This form, "some S is all P," as a variant of the exclusive proposition, is equivalent to PaS. Opposed to it is the negative statement: "None of those on business are some of those admitted," an instance of the form: "No S is some P," or "not only S is P." This may be translated into the form: PoS. Finally, there is the unimportant form: "Some S is not some P." For example, "some of our wealthy friends are not some of our influential friends."

Let the reader therefore, at this point, impress upon his memory the rules: Universal propositions distribute their subject terms; particular propositions leave their subject terms undistributed. Negative propositions distribute their predicate terms; affirmative propositions leave their predicate terms undistributed.

DRILL

Explain and illustrate: exclusive propositions; exceptive propositions; the distribution of subject and of predicate terms in A, E, I, O propositions. (See Exercises VII, 29-40.)

29. COMPOUND PROPOSITIONS

We may connect two or more affirmations or denials in compound propositions. Of the latter, we may distinguish four main types, according as the relation between the constituent propositions is expressed by such words as: and; if-then; either-or (or both); either-or (but not both).

"The brave gave their lives, and the Quislings enriched themselves." This is a conjunctive proposition (p and q), in which one conjunct (p) is related to another (q) by means of the conjunction "and." The conjuncts are taken not separately but together; so that inferences from a conjunctive proposition may naturally be other than those drawn from a conjunct alone. If either conjunct is false, the conjunctive is itself false. The two simple propositions p and q , taken affirmatively and negatively, enter into four distinct conjunctive forms: p and q ; p and not- q ; not- p and q ; not- p and not- q .

In "if-then" or hypothetical propositions (if p then q), a statement is made subject to a condition. "If we really want universal peace, then we must be prepared to pay a high price for it." "If we had no imagination, we would never be discouraged." Here a connection is asserted such that the "if" clause, or the antecedent, cannot be true without the "then" clause, or the consequent, being true also. When the relation expressed is circumstantial, coincidental, spatio-temporally restricted, the proposition may be said to be contingent-conditional. "If it rains, the scheduled open-air concert will be given in the town hall." Hypothetical propositions may, however, formulate an inherent or necessary relation, a meaning in a system of meanings; in which case they are abstract-universal hypothetical propositions. "If a creature is human, it is mortal." "If two rectangles

have their corresponding sides equal, then their areas are equal." Often the context alone discloses how a hypothetical proposition is to be classified in this respect. "If a commodity is scarce, it is costly." Incidentally, hypothetical propositions, like: "If children are spoiled, they are nuisances to themselves and to others," are readily translatable into categorical form: "All spoiled children are nuisances to themselves and to others."

A disjunction, indicated in an "either-or" proposition, may be loose or strict, according as "either-or" is taken inclusively in the sense of *vel*: "and perhaps both," or exclusively in the sense of *aut*: "but not both." The saying that "X is either a fool or a knave," does not rule out the possibility that he may be both. Such propositions may be called "alternative" (either p or q), each constituent proposition being designated as an "alternant." The alternatives presented are compossible. "Either this man is a college graduate, or he reads widely." At least one of the alternants is declared to be true.

When "either-or" is intended to signify: "but not both," the proposition is strictly "disjunctive" (not both p and q); and each constituent proposition is known as a "disjunct." "Arches are either circular or pointed." This is tantamount to the denial of the conjunctive proposition: "Arches are both circular and pointed." In a disjunctive proposition, at least one of the disjuncts is declared to be false. "Your reasons reflect your convictions or are insincere"—but not both. Disjunctive propositions are matter-of-fact or contingent when they are materially conditioned, so that the exhaustiveness of the disjunction may be or at some time become open to question. "We must have a leader, or our affairs will be mismanaged." In abstract-universal disjunctive propositions, valid disjunctions are

necessarily exhaustive. "Rectilineal figures are triangles or quadrilaterals or polygons."

Our brief survey of compound propositions enforces an important observation: two things in some ways alike may be in other ways unlike; just as two things in some ways unlike may be in other ways alike. "If-then" may express a relation of antecedent and consequent: "If iron is impure, it is brittle"; where the evidence is factual. Or "if-then" may express a relation of premises and conclusion: "If impure iron is brittle, and this is impure iron, then this is brittle." Here the conclusion follows from the premises because of the ways in which the terms are interrelated. The use of "if-then" to express both relations, that between antecedent and consequent and that between premises and conclusion, brings out a common trait: neither relation holds when the "if" clause is true and the "then" clause is false.

Modern logic has brought into prominence the distinction between "strict" and "material" implication. When I say that, "if (p) you swallow this water, then (q) it will quench your thirst," I intend to assert a relation of meaning between water swallowed and water quenching thirst; a formal implication such that it is impossible for the implicating proposition to be true and the implied one false; a connection between the three cases in which p is true and q is true, or p is false and q is true, or p is false and q is false, such that, if one of these cases does not happen, one of the others is bound to happen.

However, when I say that, "if (p) Hitler was democratic, then (q) I'll eat my hat," or, "if (p) twice two is four, then (q) Dreyfus was innocent," I simply adjoin p and q without asserting a connection between them. I consider p and q , not so much for any specific meaning, but

with respect to their truth-values. I state that either p happens to be false or else q happens to be true; or that p happens not to be true with q at the same time false. Consequently, I am saying that "if p then q " holds in this sense in any one of the three cases: p is true and q is true; p is false and q is true; p is false and q is false. On this interpretation so long as p is false, p materially implies q , and a false proposition (e.g., "Socrates was a good diviner") implies any proposition (e.g., "Socrates had enough religion for his own use"); and, so long as q is true, q is materially implied by p , and any proposition (e.g., "the speaker is not right") implies a true proposition (e.g., "he is very persuasive").

These and further considerations pertaining to compound propositions may be traced at the hand of "truth tables" (where T means true, and F means false). Let 1 represent: p and q ; 3, q true; 5, p or q ; 7, q implies p ; 9, p implies q ; 11, p is equivalent to q (or, p implies q and q implies p ; or, if and only if p then q); 13, p true; 15, an "always true" formula, sometimes called a "tautology," which would be a "law" of the "system" (e.g., if p implies q , then q false implies p false). Even-numbered columns may be added for negations of these statement-forms: 2, for: not both p and q (reading: FTTT); etc.

p	q	1	3	5	7	9	11	13	15
T	T	T	T	T	T	T	T	T	T
T	F	F	F	T	T	F	F	T	T
F	T	F	T	T	F	T	F	F	T
F	F	F	F	F	T	T	T	F	T

Consider also whether: if p and q imply r , then p implies either q false or r true. (Example: p , "Men believe absurdities"; q , "Men commit atrocities"; r , "Men need a genuinely liberal education.") Let 1 represent: p and q ;

HOW MAN THINKS

2, q false; 3, either q false or r true; 4, p and q imply r ; 5, p implies either q false or r true; 6, the statement-form to be verified.

p	q	r	1	2	3	4	5	6
T	T	T	T	F	T	T	T	T
T	T	F	T	F	F	F	F	T
T	F	T	F	T	T	T	T	T
T	F	F	F	T	T	T	T	T
F	T	T	F	F	T	T	T	T
F	T	F	F	F	F	T	T	T
F	F	T	F	T	T	T	T	T
F	F	F	F	T	T	T	T	T

DRILL

Analyze the structure of conjunctive, hypothetical, alternative, and disjunctive propositions. What is meant by a truth-value system? (See Exercises VII, 41-52.)

The Opposition of Propositions

30. CONTRARY OPPOSITION

Imagine a four-cornered conversation on the question: Is war justified? Four interlocutors contend, respectively: All wars are justified (SaP) ; no wars are justified (SeP) ; some wars are justified (SiP) ; some wars are not justified (SoP). How are these propositions interrelated?

If we accept any one of these declarations as true or reject it as false, we are committed to the truth or falsity of one or more of the others. Only in certain cases is the truth-value of others undetermined. Once we have taken a positive or negative stand, we are no longer free to take what attitude we will to any and every other suggestion.

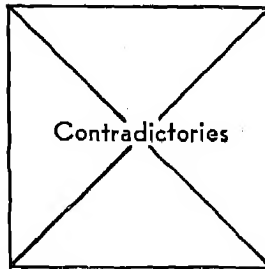
In the illustration chosen, all propositions have the same subject and predicate, but differ in quantity or quality or both. There is among them no pair in which one is independent of the other ; nor a pair in which one is equivalent to the other. Comparing them in pairs leads to the recognition of five (of the seven possible) logical relations between propositions ; as brought out in the so-called "square of opposition."

HOW MAN THINKS

Superaltern SaP

Contraries

SeP Superaltern



Subaltern

SiP

Subcontraries

SoP

Subaltern

What is the logical relation between an affirmation and a denial both of which are universal; namely, in the forms SaP and SeP? Are all wars, or none of them, justified? If SaP is true, SeP is false; and if SeP is true, SaP is false. But if SaP is false, SeP is undetermined; and if SeP is false, SaP is undetermined. Accordingly, SaP and SeP are contraries. *Of contraries, both cannot be true together; but both may be false together.* The same comments are pertinent to: "All virtue can be taught"; "no virtue can be taught."

Similarly, there are contrary compound propositions. "My car is either in the garage or in the parking-lot"; "it is neither in the garage, nor in the parking-lot, nor in my brother's keeping." We cannot maintain both of these statements. Yet we may reject both; for the car may actually be in my brother's keeping. In general, contrary opposition obtains between an alternative proposition, "either p or q ," and a conjunctive proposition of the form: "neither p nor q but r ."

Further examples of contrariety may fall under our notice. To list a few: I am a plutocrat; I am a pauper. You have earned a mark of "A"; you have earned a mark of "B." Paul wrote the Epistle to the Hebrews; Barnabas wrote

the Epistle to the Hebrews. This book is in English; this book is in Spanish. It is not possible (not contingent, impossible) that X be not elected; it is not possible (not contingent, impossible) that X be elected. It is necessary that X be elected; it is necessary that X be not elected. It is necessary that every man be a logician; it is impossible that any man be a logician.

What part do contrary propositions play in our inquiries? Examine again the universal propositions in our original illustration: All wars are justified; no wars are justified. Contraries like these set the limits within which inquiry proceeds to more adequate affirmations and denials. Hence propositions so related are not final, but give direction to further observation and reflection. Did we not formulate opposed universal affirmations and denials, our thinking would be without boundaries to control it. But, in holding to them as final, we should cover the same ground again and again, without coming to a conclusion; for, although both cannot be true together, they may very well both be false together. Hence the need of formulating contraries at an early stage of an inquiry.

A wise mother had, for the benefit of her quarrelling sons, placed a motto on the wall: "Brother! Brother! We are both in the wrong!" This genuine possibility is often overlooked in controversies. Refuting our opponent does not suffice to proving ourselves in the right. Is human conduct determined by heredity alone or by the environment alone? Or does the issue, so stated, not rather call for study of specific conditions, out of which may grow a better articulation of our theory? Is "the truth, the whole truth, and nothing but the truth" with either radicals or reactionaries as if we must once for all come to a decision between them? Or ought not whatever may be worthwhile in the protesta-

tions of both be woven together into a point of view which may do justice to a wider set of relevant facts?

DRILL

Enumerate the seven possible logical relations between propositions. Which five are represented in the "square of opposition"? Define and illustrate contrary propositions, and note their role in inquiry and in controversy. (See Exercises VIII, 1-10.)

31. SUBCONTRARY PROPOSITIONS

How are an affirmation and a denial related when both are particular; namely, in the forms SiP and SoP? To exemplify: Some wars are justified; some are not. If SiP is false, SoP is true; and if SoP is false, SiP is true. But if SiP is true, SoP is undetermined; and if SoP is true, SiP is undetermined. Accordingly, SiP and SoP are subcontraries. *Of subcontraries, both cannot be false together; but both may be true together.* Take also: "Some virtue can be taught"; "some virtue cannot be taught."

"Some," in this context, does not mean "not more nor all." If such were our intention in saying: "Some politicians are honest," we would be implicitly declaring at the same time: "Some politicians are not honest." Careful interpretation suggests that, in predicating honesty of some politicians, we are leaving the character of the others unaffected. Unless further qualified, "some" is, in other words, to be understood in its minimum sense of "some and perhaps (or perhaps not) more or all."

Additional examples of subcontraries would include such as the following: I agree with some of your views; but not with others. Hydrogen is not the lightest element; neither is helium. There is a man in this room who over-

awes us; there is a man in this room who does not overawe us. It is possible (contingent, not impossible) that X be elected; it is possible (contingent, not impossible) that X be not elected. It is not necessary that X be not elected; it is not necessary that X be elected. It is possible for some man to be a logician; it is possible for some man not to be a logician.

Like contraries, so subcontraries are not final in inquiry; although the latter are indeed more determinate than the former. While summarizing partial results, subcontraries raise the problem of determining the conditions under which both may be true.

Controversialists frequently misconceive subcontraries as contraries or even as contradictories. Is learning a matter of interest or of discipline? Shall we be guided by the will of the middle or of the lower classes? Is our way of life to be individualistic or collectivistic? Are tariffs a benefit or a detriment to a nation?

DRILL

Define and illustrate subcontrary propositions, and note their role in inquiry and in controversy. (See Exercises VIII, 11-18.)

32. SUPERALTERNS AND SUBALTERNS

After contraries and subcontraries, we may appropriately discuss the relation between universal and particular propositions of the same quality. Here again, "some" means not "some only" but "at least some." To maintain, therefore, that "some wars are justified," is to leave open the question whether "all wars are justified." So, to take the stand that "some virtue cannot be taught," does not commit one to the conviction that "no virtue can be taught."

A universal proposition is said to be superaltern to the same proposition restricted to the particular; the latter, subaltern to the former. Now if SaP is true, SiP is true; but if SaP is false, SiP is undetermined. Similarly related are SeP and SoP. In general, *if the superaltern is true, the subaltern is true; but if the superaltern is false, the subaltern is undetermined.* On the other hand, if SiP is false, SaP is false; but if SiP is true, SaP is undetermined. So for SoP and SeP. Accordingly, *if the subaltern is false, the superaltern is false; but if the subaltern is true, the superaltern is undetermined.*

Among transformations falling under the relation of super- and sub-alternation are those known as "inference by added determinants" and "inference by complex conception." Subject and predicate are limited by the same determinant when, from the inclusion of bankers in financiers, we deduce the inclusion of American bankers in American financiers. If a writer is a human being, it does not follow, however, that a first-rate writer is a first-rate human being; for the added determinant must have the same meaning throughout the same argument. Analogously, subject and predicate become parts of a more complex conception when we say that, inasmuch as the United States is a democracy, the constitution of the United States is the constitution of a democracy. But the validity of this type of argument also depends upon the caution of applying the same standard throughout: it would be fallacious to take for granted without evidence that, because journalists are informed, the élite of journalists therefore make up the élite of those informed.

Turning our attention briefly to valid syllogisms, we observe that their conclusions are subaltern to the respective premises and their premises are superaltern to the respective conclusions. These relations may be verified in

the abbreviated syllogism: "Philosophers need to promote philosophy; for all men need to promote philosophy."

Further reflection on examples previously given of contraries and subcontraries will lead to the recognition of additional types of propositions related as superaltern to subaltern. It is not possible (not contingent, impossible) that X be not elected (or, it is necessary that X be elected); it is possible (contingent, not impossible) that X be elected (or, it is not necessary that X be not elected). It is not possible (not contingent, impossible) that X be elected (or, it is necessary that X be not elected); it is possible (contingent, not impossible, that X be not elected, (or, it is not necessary that X be elected). It is necessary that every man be a logician; it is possible for some man to be a logician. It is impossible that any man be a logician; it is possible for some man not to be a logician.

In inquiry, a proposition of the form of "some" stimulates the thinker to a course of procedure by which he may render more precise his statement of a proposition in universal form. One who insists that "some Anglicans are Protestants" may be reminded why, at least in a Romanist's view, "all Anglicans are Protestants"; and the truth of the superaltern implies the truth of the subaltern. But, more significantly, a testator, supposing he may bequeath his property to whom he pleases, may find himself mistaken because a will which fails to provide for his wife is defective. This discovery may prompt him to ask: Must I then provide for all my close relatives?

A confusion, common in controversies, occurs when two propositions not equivalent are held mutually opposed or exclusive. By saying that statesmen are interested in political problems, one does not necessarily deny that citizens generally share that interest. The two are not identical,

of course; but they are not therefore incompatible. The subject of the subaltern is, in the case before us, a part of the whole which is the subject of the superaltern.

DRILL

Define and illustrate superaltern and subaltern propositions, and note their role in inquiry and in controversy. (See Exercises VIII, 19-26.)

33. CONTRADICTORY OPPOSITION

What logical relation holds between propositions which differ both in quantity and quality? All wars are justified; some are not. Or: No wars are justified; some are. If SaP is true, SoP is false; if SoP is true, SaP is false. If SaP is false, SoP is true; if SoP is false, SaP is true. SaP and SoP, then, contradict each other; as do also SeP and SiP. *Of contradictories, both cannot be true together; and both cannot be false together.* Mutually contradictory therefore are also: "All virtue can be taught," and "some virtue cannot be taught." Likewise, "no virtue can be taught," and "some virtue can be taught."

In order to contradict a singular proposition with a single subject and predicate, we need but change its affirmative or negative quality: "Socrates is wise"; "Socrates is not wise." Similarly, a proposition in which a universal subject term is taken as one: "Man is the noblest of creatures"; "man is not the noblest of creatures."

The contradictory of an alternative proposition: "either p or q ," is a conjunctive proposition in which the conjuncts contradict the corresponding alternants: " p false and q false" (or: "neither p nor q "). How then would we contradict the contention: "Either all the measures proposed will be adopted, or all of us will go down into destruc-

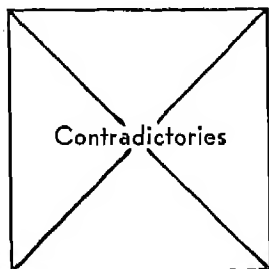
tion"? The contradictory is: "Some of the measures proposed will not be adopted, and some of us will not go down into destruction."

Another form of this relation is the contradiction of the conjunctive: " p and q ," by the alternative: "either p false or q false." Hence, to contradict: "All he writes appeals to the eye and to the ear," we would declare: "Either some of his writing does not appeal to the eye, or some of his writing does not appeal to the ear."

Some examples, previously given in other arrangements, may be brought together in a square of opposition; where the diagonals represent contradiction.

(1) For x to be,
is necessary

(2) For x not to be,
is necessary



(3) For x not to be,
is not necessary

(4) For x to be,
is not necessary

Equivalent propositions which may find a place in this figure would include, respectively: (1) for x not to be, is not possible (not contingent, impossible); (2) for x to be, is not possible (not contingent, impossible); (3) for x to be, is possible (contingent, not impossible); (4) for x not to be, is possible (contingent, not impossible).

The relation of contradiction holds also between the statements that "it is necessary that every man be a

logician" and that "it is possible for every man not to be a logician"; as well as between the statements that "it is impossible that any man be a logician" and that "it is possible for some man to be a logician." Likewise mutually contradictory are the statements that "it is necessary that some man be a logician" and that "it is possible for every man not to be a logician"; as well as the statements that "it is impossible that some man be a logician" and that "it is possible for every man to be a logician."

Wherein lies the importance of contradictories within the process of inquiry? Of course, a flat contradiction does not bring an inquiry to an end. The discovery of negative instances rather leads to an attempt to revise the generalization contradicted. But contradictory propositions, of which one must be true and the other false, are more determinate than are contraries and subcontraries.

Moreover, a debater who contends for a universal proposition risks the necessity of explaining away or disproving every exception to which attention may be called. He is therefore on safer ground when he commits himself to a particular proposition which can be refuted only by a universal proposition of opposite quality. By the same token, an opponent will find it easier to contradict the universality of a statement than to oppose to it a contrary, or a universal proposition of opposite quality.

DRILL

Define and illustrate contradictory propositions, and note their role in inquiry and in controversy. (See Exercises VIII, 27-39.)

Equivalent and Independent Propositions

34. EQUIVALENT PROPOSITIONS

John Calvin was impressed by the doctrine that many are called but few chosen. According to Charles Darwin, there is a struggle for existence in which the fittest survive. Are these teachings opposed? Occasions arise when we properly point out differences in modes of expression and in their settings and backgrounds. But how can we avoid futile controversy when we forget that differing utterances may convey essentially the same idea? Although each of the sayings cited may serve different purposes in different connections, both strike a common human note, are relevant to a single universe of facts, and felicitously communicate the same fundamental observation.

Most of us have a decided preference for our own ways of saying things. Care in observing the niceties of language is indeed a desirable virtue. But, unfortunately, we are also often unwilling to examine to what extent different vernaculars may agree in the meanings they are intended to express. Despite variety in their formulation, *two propositions are equivalent in truth-value when, if either is true, the other is true; and if either is false, the other is false.*

A valuable logical formula is that for equivalent compound propositions. Consider the hypothetical: "If you drop that bomb, there will be an explosion." We cannot take the "then" clause to be false without taking the "if" clause to be false also. Hence we would have to say: "If there is no explosion, you have not dropped that bomb." These two equivalent hypothetical propositions are said to be contrapositives of each other: if p then q ; if q false then p false. Instead of making our original statement subject to a condition, we may make it subject to alternatives: "Either you do not drop that bomb, or there will be an explosion." What we are now saying is that we must accede to at least one of the alternants: either p false or q true. Nor can we at the same time take our original "if" clause to be true and our original "then" clause to be false. Accordingly, the disjunctive is declared true: "It is not the case that both you drop that bomb and there will not be an explosion." At least one of the disjuncts is false: not both p true and q false. The four compound propositions given are equivalent. "If p then q " is equivalent to: "if q false then p false," which is equivalent to: "either p false or q true," which is equivalent to: "not both p true and q false." Let us bring this formula and a fresh example together into a table.

If p then q :	If a democracy is to be noble, its citizens must have virtue.
If q false then p false:	If the citizens do not have virtue, a democracy will not be noble.
Either p false or q true:	Either a democracy will not be noble, or its citizens will have virtue.

Not both p true and q false:	It is not the case that both a democracy will be noble and its citizens will not have virtue.
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Each of the compound propositions may, of course, be contradicted or declared false; and these contradictories will be equivalent to each other. The contradictory of: if p then q , is equivalent to the contradictory of: if q false then p false, which is equivalent to the contradictory of: either p false or q true, which is equivalent to: p true and q false. The last is a conjunctive proposition.

Several statements are made conjointly when it is said that "my friend was sent to the hospital yesterday in order to be treated for a broken leg." The contradictory of this conjunctive proposition is therefore: "Either my friend was not sent to the hospital, or he was not sent yesterday, or he was not sent to be treated, or he was not sent to be treated for a broken leg."

As previously noted, the denial of a conjunction: "We shall all lose money but not our reputations," is an alternative with the elements separately denied: "Either some of us will not lose money, or some of us will lose our reputations." The denial of an alternative: "Either he marries the girl, or he will look like a fool," is a conjunction with the elements separately denied: "He need not marry the girl nor look like a fool."

DRILL

Define and illustrate equivalent propositions. State the formula for equivalent compound propositions. Show how equivalent compound propositions may be contradicted. (See Exercises IX, 1-15.)

35. OBVERSION

When someone makes a significant affirmation, it becomes important to ask: What does he intend to deny? The reverse is equally true. For equivalent affirmations and denials illuminate and re-enforce each other.

In the interest of precision or emphasis, we often, in our ordinary speech, put an affirmation into an equivalent negative form: it is not unlikely, not unnatural, not in vain, not bad, not a small matter, etc. We may give double assurance: "This lesson is not complicated, it is quite easy"; "Everyone was present, not a single one was absent."

If "all physicists are mathematicians," what is the relation between physicists and non-mathematicians? It would be valid to say that "no physicists are non-mathematicians." The two propositions are equivalent, since both declare that no one is a physicist and a non-mathematician. The passage from an affirmation (or denial) to an equivalent denial (or affirmation, respectively) is called "obversion"; the original proposition, the "obvertend"; and each proposition, the "obverse" of the other. While the original subject remains the same, the quality of the proposition is altered (from affirmative to negative, or vice versa); with the result that the predicate term must be changed to its negative or contradictory. For the purpose of obversion is that we may have an equivalent affirmation and denial about the same subject term.

Obversion may be applied to A, E, I, and O propositions. If " a is equal to b ," then " a is not unequal to b ." If "no crows are white," then "all crows are other-than-white." If "some products of art are ornamental," then "some products of art are not non-ornamental." If "some exhorters are not unpopular," then "some exhorters are popular." Hence,

to obvert a proposition, we first change the quality of the proposition and then change the predicate term to its contradictory.

Obvertend:	SaP	SeP	SiP	SoP
Obverse:	SeP'	SaP'	SoP'	SiP'

A careful distinction is to be made between an affirmative or negative proposition and a positive or negative term. In the symbols given above, S and P are positive terms; P' is a negative term ("non-P"); A and I indicate affirmative propositions ("all are," "some are"); E and O indicate negative propositions ("none are," "some are not"). What distinguishes a proposition as affirmative or negative is not the positive or negative character of the terms it contains, but the function of the proposition to affirm or deny a predicate of a subject. Obviously, what is denied must be the negative of what is affirmed; and vice versa.

There is another caution to be observed in obverting a proposition. The predicate term must be changed into its contradictory, not into its contrary. To make sure of this, we may simply add to the predicate term the prefix "non." May we, from: "All crows are black," derive: "No crows are white"? Yes; but this is a one-way process. The two propositions are not equivalent; for, from the latter, we may not derive the former. Black and white are contrary terms, mutually exclusive; whereas black and non-black are contradictory terms, not only mutually exclusive but also exhaustive of the universe of colors. Had we given the correct obverse, that "no crows are non-black," we should have had an affirmation and a denial that are equivalent; we should have been able, from either proposition, to obtain the other. Another illicit obversion would be the passage from: "All

roads lead to Rome," to: "No roads lead elsewhere than to Rome."

Obversion also raises interesting questions about the distribution of terms. In SaP , S is distributed because the affirmation that it is a P is made about every S . The distribution of S has a different meaning in the negation SeP' . The negation may be expanded to read: Since S excludes everything else outside of S , and since P' is somewhere outside of S , S excludes that which is included in whatever is other than S ; namely, P' . Again, in SaP , so far as S is concerned, P has an indefinite range and is therefore undistributed. Not so, however, so far as P' is concerned. In SeP' , therefore, P' is distributed.

DRILL

Define and illustrate: obversion; obvertend; obverse. To what propositions may obversion be applied? How are obvertend and obverse related in truth-value? What cautions must be observed in obversion with respect to the quality of propositions and terms; with respect to the relation between the predicate terms? (See Exercises IX, 16-25.)

36. CONVERSION

Having made an affirmation or a denial about a subject term, what may we affirm and deny about the predicate term? If "no reptiles are feathered," then "no feathered creatures are reptiles." Here subject and predicate are transposed; the quality of the propositions remains the same; the statements made are equivalent. The process involved is called "conversion"; the original proposition, the "convertend"; and each proposition, the "converse" of the other. Like E propositions, so I propositions may be simply con-

verted: for if "some who work in the cities live in the suburbs," then "some who live in the suburbs work in the cities."

May we assert that, if "all wounds are injuries," then "all injuries are wounds"? This transformation would be an illicit conversion. What follows is rather that "some injuries are wounds." Though we may reason from all to all of a term, from some to some of a term, and from all to some of a term, we cannot cogently reason from some to all of a term. *No term is to be distributed in the converse unless that term was distributed in the convertend.* Accordingly, A propositions cannot be simply converted, but are converted by limitation (*per accidens*); their quantity is changed from universal to particular. PiS is primarily the converse of SiP; secondarily, of SaP. The limited converse of A propositions is equivalent to the subaltern of the convertend (e.g., to: "some wounds are injuries").

The reader may object: But may not certain universal affirmations be simply converted? Take: "Every square is a right-angled rhombus; and every right-angled rhombus, a square." Propositions so related are known as reciprocals. We have previously agreed to treat "all S is all P" as two propositions: SaP, and PaS. Note that, whereas a partial converse expresses the same truth expressed in the convertend, a reciprocal expresses another truth, which must be proved on its own account.

As to the conversion of an O proposition, this would require passing from "some S is not any P" to "some P is not any S." But we have ruled out as invalid a passage from some S to all or any S. Accordingly, we may not, from SoP, conclude PoS. In sum, *E and I propositions are converted simply; A propositions are converted by limitation; O propositions cannot be converted.*

We take another step forward in our study of affirming and denying when we learn that every converse has an obverse equivalent to it. In the illustrations given, the obverted converse would be, respectively: "All feathered creatures are non-reptiles"; "Some who live in the suburbs are not those who do not work in the cities"; "Some injuries are not other than wounds."

Convertend:	SaP	SeP	SiP	SoP
Converse:	PiS	PeS	PiS	...
Obverted Converse:	PoS'	PaS'	POS'	...

A very simple type of a pair of equivalent propositions results from "inference by converse relation." If the color of the sky is lighter than that of the ocean, the color of the ocean is darker than that of the sky. Since Chicago is north of New Orleans, New Orleans is south of Chicago. Since two is greater than one, one is less than two. Symbolically, if "a" is related to "b," then "b" stands in the converse relation to "a" ($aRb, \therefore b\bar{R}a$).

DRILL

Define and illustrate: conversion; convertend; converse. Which propositions can be converted; how; with what results in logical relations? Which propositions have an obverted converse? What is inference by converse relation? (See Exercises IX, 26-34.)

37. CONTRAPOSITION

Let us say that there are statesmen (S), leaders (P), non-leaders (P'), and non-statesmen (S'). All four of these terms figure in a set of propositions in which we obvert, convert, and obvert. An affirmation or a denial about non-P, obtained from an affirmation or a denial about S, is known as a "contrapositive."

HOW MAN THINKS

Original (SaP) : All statesmen are leaders.

Obverse (SeP') : No statesmen are non-leaders.

Partial Contrapositive (P'eS) : No non-leaders are statesmen.

Full Contrapositive (P'aS') : All non-leaders are non-statesmen.

Comparing the first and the last of these propositions, we note that the subject and the predicate are contradicted and transposed. Hence the name "full contrapositive." The converted obverse is known as the "partial contrapositive." We may, in similar manner, obtain an affirmation and a denial about non-P if we begin with a negation, whether universal or particular.

Original (SeP) : No men are perfect.

Obverse (SaP') : All men are imperfect.

Partial Contrapositive (P'iS) : Some imperfect beings are men.

Full Contrapositive (P'oS') : Some imperfect beings are not non-men.

Original (SoP) : Some employees are not reliable.

Obverse (SiP') : Some employees are unreliable.

Partial Contrapositive (P'iS) : Some unreliable persons are employees.

Full Contrapositive (P'oS') : Some unreliable persons are not non-employees.

The obverse and the contrapositives of SaP are equivalent to the original; as are also the obverse and the contrapositives of SoP. Since the obverse of SeP is converted by limitation, the contrapositives of SeP, though equivalent to each other, are subaltern to the original and its obverse. The observe of SiP being an O proposition, which is unconvertible, an I proposition has no contrapositive.

In short, A and O propositions are contraposed simply; E propositions are contraposed by limitation; I propositions cannot be contraposed. A violation of these rules would constitute a fallacy of illicit contraposition.

DRILL

Define and illustrate the partial and the full contrapositives of A, E, and O propositions. (See Exercises IX, 35-40.)

38. INVERSION

An affirmation or a denial about non-S, obtained from an affirmation or a denial about S, is known as an "inverse." From SaP: "All statesmen are leaders," we have obtained the full contrapositive P'aS': "All non-leaders are non-statesmen." The latter may be converted into S'iP': "Some non-statesmen are non-leaders," which is the full inverse of the original. By obverting this form, we get the partial inverse S'oP: "Some non-statesmen are not leaders."

So too, from SeP: "No reptiles are feathered," we have derived the obverted converse PaS': "All feathered creatures are non-reptiles." By conversion, "Some non-reptiles are feathered" (S'iP, the partial inverse); and, by obversion, "some non-reptiles are not featherless" (S'oP', the full inverse).

Inversion, whether of an A or an E proposition, is by limitation. If every *s*(ycophant) is a *p*(arasite), then it is some non-*s*(ycophant) who is a non-*p*(arasite). If no *s*(abotage) is *p*(atriotic), then we must look somewhere among acts not *s*(abotage), if anywhere, for something *p*(atriotic). But neither an I nor an O proposition has an inverse; for, in obverting and converting, or in converting and obverting, we soon come to a stop at an O proposition, which cannot be converted.

Do we, from the assertion that "all good men are mortal," derive the information that "some bad men are immortal"? Illicit inversion in this form is due to lack of care in the forming of negative terms. The full inverse is properly: "Some of the things that are not good men are some of the things that are not mortal." Patently, among "things not good men" are "things not mortal," things which do not come into being or pass out of being; e.g., numbers.

Another objection is frequently urged against inversion. In SaP, the predicate term P is undistributed; whereas, in the inverse: S'oP, P has become distributed. But, as previously noted, change in the quality of a proposition requires a different interpretation of distribution. It is indeed the same P that is considered first with S and then with non-S as subject. But, so far as S is concerned, P has an indefinite range; whereas, in relation to non-S, we deal with P as bounded or limited. There is nothing to hinder the same term from being distributed and undistributed in different respects.

Let us summarize in a table the results of alternate obversion and conversion. Reading downward, we begin with obversion; reading upward, with conversion. The process stops when an O proposition is to be converted.

Original	SaP	SeP	SiP	SoP
Obverse	SeP'	SaP'	<u>SoP'</u>	SiP'
Partial Contrapositive	P'eS	P'iS		P'iS
Full Contrapositive	P'aS'	<u>P'oS'</u>		<u>P'oS'</u>
Full Inverse	S'iP'	<u>S'oP'</u>		
Partial Inverse	<u>S'oP</u>	S'iP		
Obverted Converse	<u>PoS'</u>	PaS'	<u>PoS'</u>	
Converse	PiS	PeS	PiS	
Original	SaP	SeP	SiP	<u>SoP</u>

DRILL

Define and illustrate the partial and full inverse of A and E propositions. Review obversion, conversion, contraposition, and inversion; also formal fallacies of illicit obversion, conversion, contraposition, and inversion. (See Exercises IX, 41-46.)

39. INDEPENDENT PROPOSITIONS

One of the seven possible logical relations among propositions remains to be discussed as such: namely, independence. *Two propositions p and q are independent when the truth-value of neither is in any way determined by the truth-value of the other.* Be it true or false that life will some day be completely explained (p), life as it is lived imposes upon us in any case the necessity of distinguishing right from wrong (q). Whether or not our successes are to be attributed to our own efforts (p), we can hardly blame all our troubles on circumstances beyond our control (q). Regardless of the merits of our arguments (p), these do not of themselves determine the causes that lead to our convictions (q). As here suggested, any two propositions p and q are independent when, regardless of whether p is true or false, it is possible for q to be true or false or undetermined in truth-value.

We have seen that, for premises to imply a conclusion, it must be impossible both for the premises to be true and for the conclusion to be at the same time false. If, on the other hand, it is possible both for the conclusion to be false and the premises true, then the premises do not imply the alleged conclusion. By means of this simple logical principle, a Euclidean assumption, that through a point outside of a given line only one line may be drawn parallel to the

given line, has been shown to be independent of other Euclidean assumptions. To be sure, denial of that postulate left intact certain Euclidean theorems: that the base angles of an isosceles triangle are equal, etc. But the denial also implied other theorems incompatible with Euclidean theorems: with the theorem that the angle sum of a triangle is equal to two right angles, etc.

If a set of consistent propositions P imply a proposition Q , then the propositions P together with a proposition contradictory or contrary to Q are inconsistent with each other. If the inconsistency appears in the set of axioms, then Q is not independent of P . Similarly, if the inconsistency appears in the incompatibility of some properly derived theorem with some axiom or other properly derived theorem. On the other hand, if P does not imply Q , then P together with the contradictory or contrary of Q form a consistent set. Such, briefly stated, is a test of the independence of propositions.

DRILL

Define and illustrate propositions logically independent. Review the seven possible logical relations among propositions. (See Exercises IX, 47-50.)

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Supporting Our Conclusions

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Categorical Syllogisms

40. SYLLOGISTIC STRUCTURE

"John is unstable." On what evidence? "He is double-minded." What is the basis of the inference? "All double-minded persons are unstable." And have we thus proved our allegation? No; but we have given grounds for it, exhibited assumptions on which the reasoning rests, stated conditions underlying the judgment we have pronounced. Affirmation or denial calls for such support. Hence we look for premises which may warrant a conclusion.

To analyze the argument given, we may reproduce it in reverse order: If (or since) "all double-minded persons are unstable" (p), and if (or since) "John is double-minded" (q), then (or therefore) "John is unstable" (r). Thus we have applied a rule (p) to a case (q) with a result (r). Granted the truth of the premises (p and q), the truth of the conclusion (r) necessarily follows; although the conclusion, being subaltern to the premises, does not prove the premises true.

Whether the statements made in such an argument are true, or are but supposed to be true, does not affect the validity of the inference taken by itself. "A syllogism," says Aristotle, "is an argument in which, certain things being laid down, something other than what is laid down follows necessarily because they are what they are."¹ In particular,

¹ Aristotle *Prior Analytics* i.1.24b18-20.

a categorical syllogism is one made up of categorical propositions (of the form A, E, I, or O).

Examine the argument: "Knowledge is power; for knowledge is foresight, and foresight is power." The conclusion is readily recognized, since it may be introduced by "then" or "therefore"; as the premises may be introduced by "if" or "since." The subject term of the conclusion is called the minor term (S); the predicate term of the conclusion, the major term (P); the term which does not get into the conclusion but is common to the premises, the middle term (M). The premise which shares with the conclusion the minor term is the minor premise; the premise which shares with the conclusion the major term, the major premise.

In the conclusion, a minor and a major term are compared as to whether they agree or disagree in some respect. The comparison is supported in the syllogism inasmuch as each of these terms is, in the premises, compared with the middle term. By means of the middle term therefore, the minor and major terms may be said to be brought together in the conclusion. Each being related in some way with the middle term, there results a certain relation between the minor and the major terms themselves, which is asserted in the conclusion. So in the syllogism: "Birds are (totally included in) bipeds, and robins are (totally included in) birds; hence robins are (totally included in) bipeds." A syllogism therefore requires three terms, each of which is used twice, and two relations, which are combined in the conclusion.

$$\begin{array}{l} MR_2P \\ SR_1M \\ \therefore S(R_1R_2)P \end{array}$$

Where valid reasoning employs more than three terms, it may be either transformed into a regular categorical syllogism or analyzed according to rules other than those pertaining to the regular syllogistic form. "If acids are compounds, and hydrogen is an element, then hydrogen is no acid." But, by obverting the minor premise to read: "Hydrogen is not a compound," we obtain a syllogism in the regular form. Hardly anyone would expect to obtain a syllogistic conclusion from premises with four insufficiently related terms: "George Washington was the Father of his Country, and Abraham Lincoln was the Great Emancipator." Yet the fallacy of four terms does occur in the somewhat more subtle form of the fallacy of an ambiguous middle: "Killing (in the sense of murder) is wrong; what then about the killing done by a soldier on the field of battle?"

The validity of a syllogism is not affected if we alter the usual order of major premise, minor premise, and conclusion. But care must be taken in the proper identification of terms and propositions in the syllogism. Nor does an inference become invalid when a syllogism is abbreviated by the omission of the major premise, or of the minor premise, or of the conclusion. In such abbreviated or incomplete syllogisms, we have what is traditionally called an "enthymeme" of the first, or second, or third order, respectively. Examples: "Our fathers were members of the same class and are therefore probably acquainted." "All bodies attract; therefore the earth attracts." "Anyone who professes not to believe anything indulges in make-believe, and Descartes at one time professed not to believe anything."

Although we ordinarily shorten our arguments by suppressing one or another part, putting them into syllogistic form is an aid in articulating and testing our reasoning.

Sometimes indeed our reasoning spontaneously takes the form of a syllogism: "The entire second regiment was taken prisoner; my cousin was enlisted in the second regiment; therefore my cousin has probably been taken prisoner." "I shall never be a successful banker, for I do not think in financial terms, and that is a prime necessity of bankers."

DRILL

Analyze the structure of a categorical syllogism, showing the interrelation of its parts. Explain the fallacy of ambiguous middle. What is an "enthymeme" (in the traditional sense)? (See Exercises X, 1-20.)

41. SYLLOGISTIC RULES

May we base the judgment that "X is a liberal" on the ground that "he (is one who) holds certain opinions" and that "all liberals (are some who) hold such opinions"? Indeed not! It is quite possible for X to share some opinions of liberals without being one of them. To warrant his inclusion in liberals, we must compare both X and liberals with the same part of those who hold the opinions in point. This the argument does not guarantee, since it leaves the middle term twice undistributed. Hence the rule: *The middle term must be distributed at least once.* To violate this rule, is to commit the fallacy of undistributed middle. An exception to the rule is an argument in the form: Most M is P, most M is S, therefore some S is P. Here both S and P are compared with the same M's, at least in part. "If most social relations are affected by government, and most of them are non-political, then some non-political relations are affected by government."

Our study of conversion has familiarized us with another rule, which we may here repeat in slightly altered

form: *No term is to be distributed in the conclusion unless that term was distributed in its premise.* This rule applies both to the minor and to the major term. A violation of this rule is an illicit process. The fallacy of illicit minor would be committed if it were argued that, "inasmuch as all good citizens are socially minded and are (some of the) regular voters, therefore all (instead of some) regular voters are socially minded." Correspondingly, it is a fallacy of illicit major to argue that, "since perfect men (are some who) can live together without law, and no men are perfect, therefore no men (are any who) can live together without law." As an undistributed middle is too wide in range to serve as evidence, so an illicit minor or major grounds a conclusion upon subject matter in need of supplementation.

Besides rules for the distribution of terms, there are criteria pertaining to the quality and to the quantity of syllogistic premises. First, *from two negative premises, no valid syllogistic conclusion can be drawn.* Agreement or disagreement of two terms cannot be established by their disagreement with a third common term. It would be a fallacy of negative premises to try to demonstrate that Pennsylvanians are Americans because neither are Europeans; or that, for similar reasons, Australians are (or are not) colonists. Note that obversion of the premises would, in these pseudo-syllogisms, yield undistributed middle: Americans are non-Europeans, Pennsylvanians are non-Europeans; etc. Secondly, *if one premise is negative, the conclusion must be negative; and vice versa.* Two terms must differ when one agrees and the other disagrees with a third common term.

So too, *from two particular premises, no valid syllogistic conclusion can be drawn.* A fallacy of particular premises would violate the canon either against negative premises, or against undistributed middle, or against illicit

process. Similar results ensue on transgression of the rule that, *if one premise is particular, the conclusion must be particular, and vice versa*. As in the case of one negative premise, so here the conclusion follows the weaker premise.

To these six rules, which ought to be memorized, we may add that a particular affirmative major premise and a universal negative minor premise do not yield a valid conclusion. For the major term would be undistributed (whether as subject or as predicate) in the major premise, but distributed in the conclusion (as the predicate term of a negative proposition). In fine, there are sixteen conceivable combinations of premises in categorical syllogisms: "aa," "ae," "ai," "ao"; "ea," "(ee)," "ei," "(eo)"; "ia," "(ie)," "(ii)," "(io)"; "oa," "(oe)," "(oi)," "(oo)." The eight indicated by parentheses are eliminated by the general rules of the categorical syllogism.

DRILL

Formulate accurately two syllogistic rules each: for the distribution of terms; for the quality of the premises; for the quantity of the premises. (See Exercises X, 21-27.)

42. SYLLOGISTIC FIGURES AND MOODS

Further light is shed on the general rules when they are applied to the figures and moods of the categorical syllogism. The moods are identified by means of the propositions which constitute major and minor premise and conclusion: aaa, cae, etc. The figures, of which there are four, are determined by the position of the middle term, as shown in the accompanying diagram. (A somewhat distorted W, formed by drawing a line from one M to another, may serve as a crude mnemonic device for the four figures of the categorical syllogism.)

Figure I	Figure II	Figure III	Figure IV
M-P	P-M	M-P	P-M
S-M	S-M	M-S	M-S
<hr/>	<hr/>	<hr/>	<hr/>
S-P	S-P	S-P	S-P

"If every member of a class has (or lacks) a certain property, and certain objects are included in that class, then these objects must have (or lack) that property." It is only in the first figure that both the subject of the conclusion is subject, and the predicate of the conclusion is predicate, in the premises. Under what conditions then can we support a conclusion by predicating P of M and predicating M of S? If, *in the first figure*, the minor premise were negative, the conclusion would have to be negative; the major premise, affirmative. But this would distribute the major term in the conclusion while leaving it undistributed in the major premise. Hence *the minor premise must be affirmative*. The middle term, thus left undistributed in the minor premise, must accordingly be distributed in the major premise. Hence *the major premise must be universal*. By these considerations, all but six moods are eliminated in the first figure. The valid six are: "aaa," "(aai)," "eae," "(eao)," "aai," "eio." The first figure is the only one which can demonstrate a universal affirmative conclusion and conclusions of all the forms A, E, I, and O.

"A certain disease is characterized by such and such symptoms; but, since this disease is not characterized by those symptoms, it is not the disease in question." This is a syllogism in the second figure. It suggests the question: When do we find grounds for a conclusion by predicating M of both P and S? In order that the middle term, both times predicate *in the second figure*, be at least once distributed, *at least one premise must be negative*. The con-

clusion must therefore be negative and accordingly also distribute the major term. In order that the major term may therefore be distributed in the major premise, *the major premise must be universal*. Again but six moods conform to the rules: "eae," "(eao)," "aee," "(aeo)," "eio," "aoo." Demonstrating as it does only negative conclusions, the second figure is valuable for disproof.

The fact that "Masaryk and others were both philosophers and statesmen" may be cited as evidence that "at least some statesmen have been philosophers." Syllogisms in the third figure raise the problem: What conditions must be fulfilled if, in seeking a basis for a conclusion, we predicate both P and S of M? The first rule of *the third figure* may be established in the same manner as the first rule of the first figure. *The minor premise must be affirmative*. Since the minor term is thus left undistributed in the minor premise, it must also be left undistributed in the conclusion. Hence *the conclusion must be particular*. With these rules as criteria, but six moods survive elimination: "[aai]," "iai," "aii," "[eao]," "oao," "eio." The third figure is useful especially if the middle term is a singular term or if we desire to establish an instance by which a universal proposition may be refuted.

Aristotle distinguished but three figures on the basis of the "width" of the middle term: $S < M < P$, $S < P < M$, $M < S < P$. Moods of *the fourth figure* were treated as variations of the first.² To determine a figure by the position of the middle term, involves separate recognition of the fourth figure. To be sure, the value of the latter has been questioned: "Arguments worthy of consideration must occur in ordinary discourse; but, since no argument in ordinary discourse is in the fourth figure, no arguments in the

² *Ibid.* i.7; iii.1.

fourth figure are worthy of consideration." This very argument, however, is in the fourth figure.³ Here we undertake to demonstrate a conclusion by predicating M of P and S of M.

The fourth figure is somewhat more complicated than the rest. An affirmative major premise would leave undistributed the middle term, which must then be distributed in the minor premise. Hence, *if the major premise is affirmative, the minor premise must be universal*. A negative premise would necessitate a negative conclusion, which distributes the major term; so that the latter must then be distributed in the major premise. Accordingly, *if either premise is negative, the major premise must be universal*. An affirmative minor premise leaves undistributed the minor term, which therefore cannot be distributed in the conclusion. Hence, *if the minor premise is affirmative, the conclusion must be particular*. Here too the special rules result in the discarding of all but six moods: "[aai]," "ace," "(aeo)," "iai," "[cao]," "eio."

The five moods in parentheses are weakened syllogisms inasmuch as the conclusion is particular when it may be universal. Take AAI in the first figure: the premises that "all radicals are dangerous," and that "all agitators are radicals," warrant the conclusion that not only "some" but rather "all agitators are dangerous." The four moods included in brackets are strengthened syllogisms inasmuch as one premise is stronger than it need be. This we may glean from comparison with other moods listed for the same respective figure. Various mnemonic lines have been invented for categorical syllogisms in the four figures. The lines given herewith, partly for their antiquarian interest, include the strengthened but not the weakened syllogisms.

³ W. E. Johnson *Logic* II.89.

Barbara, Celarent, Darii, Ferio-*que prioris*;
 Cesare, Camestres, Festino, Baroco *secundae*;
Tertia Darapti, Disamis, Datisi, Felapton,
 Bocardo, Ferison *habet*; *Quarta insuper addit*
 Bramantip, Camenes, Dimaris, Fesapo, Fresison.

Words in italics mean: and (Ferio are) of the first (figure); (are) of the second; the third has; the fourth adds to them. Vowels in the artificial names designate the respective moods. Initial and certain other consonants are explained in the sequel.

DRILL

How may we identify the figures and the moods of categorical syllogisms? (See Exercises X, 28-45.)

43. SYLLOGISTIC REDUCTION

Aristotle based the various forms of the syllogism on a principle which has come to be known as the *dictum de omni et nullo*. The "statement concerning all and none" is to this effect: Whatever we affirm or deny about a term distributed, that we must in like manner affirm or deny about everything contained under that term. More briefly, "that which is in the contained is in the containing."⁴

Let us affirm of all intellectuals that they promote a free mind, and let us include all (or some) teachers among intellectuals; then we shall have to make the same affirmation about all (or some) teachers. Let us, on the other hand, deny of any fanatics that they ought to be followed, and let us include among fanatics all (or some) demagogues; we have then committed ourselves to the negative conclusion that no (or, respectively, that not all) demagogues ought to be followed.

⁴ Father Buffier, quoted in H. L. Mansel *Artis Logicae Rudimenta*, from the text of Aldrich (2d ed. cor. & enl.; Oxford: W. Graham [etc., etc.] 1852), p. xlv.

Essentially, a syllogism identifies, in some respect, two terms with a same third term; or differentiates two terms, of which one is and the other is not, in some respect, identical with a same third term. The syllogism proceeds from one universal term to another which is equally or less extended. Aristotle's *dictum* provides a simple means by which the form of a syllogism may be validated.

This principle can, however, be directly applied only in the first figure—one reason why Aristotle called it the “perfect” or “complete” figure. From this point of view, moods in the “imperfect” figures are to be “reduced” to the first figure. One of two methods is by “direct reduction”: Convert one or more propositions and, in certain cases, transpose the premises. When this is to be done, is specified in the mnemonic lines, where *s* stands for simple conversion (*conversio simplex*); *p*, for conversion by limitation (*per accidens*); *m*, for transposition of premises (*mutare*, or *metathesis praemissarum*); and an initial consonant, for the corresponding mood of the first figure.

Take Festino: “No Frenchmen are Germans; many Parisians are Germans; therefore many Parisians are not Frenchmen.” By converting simply the major premise, we obtain a syllogism in Ferio, to which we may apply the *dictum*: “No Germans are Frenchmen; etc.” So, in Darapti, the minor premise is to be converted by limitation: “All airplane flights are expensive; all airplane flights are risky (some risky ventures are airplane flights); therefore some risky ventures are expensive.” Next, consider an example of Camenes: “All perfectionists are extremists; no extremists are well adjusted; therefore no well adjusted persons are perfectionists.” Camenes calls for transposition of premises and for simple conversion of the conclusion: “No extremists are well adjusted; all perfectionists are extremists; there-

fore no perfectionists are well adjusted." Where conversion of a conclusion is signified in the mnemonic lines, it is, strictly speaking, the new conclusion that is converted for the purpose of demonstrating the original conclusion. A peculiar case is Bramantip: PaM, MaS, SiP; which calls for reduction to Barbara: MaS, PaM, PaS. (Similar to Bramantip are Camestrop and Camenop.)

The second method is "indirect reduction," symbolized in the mnemonic lines by a small *c*: *per contradictionem*. Take Baroco: "Every courteous person shuns gossip, which some church members do not do; some church members, therefore, are not courteous." To an opponent who admits the premises but denies the conclusion, one may reply in this manner: Use the contradictory of the conclusion as a premise of a new syllogism; the opponent would then consistently have to contradict one of the original premises. We thus obtain Barbara: "Since all courteous persons shun gossip, and all church members are courteous, all church members shun gossip." On the assumption that the original premises are true, the conclusion of the second syllogism cannot be true. Consequently, the minor premise of the second syllogism cannot be true; which is to say that the original conclusion cannot be false, but must be true. The principle underlying this demonstration is known as *reductio ad absurdum*.

By means of the same principle, we may demonstrate the validity of Bocardo. If, from (*p*) MoP, and (*q*) MaS, we may infer (*r*) SoP, then we may, from the contradictory of (*r*): SaP, together with (*q*): MaS; infer the contradictory of (*p*): MaP. The principle of *reductio ad absurdum* indeed generalizes the idea of the contrapositive of the hypothetical proposition. The hypothetical proposition: if *a* then *b*, is equivalent to its contrapositive: if *b* false then *a*

false. So the syllogism: if p and q then r , is equivalent to the syllogism: if q true and r false then p false; and is equivalent to the syllogism: if p true and r false then q false.

Direct and indirect reduction of syllogisms were devised to bring all syllogisms immediately under the *dictum de omni et nullo*. However, there is nothing to prevent indirect reduction from being applied to syllogisms other than Baroco and Bocardo. Moreover, the latter may themselves, with the aid of obversion and contraposition, be directly reduced to the first figure: Baroco, to Ferio; Bocardo, to Darii. By choosing appropriate processes, we may indeed, within limits, reduce any mood to almost any other mood.⁵

A recently proposed extension of indirect reduction is the antilogism, an inconsistent triad of propositions formed by the premises of a syllogism and the contradictory of the conclusion: (p) "No bigots are good citizens"; (q) "all rabble-rousers are bigots"; (r) "some rabble-rousers are good citizens." We may, from the antilogism, obtain a triad of valid syllogisms with new conclusions formed by contradicting r , q , and p , respectively. Rules have been formulated for the antilogism on the basis of the symbolic notation: There is no M which is also P ; there is no S which is also non- M ; there is an \bar{S} which is also P . Analyzed in terms of such symbols, an antilogism is composed of two universal propositions and one particular; the two universals have a common term, which is once positive and once negative; the particular proposition contains the other two terms. Apart from strengthened and weakened syllogisms, which require other assumptions and conditions, a syllogism is valid if it corresponds to an antilogism whose structure conforms to the three conditions stated.

⁵ On the theory of "modal" syllogisms (concerning the necessary, possible, and impossible), see C. Prantl *Geschichte der Logik* (Leipsic: Gustav Fock 1927) I.278-94.

DRILL

Explain and illustrate: *dictum de omni et nullo*; direct reduction of a categorical syllogism to the first figure; indirect reduction, or *reductio ad absurdum*. (See Exercises X, 46-65.)

Conditional Syllogisms

44. HYPOTHETICAL SYLLOGISMS

An often-quoted saying has it that, "if the blind lead the blind, they will both fall into the ditch." On this assumption as a major premise, it may be suitable to argue: "Here is one blind man leading another; they will surely come to grief." Or we may properly reason: "The expected disaster is not menacing these men; hence their association must be other than that of blind leader and blind follower." There is no warrant, however, for the inference: "Since these people have fallen into a ditch, they must have been bound together as blind followers of a blind guide." Nor could we rightfully maintain: "Inasmuch as we are neither blind nor led by the blind, we shall escape the threatened calamity."

Another possible mode of reasoning, on the assumed major premise, would be: "If incompetent and unprincipled officials administer the affairs of indifferent citizens, the blind are leading the blind; therefore if such is the state of a nation, that nation will be ruined." This is a pure hypothetical syllogism, all three of whose propositions are hypothetical. But when a hypothetical major premise is combined with a categorical minor premise and conclusion, as in the previous examples, the argument is called a mixed hypothetical syllogism.

Take another illustration of a mixed hypothetical syllogism: "If men are ignorant concerning war, they cannot expect soon to abolish it; but as yet they do not know enough about it to realize this happy consummation; therefore they cannot reasonably expect soon to abolish war." Having committed ourselves to the truth of the hypothetical major premise and of its antecedent, we are necessarily committed to the truth of the consequent. The actual or supposed, certain or probable, truth of the consequent is indirectly established; namely, as the conclusion of an inference from the actual or supposed, certain or probable, truth of the premises. Schematically expressed, the syllogism is of the form: If p then q ; but p true; therefore q true. A hypothetical syllogism having this form, in which the minor premise affirms the antecedent of the major premise, is said to be constructive. Such a syllogism exemplifies a valid *modus ponendo ponens*: a "mood which, by affirming," in the minor premise, the antecedent of the major premise, "affirms," in the conclusion, the consequent of the major premise. That this mood is valid, may be corroborated by re-statement as a categorical syllogism in Barbara: All cases of p are cases of q ; the present case is a case of p ; therefore the present case is a case of q .

It would be fallacious, however, to argue that, granting the major premise, "if men do not expect soon to abolish war, they are necessarily ignorant about it." What the major premise asserts is not that the truth of the consequent certifies the truth of the antecedent; but rather the reverse. Nor does the major premise given present the condition stated in the antecedent as the only condition under which the consequent is true. Therefore no valid conclusion can be drawn when the minor premise asserts the truth of the consequent of the major premise. Such an argument would

commit the fallacy of affirming (in the minor) the consequent. The inference would exemplify the invalid *modus ponendo ponens*: if p then q ; but q true; therefore p true. This type of fallacy corresponds, in the categorical syllogism, to the fallacy of undistributed middle: Every case of p is a case of q ; the present case is a case of q ; therefore the present case is a case of p .

A valid conclusion may be had from a denial of the consequent (always granting the major premise): "if men reasonably expect soon to abolish war, they have ceased to be ignorant about it." By this kind of inference, we disprove a suggested theory. A hypothetical syllogism of this form, the minor premise denying the consequent of the major premise, is said to be destructive. The form may be represented by the scheme: if p then q ; but q false; therefore p false. This is a valid *modus tollendo tollens*: a "mood which, by denying," in the minor premise, the consequent of the major premise, "denies," in the conclusion, the antecedent of the major premise. The corresponding mood in the categorical syllogism is Camestres: every p is q ; the present case is not q ; therefore it is not p .

The falsity of the antecedent may not, however, be offered as evidence for the falsity of the consequent. We cannot validly infer, on the basis of the major premise laid down, that "human ignorance concerning war has ceased and that we may therefore confidently expect soon to abolish war." May not fuller knowledge, at least conceivably and so far as the premises are concerned, lead to the conviction that war of some kind must be reckoned with? The denial, in the minor premise, of the antecedent of the major premise, so far from establishing a valid conclusion, constitutes the fallacy of denying the consequent. The result is an invalid *modus tollendo tollens*: if p then q ; but p false;

therefore q false. Translation into categorical form shows a correspondence with the fallacy of illicit major: all p 's are q 's; this is not p ; therefore it is not q .

For mixed hypothetical syllogisms, we may then state the rule: The minor premise may *either affirm the antecedent or deny the consequent* of the major premise. The rule is, moreover, directed against the fallacies of affirming the consequent and of denying the antecedent. "If burly sinners run the world, our social problems are indeed far from solution." But it does not follow that removal of reprehensible rulers will automatically solve those problems; or that persistence of social problems proves the unworthiness of rulers. Evidence of bad leadership is ground, on the assumption made, for the inference that the problems will continue; as the solution of the problems would point to the accomplished "liquidation" of the "burly sinners."

How can we tell whether a given pure hypothetical syllogism is valid or invalid? Most readily, perhaps, by transforming it into a categorical syllogism. Take the form: if q then r ; if p then q ; therefore if p then r . This may be translated: QaR ; PaQ ; therefore PaR . By such a method, we may examine: "Unless men improve their thinking, they will believe absurdities; if men believe absurdities, they will commit atrocities; therefore, unless men improve their thinking, they will commit atrocities."

Conditional syllogisms, in general, include hypothetical, alternative, and disjunctive syllogisms, distinguished as such by the forms of the major premise. And principles of conditional syllogisms are combined in the dilemma.

DRILL

What is a pure hypothetical syllogism; a mixed hypo-

thetical syllogism; the constructive mood of the latter; its destructive mood; the fallacy of affirming the consequent; the fallacy of denying the antecedent? (See Exercises XI, 1-14.)

45. ALTERNATIVE AND DISJUNCTIVE SYLLOGISMS

If I promise a friend either to visit him or to write him a letter, I must, in order to keep my promise, perform at least one of these actions; but my promise is not broken if I do both. Failing to receive the visit, my friend has ground for expecting a letter; or vice versa. The visit would not, however, of itself indicate that a letter will not be forthcoming; or vice versa.

Again, popular psychoanalysis often characterizes a person either as an introvert or an extrovert; or, it may be, as both (i.e., an ambivert). With an alternative proposition like this as a major premise we may combine a categorical minor premise and conclusion so as to form a mixed alternative syllogism. Since, according to the major premise, at least one of the alternants is true, but the possibility of both being true is not ruled out, it would not be valid to infer: "Benjamin Franklin was an extrovert and therefore not an introvert." This would exemplify the fallacy of affirming (in the minor) an alternant, or an invalid *modus ponendo tollens*: either p or q ; but p true; therefore q true. A minor premise which, however, denies one or more alternants, will yield a valid conclusion affirming the rest: "Benjamin Franklin was not an introvert; hence he was an extrovert" (provided, of course, that the major premise is true). Accordingly, a valid mixed alternative syllogism is in a "mood which by denying (in the minor premise) affirms (in the conclusion)"; a valid *modus tollendo ponens*. The argu-

ment has the form: Either p or q ; but p false; therefore q true. Mixed alternative syllogisms eliminate suggested possibilities, factors, or hypotheses; so that, provided the alternative premise be true, the desired solution will be found among the remaining possibilities, factors, or hypotheses.

Pure alternative syllogisms occur rarely. To test them, we may translate them into pure hypothetical syllogisms. Take the argument: "Either business is good, or the party in power does not merit support; either business is not good, or prosperity is general; therefore either prosperity is general, or the party in power does not merit support." The pure hypothetical syllogism equivalent to this would be: "If business is not good, the party in power does not merit support; if prosperity is not general, business is not good; therefore if prosperity is not general, the party in power does not merit support." To cite another example: "Either an article is produced in large quantities, or it is expensive; either it is in great demand, or it is not produced in large quantities; therefore either it is in great demand, or it is expensive."

According to a common proverb, "you cannot eat your cake and have it too." This is a strict disjunction, in which one alternative excludes the other. A strict disjunction asserts that at least one of the disjuncts is false. But it would be fallacious, from the falsity of one of the disjuncts, to infer the truth of the other. Such an inference would commit the fallacy of denying a disjunct and would represent an invalid *modus tollendo ponens*: Not both p and q ; but p false; therefore q true. The proper procedure is for the minor premise to assert one or more disjuncts and for the conclusion to deny the rest: "Since I want to hold on to my cake, I must not eat it" (or vice versa). The resulting syllogism represents a valid "mood which by affirming (in

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the minor premise) denies (in the conclusion)"; in other words, a valid *modus ponendo tollens*. This has the form: Not both p and q ; but p true; therefore q false.

The principles by which we distinguish valid from invalid mixed syllogisms are employed in life whenever we reflect. What the logician does is to formulate these principles. In particular, it becomes important, for right reasoning, to make explicit all the premises involved. Let us at this point bring together into a table the symbols summarizing the various forms of the mixed syllogism.

	<u>VALID</u>	<u>INVALID</u>
<i>Modus ponendo ponens</i>	If p then q p $\therefore q$	If p then q q $\therefore p$
<i>Modus tollendo tollens</i>	If p then q q false $\therefore p$ false	If p then q p false $\therefore q$ false
<i>Modus tollendo ponens</i>	Either p or q p false $\therefore q$	Not both p and q p false $\therefore q$
<i>Modus ponendo tollens</i>	Not both p and q p $\therefore q$ false	Either p or q p $\therefore q$ false

One mood of a mixed syllogism may be readily resolved into another. To this end, we may simply combine a minor premise (p) and a conclusion (q) with equivalent major premises: if p then q ; if q false then p false; either p false or q true; not both p true and q false. Thus, a banker deciding to give up his investment business, because of a desire to retain his deposit business, would base his decision upon a major premise expressible in various ways: "If I

want to be a deposit banker, I cannot also be an investment banker." Or, "if I want to be an investment banker, I cannot also be a deposit banker." Or, "I do not want to be a deposit banker, or I shall give up my investment business." Or, "it is not the case that both I can retain my deposit business and not give up my investment business."

DRILL

What is a mixed alternative syllogism; a mixed disjunctive syllogism; the rule for the former; the rule for the latter; the fallacy of affirming an alternant; the fallacy of denying a disjunct? (See Exercises XI, 15-22.)

46. DILEMMAS

"Culture," writes Mr. Santayana, "is on the horns of this dilemma: if profound and noble it must remain rare, if common it must become mean."¹ Obviously, the reference is to a practical dilemma. In order to frame a dilemma in the logical sense, we may take this conjunctive assertion of two hypothetical propositions as a major premise. Let us then, with this major premise, unite a minor which would alternatively assert the antecedents of the major: "Culture either is profound and noble; or is common." The conclusion follows: "Culture must either remain rare; or it must become mean." Thus we obtain a complex constructive dilemma: constructive, because the minor premise alternatively affirms the antecedents of the major; complex, because the conclusion is a compound proposition.

The same major premise may be utilized for a complex destructive dilemma. The latter requires a minor premise which alternatively denies the consequents of the major: "Culture either need not remain rare; or it need not become mean." These premises would yield the con-

¹ George Santayana *Reason in Society*, in *Works* (Triton ed.; New York: Charles Scribner's Sons 1936-40) III.306.

clusion: "Culture is either not profound and noble; or it is not common." In destructive dilemmas, the consequents of the major premise are different propositions; whereas in constructive dilemmas, it is the antecedents of the major premise which are different.

Consider another striking utterance by Mr. Santayana: "Absolute pessimism and absolute optimism are opposite sentiments attached to a doctrine identically the same. In either case no improvement is possible, and the authority of human ideals is denied."² In the terms of a simple constructive dilemma: "If the universe is thoroughly evil, as well as if the universe is thoroughly good, no improvement is possible; but the universe is either thoroughly evil or thoroughly good; therefore, in either case, no improvement is possible." When a simple dilemma is constructive, the conclusion affirms consequents which are identical in the major premise; as, when a simple dilemma is destructive, the conclusion denies antecedents which are identical in the major premise.

The simple destructive dilemma may be exemplified by a famous argument which Zeno of Elea maintained against motion: "If a body moves, it must either move where it is or where it is not; but neither can it move where it is, nor can it move where it is not; therefore, in either case, it cannot move." On the ground that no alternative hypotheses are here entertained, this form of argument may be designated as a hypothetical syllogism with a disjunctive consequent in the major premise. Nevertheless we may define the dilemma so as to include the simple destructive form.

When do we take recourse to dilemmatic reasoning? When we are unable to declare any one of the antecedents true, or any one of the consequents false; but are able to

² George Santayana *Reason in Religion*, in *Works* (Triton ed.; Charles Scribner's Sons) IV.153.

declare alternatively the truth of the antecedents, or the falsity of the consequents. Principles of conditional syllogisms are thus combined in the dilemma. Naturally, proper use of a dilemma depends upon due care in its construction. Yet not every dilemma is fallacious. What we may do in opposing a dilemma is to call into question the truth of its constituent propositions. There are, in general, three possible ways in which we may avoid being impaled on the horns of a dilemma and thus avoid admitting its conclusion.

First, escape between the horns of the dilemma; i.e., object that the alternatives in the minor premise are not exhaustive. The universe, for example, may be neither thoroughly evil nor thoroughly good, but may contain both what is good and what is evil. An appropriate attitude to the universe may therefore be neither an absolute pessimism, nor an absolute optimism; but a relativistic meliorism, in which we seek to increase the good and to decrease the evil.

Secondly, take the dilemma by the horns; i.e., attack the truth of the major premise. Some one or other of the antecedents may have consequences different from those stated. Thus, as against the arguments of Zeno of Elea, Plato pointed out that a top moves in the place where it is; although hardly anyone, including Plato himself, would regard this an altogether adequate refutation.

Finally, rebut the dilemma; i.e., construct a counter-dilemma whose conclusion contradicts the original conclusion. Usually, however, the contradiction is apparent only; and therefore without much value, except for impressing the untrained. Compare with the first example the rebutting dilemma: "If culture is profound and noble, it will be honored; if common, it will raise standards everywhere; therefore, since culture has either the one or the other

characteristic, it will either be honored, or it will raise standards everywhere." The original conclusion would be genuinely contradicted if we said: "At least some culture need neither remain rare nor become mean."

DRILL

What are the four forms of the dilemma; three ways of meeting the dilemma? (See Exercises XI, 23-30.)

47. THE SYLLOGISM IN GENERAL

Does syllogistic inference beg the question? Not if to beg the question is to assume demonstrated what one sets out to demonstrate and if a syllogistic conclusion is drawn from two premises having in common a genuine middle term. It would admittedly be question-begging to argue that all men are mortal, since they are doomed to die. But that is not our procedure when we point to the evidence that all living organisms, including human beings, disintegrate with time. Moreover, it is a misconception that universal propositions can be established only by an examination of all their instances. Such a requirement holds only for enumerative universals: "All the known planets revolve around the sun." The requirement certainly does not apply to a resolution: "All trespassers will be fined"; or even to warranted generalizations: "All men are mortal."

Must we really be acquainted with every triangle in existence before employing the premise that the angle-sum of every triangle is equal to two right angles? Must we have complete insight into every property of a triangle before discriminating a given triangle from other figures? And is our acceptance of premises dependent upon a previous knowledge of the truth of a conclusion; or do we not rather on occasion fail to see just what our premises imply?

At first blush it may seem paradoxical that the conclusion of an inference must be different from the premises, yet must itself be contained in the premises. But, for one thing, people vary as to the exact point at which they distinguish the direct import or conventional meaning of a proposition from its logical implications. Many have agreed that tax-payers are entitled to vote; they have also acknowledged that women are tax-payers; but it has come to them as a distinct surprise to learn they had virtually committed themselves to woman suffrage. Thoughtful inspection of the assumptions of a geometrical system does not at once bring before everyone all their consequences. So the drawing of a conclusion may well be accompanied by the feeling of novelty.

Given premises, on the other hand, do not "contain" a conclusion spatially; they "imply" it rather. The connection is logical and is made explicit through the passage from the premises to the conclusion which necessarily follows from them. The "paradox of inference" loses its seriousness when we reflect that, whatever the inferences we make, there are implications for us to discover.

Syllogisms may indeed be constructed which represent little more than verbal transformations or dialectical jugglery. Yet even dialectic may have its uses. To one who maintains that no generalization can be established, we may reply that this objection (that no generalization can be established) is itself a generalization, which is thus placed beyond the pale of confirmation. Perhaps few will derive much satisfaction from this sort of obvious dialectic; yet by such means it is possible now and then to make a point or to disclose a trap into which we may fall.

It is worthwhile to examine several types of inference closely allied to the syllogism. What Aristotle called an "en-

thymeme" is a *rhetorical syllogism* in which, on the basis of a general probability, a fact is treated as indicating the truth of some further statement: "The envious hate; this man envies; therefore he probably hates." In a *polysyllogism* the conclusion of the supporting syllogism or prosyllogism becomes a premise of the supported syllogism or episyllogism: "Products of art are less remarkable than the human body; inventions are products of art; inventions are therefore less remarkable than the human body; the radio is an invention; the radio is therefore less remarkable than the human body." Stated as a passage from prosyllogism to episyllogism, the reasoning is said to be "progressive" or "synthetic"; as a passage from episyllogism to prosyllogism, "regressive" or analytic." A polysyllogism in which one of the constituents is briefly indicated (as an "enthymeme" in the usual sense) is known as an *epicheirema*: "Inventions are less remarkable than the human body, for they are products of art; the radio is an invention; therefore the radio is less remarkable than the human body."

Abridged syllogisms are said to form a *sorites* (from *soros*, heap) when the conclusion of one syllogism is a premise in another, when two premises contain a common term, and when all the conclusions except the last are unexpressed. To exemplify one form, somewhat inappropriately called the Aristotelian sorites: "The radio is an invention; an invention is a product of art; a product of art reflects human skill; what reflects human skill is less remarkable than a product of nature; hence the radio is less remarkable than a product of nature." When this order of premises is reversed, the chain formed by the abridged syllogisms is called a Goclenian sorites. (Rudolf Goclenius of Marburg, 1547-1628.)

Aristotelian Sorites

All A is B

All B is C

All C is D

All D is E

∴ All A is E

Goclenian Sorites

All D is E

All C is D

All B is C

All A is B

∴ All A is E

Although a sorites may be broken up into its constituent syllogisms and thus tested for its validity, special rules may be given for it. In the Aristotelian sorites, only the last premise can be negative; only the first, particular. In the Goclenian sorites, only the first premise can be negative; only the last, particular. The following is a hypothetical sorites.

If the quantity of money increases, the rate of interest will be reduced;

If the rate of interest is reduced, the volume of investment will be enlarged;

If the volume of investment is enlarged, employment will spread;

If employment spreads, prices will rise;

If prices rise, a given rate of interest may be maintained;

Therefore if the quantity of money increases, a given rate of interest may be maintained.

One type of formal inference which looks like a syllogism, but does not (unless re-formulated) fall under the rules of an ordinary syllogism, is the argument *a fortiori*: "B is greater than C; A is greater than B; therefore A is greater than C." There are numerous other valid arguments containing four terms: "B is a brother of C; A is a brother of B; therefore A is a brother of C." To be sure, we may analyze this as a three-term argument; but then the terms are not connected by means of the copula. If then we class

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such inferences as *relational syllogisms*, it must be acknowledged at the same time that all syllogisms are in one way or another "relational." We are thus led to sketch the elements of a logic of relations.

DRILL

Does a syllogism necessarily beg the question? Explain and illustrate: rhetorical syllogism; polysyllogism; epich-eirema; sorites; *a fortiori* argument. (See Exercises XI, 31-45.)

Formal Relations

48. FORMAL RELATIONS OF TERMS

How one term is related to another, has an important bearing on the validity of an inference. If you are "older than" a friend who is "older than" I, then you are "older than" I. But if you "hold membership" in a local organization which in turn is a "member" of a national organization, it does not follow necessarily that you personally "hold membership" in that national body.

An object is said to be in a relation if, when we make an affirmation or denial about it, we must explicitly refer to another object. Thus, when Reubens is declared to have been a contemporary of El Greco, "being a contemporary of" is a relation which connects "Reubens" and "El Greco." The term from which the relation goes, "Reubens," is the *referent*. The term to which the relation goes, "El Greco," is the *relatum*. According to the number of terms connected, a relation may be said to be dyadic (the Athenians "executed" Socrates), triadic (the Athenians "gave" poison "to" Socrates), tetradic (the Athenians "provided" their jailer "with" hemlock "for" Socrates), etc.

We may also distinguish between concrete "relative" terms which have many connections other than that specified (father of, short, small, near, next) and abstract-universal "relational" terms whose meaning is exhausted in

the specified relation (fatherhood, length, magnitude, nearness, nextness, or, for, minus). From this point of view,¹ factual propositions, holding for specified circumstances of time and place, may be regarded as polyadic. Propositions stating a necessary relation, independently of temporal and spatial reference, would then be considered dyadic: $(x+y)^2 = x^2 + 2xy + y^2$.

Note, in this connection, the numerous logical relations or functions, other than class-inclusion, which the verb "to be" may express. Let *ABC* "be" (identified as) a triangle. Six square yards "are" (equivalent to) 7,776 square inches. Man "is" (by definition) a rational animal. Wine "is" (in one of its attributes) red. There "is" a church-window (in existence). The spiritual "is" (as an intrinsic actuality). This spark "is" (as to its derivation) a fire-fly.²

How can terms be logically ordered so as to yield warranted conclusions? One basis of a valid inference is that a given relation be *transitive*. Persons or objects may be arranged according to comparative degrees in a designated trait, so that "if A ranks higher than B, and B ranks higher than C, then A ranks higher than C"; or according to spatial or temporal succession in a serial order, so that "if A comes after B, and B comes after C, then A comes after C." Similarly, kinds may be determined with respect to inclusive kinds in an order of progression: "If whales are mammals, and mammals are vertebrates, then whales are vertebrates." Also principles underlying such orders may themselves be ordered in a system of transitive relationships; as in a legal system in which acts of "killing" are distinguished according to a graded series of kinds (accident, self-defense, murder, etc.).

¹ John Dewey *Logic*, chap. xvii: "Formal Functions and Canons."

² George Santayana "Some Meanings of the Word 'Is,'" in *Works* (Triton ed.; Charles Scribner's Sons) XIII.277-96.

Relations not satisfying this condition are said to be "intransitive": If A is the "father of" B, and B is the "father of" C, it cannot be concluded that A is the "father of" C; even though "paternity" as such constitutes a system of transitive relationships. In general, specific acts and changes (as when "A killed B") are intransitive until appropriately determined, along with correlated changes, in such systems of relationships as spatial and temporal succession and graded series of kinds. Nor is the condition of transitivity satisfied by relations which are sometimes transitive, sometimes not, and which are therefore said to be "nontransitive." At a given time, the United States may be "allied with" other nations, including Russia, in warfare against Germany, but not so with Russia against Japan.

Certain inferences depend for their validity upon a relation being *symmetrical*; that is, the same as its converse. Thus, in a marriage, each partner is the "spouse" of the other. The relations of total exclusion, in E propositions, and of partial inclusion, in I propositions, are symmetrical; which is why such propositions can be simply converted. Inference by converse relation, however, is based upon a relation of converse symmetry connecting two terms: if A is the husband of B, then B is the wife of A. The relation "husband-wife" is itself "asymmetrical." Relations sometimes symmetrical, sometimes not, are said to be "nonsymmetrical." Thus, one proposition may "imply" another, which may or may not imply the former; and "love" may or may not be requited.

Logical equivalence is grounded upon the conjunction of symmetry with transitivity. Typical expression is given to the transitive symmetrical relativity of terms in the formula: "Things equal to the same thing are equal to one another." By combining the distinctions based upon trans-

itivity and symmetry, we may obtain other types of relations: transitive asymmetrical ("superior to"), transitive nonsymmetrical ("brother of"), intransitive symmetrical ("twin of"), intransitive asymmetrical ("mother of"), intransitive nonsymmetrical ("nearest blood-relative of"), nontransitive symmetrical ("half-brother or half-sister of"), nontransitive asymmetrical ("son-in-law of"), nontransitive nonsymmetrical ("not including").

A third principle by which terms may be ordered in inference and discourse is the scope of the relation connecting the terms. Attention is here paid to the number of objects represented by the referent and by the relatum. The technical name designating this form of order is *correlation*. With this criterion in view, we distinguish: one-one relations (three is "greater by one" than two), one-many relations (A is the "mother" of B), many-one relations (A,B,C are "wives" of a practicing polygamist), and many-many relations (A is a "fellow-citizen" of B). Although the relation of friend to friend is a many-many relation, it is in some cases so ordered as to permit of transitivity: if A is a friend of B, who is a friend of C, then A is a friend of C.

Still another logical property of relational terms is *connexitivity*. This is a complex of formal relations of terms. Thus the asymmetrical transitive relation "greater than," together with its converse "less than," holds between every pair of integers; whereas the relation "greater than by two" does not possess this connexity. Again, when symmetrical terms are also transitive, as in the case of their equivalence, they are "connected" (in a wider sense) so as to allow of movement back and forth in inference and discourse. An asymmetrical intransitive relation, however, is deficient for this purpose. Basic therefore in all modes of logical relation is the function of transitivity.

DRILL

Distinguish types of relations according to the number of terms connected; according as they are concrete or abstract; according to their transitivity, symmetry, scope or correlation, and connexity. (See Exercises XII, 1-20.)

49. FORMAL RELATIONS OF PROPOSITIONS

Relational words may connect one term with another in a proposition. Note how we express relations of inclusion and exclusion in simple affirmations and denials: this body is my own; all bodies seek their appropriate places; no bodies are separable from their environment; some bodies are highly disciplined; some bodies are not gifted with sight; only human bodies are builders of temples.

Relational words may also connect one proposition with another. Take familiar forms of compound propositions: bodily organization and behavior are fascinating objects of study; if human bodies think and worship, then is matter dignified; either physical or mental activities or both are co-ordinated by the body; it is not the case that our bodies can be abused and also function efficiently.

Clearly, there are purely relational words whose whole office it is to institute relations between terms or between propositions. Other words may serve other purposes besides assisting in fixing such relations. In any case, a pointed question arises: Under what conditions may a proposition become a related member in an ordered set or series of propositions? Not all relational terms are equally effective in representing fulfilment of those conditions. Comparison and contrast leading to related affirmations and denials which express inclusions and exclusions as well as, above all, conjunctions and disjunctions—these must be our chief reliance.

To cite an instance of conjunction applied to singulars: "Our executive board is composed of the president, secretary, treasurer, and chairmen of standing committees." The singulars listed compose a collection; their conjunction is additive. Moreover, as "and" expresses a summation, so "or" expresses an alternation: "Any member of our executive board is either the president, secretary, treasurer, or chairman of a standing committee." To gainsay a proposition of this type, we assert that one or more singulars are to be omitted or are to be added.

Also additive is a conjunction applied to kinds. "Calls to order, objections to consideration of a question, and motions for the orders of the day do not need to be seconded." Such a summation may or may not avoid incompleteness or overlapping. So we may re-state it in alternative form: "Motions which do not need to be seconded are either calls to order, objections to consideration of a question, or motions for the orders of the day." Denial of this type of generic proposition applies, not to this or that kind, but to the relation between including and included kinds. Are the sub-kinds mutually exclusive; and the traits describing them, independent of one another? Are the sub-kinds together exhaustive; and do their traits, taken in conjunction, suffice to differentiate one kind from another as well as to determine the inclusive kind?

With reference then to singulars comprising a collection, as with reference to sub-kinds of an inclusive kind, conjunction or disjunction is additive: *a* or *b* or Not so in abstract-universal propositions. When, for example, is an animal a mammal? When it is warmblooded and viviparous and offspring-suckling. These characters are inter-related. Each modifies, and is modified by, the others. In this sense, their conjunction is not additive but multiplica-

tive: *ab*, *ac*, *bc*, *abc*. Hence, when it is said that "to be mammalian is to be warmblooded, viviparous, and offspring-suckling," the relation between subject and predicate terms in this abstract-universal proposition is a necessary relation. To replace "and" by "or" in the definition given, is to pronounce the characters mentioned sufficient and to rule out others as irrelevant and superfluous.⁹

The processes distinguished go hand in hand. Take as a simple illustration the case of a committee organized to iron out factional grievances. It becomes important to determine how the committee is to be constituted as a "collection"; whether each and every faction is represented as a "kind"; and what agreements and differences define the interrelated "characters" of the factions as such. Conceivably, divided allegiances complicate the situation; but there are instances in which this possibility must be eliminated. Certainly, in many scientific inquiries, kinds must be so related that any specimen is of one or another but not of more than one kind. That is why we try to establish both additive and multiplicative conjunctions and disjunctions.

By such a procedure, we obtain a definition that is not arbitrary but logically grounded. The conception to be defined is analyzed into interrelated meanings. Substitution of one term by another is rendered possible in discourse. Whereas repeated observation would raise recurrent questions, final judgment as the outcome of inquiry is based upon reasons. Above all, sets and series of propositions may be ordered in a scientific system so that, taken severally, they are exclusive; taken together, they are exhaustive; taken in their correspondence, they are interrelated.

DRILL

Distinguish additive conjunction and disjunction, as

⁹ Dewey *op. cit.*, p. 340.

applied to singulars and to kinds, and multiplicative conjunction and disjunction in abstract-universal propositions. (See Exercises XII; 21-29.)

50. FORMAL CANONS OF REASONING

Basic to all reasoning are the three so-called "primary laws of thought." Not only do they underlie the relations of terms and of propositions in inference and discourse and in inquiry and proof; they reach, in the nature of things, to the roots of being. These principles, apart from which thought and speech and being itself would become chaotic, have been formulated as the canons of identity, of contradiction, and of excluded middle.

According to the canon of *identity*, "if anything is A, it is A." Not that frogs' eggs do not develop into tadpoles, and tadpoles into frogs; but if anything is a frog's egg here and now, then it is a frog's egg and not, in this one of its aspects, something else. How could we affirm or deny, reason or inquire, with any confidence at all if the meaning of a term did not remain constant in a given proposition, argument, or inquiry? Knowledge is indeed progressive, and we do not bind ourselves once and for all to a fixed meaning of any one symbol. Even so, stability of meanings is an ideal which we seek progressively to approximate.

Similar considerations hold for the truth of what we affirm or deny. "If any proposition is true, it is true." That "it is raining" may, of course, be true here and now and not there and then; but time- and place-reference must be made explicit whenever necessary in the interest of the completeness of a proposition. Moreover, we must distinguish time- and place-reference *in* a predication from time and place *of* a predication as an utterance. If once true, it is always true that "an armistice occurred on November 11,

1918"; it cannot be true on that day and false on another. Truth too is an ideal which we strive progressively to approximate, with a view to which we qualify and re-qualify our statements, and without which any investigation or communication worthy of the name would disintegrate.

Then too, all ground would be removed from any practical reliance if the law of identity did not hold. "Business is business"—incidentally, a saying which is generalized in the formula: "A is A." Whether we look in the direction of the actual or of the ideal, we find ourselves driven to acknowledge that "whatever is, is." Where but in the realm of the inane would identification be possible without the principle that "everything is identical with itself"?

What the law of *contradiction* asserts is that "nothing can be both A and not-A." Were A also not-A, anything could both belong and not belong to the kind designated as A. If we hesitate about this, it is because we have not yet determined A conjunctively and disjunctively. So too it is impossible for anything to possess a certain quality and with respect to the same time and place and aspect to possess a contradictory and inconsistent quality. Though water feel warm to one hand and cold to another, or warm at one time or place and cold at another, it cannot, as respects the same time and place and aspect, be both warm and not warm to the same hand.

In another formulation of the canon, "no proposition can be both true and false." If this were not so, how could any significant statement be refuted or even made? In order to support a conclusion as sound, it is necessary for us to determine a pair of contradictory propositions such that, if one is true, the other is false. Genuinely contradictory opinions are held only at the expense of clearness and consistency. As to the question of two apparently contradictory proposi-

tions being both true, it is helpful again to distinguish time or place *in* and *of* a complete predication. I can both have and not have a handkerchief in my pocket, but not at the same time; and one and the same straight stick may appear both straight and not straight, but not under the same conditions. Such necessary information as to time and place and circumstances must be supplied if my affirmation or denial is to be complete. Even one who confesses: "I am lying," cannot at the same time and in the same respect be telling both a truth and a falsehood. His utterance as it stands is incomplete; he must state the proposition to which he has reference and which he is characterizing as false; one cannot literally and strictly "always" lie and, in saying so, tell the truth. We may attack or defend on separate grounds a primary proposition, a secondary proposition characterizing a primary one, and a tertiary proposition about a secondary one: (i) that matter exists, (ii) was denied by Berkeley, (iii) as at least some of his interpreters have understood him.

It is seldom perhaps, in any but elementary issues, that we may be able to say offhand whether the principle of contradiction is violated or not. Nevertheless the canon does express an ideal which even then guides our thinking. At the same time, any statement or inference clearly combining contradictory qualities bears on its face the stamp of impossibility and falsehood. If mushrooms are fungi, and fungi are cellular plants, then mushrooms are cellular plants or nothing at all. For no quality can be at the same time both present and absent. "Nothing can both be and not be." Existence is so constituted that anything whatever cannot be in one and the same respect otherwise than it is.

The principle of *excluded middle* states that "anything must be either A or not-A." Between the two alternatives,

there is no third or middle course: *Tertium non datur*. Mention anything whatsoever, and any quality or circumstance that may be mentioned along with it either belongs to that thing or does not. Thus, given a quantity, it must be either greater or not-greater than another quantity (where "not-greater" includes both the equal and the less). This law is, however, perhaps even more than the other two, easily misunderstood.

Consider the propositional application of the canon: "Any proposition must be either true or false." Of any two contradictory propositions, both cannot be false; at least one must be true. One frequent objection to the law is due to the confusion of contradictory and contrary propositions. It may be asked: Must one brother be older or younger than another; or may not both be of the same age? What is here overlooked is that "older" and "younger" are contraries; the contradictory of "older" is "not-older," which includes both the "younger" and the "equally old." The principle of excluded middle pertains not to contraries but to contradictories.

Another objection is to the effect that, as long as an action or development is in process of transition, it is hard to tell whether a certain change has taken place or not. Am I or am I not "educated"? However, the canon of excluded middle does not presume to inform us, in any given case, which of two contradictory alternatives is the true one. It is often a difficult task to draw sharp dividing-lines, let alone to ascertain on which side truth lies. The canon nevertheless holds. Knowledge advances as the conditions stated in the canons are progressively satisfied. To the extent that we are interested in pursuing an inquiry, it is open to us either to make further distinctions (as to what we mean by being "educated" and the like) or to agree

upon some conventional standard for discrimination (such as social or professional recognition and the like).

Not a few misapprehensions are due to ambiguities of speech, if not to verbal quibbles. "Guilty or not guilty" are contradictory alternatives, provided they are restricted to the same aspect or aspects; and provided also that the discussion is not shifted to some other contrast, as between guilt or innocence "proved" and "not proved." Then there is the trivial question whether virtue is "odd" or "even." It may be possible to give a suitable answer to an unsuitable question. But why extend the laws of thought to meaningless expressions? Why not rather distinguish the categories or types of being together with the general conditions of significance?

Incomplete or irrelevant disjunctions are judged in the light of such as are exhaustive and necessary. In every serious investigation, conjunction and disjunction, additive and multiplicative, play a united role. Insofar as inquiry may be said to aim at meeting such logical requirements, this goal is formulated in the canon of excluded middle. Examine any adequate statement of alternatives: it will be found to illustrate, in its own appropriate way, the principle that "everything must either be or not be."

The formal canons of reasoning cannot be demonstrated without being assumed in the demonstration. Yet it is by their means that we direct our affirmations, denials, and disjunctions. They are not laws of "thought" alone (or rather of valid "thought"), but principles of being, confirmed and exhibited in every conclusion that is warranted by the comparisons and contrasts we institute to determine identities, non-contradictions (or conjunctions), and exclusions (or disjunctions) in existence.

DRILL

State and explain the canons of identity, contradiction, and excluded middle, especially in opposition to misunderstandings of these canons. (See Exercises, XII, 30-55.)

51. THE EXISTENTIAL IMPORT OF PROPOSITIONS

Many of our modes of speech suggest questions about the existence of their subject matter. "Perfect men may live together without law." "Those hard of hearing are requested to sit in front." "Who steals my purse, steals trash." "He that is without sin among you, let him first cast a stone at her." "No classless society fosters injustice." "No ghost haunts my house." "No woman ministerial candidate has been ordained in our church." "Sometimes the Muses fail to sing." "There are illusions." "There are no panaceas."

Existential questions arising out of modes of speech like these have in turn given rise to different logical systems. Ultimately, questions of existence lead to analysis of the relation between sense and intellect, or between concrete and abstract terms, and the like. But, as games may be played according to different rules and conventions, so certain features of logic allow of variety in interpretation. Suffice it here to indicate how rules of elementary logic have been complicated by different constructions put upon the existential (or instantial) import of propositions.

In the first place, a reassurance is in order. It is admitted on all hands that the traditional formulas deal adequately with the affirmations and denials in which we are most interested in ordinary discourse. Without those formulas, the complications introduced into logic can hardly be understood.

Next, let us explicitly assume the existence of our subjects and predicates and of their contradictories. Take: "My savings are protected." The contradictory of SaP would be: Either some of my savings are not protected (SoP); or else S or non-S or P or non-P is nonexistent." That is to say, should the basic existential assumption we have made turn out to be inconsistent with fact, then A and O, or E and I, as usually stated, may both be false; likewise I and O.

Let us, in another system, insist upon the existence of the things about which our affirmations and denials are made: "All late-comers are fined (and there are late-comers)." We may then manipulate the square of opposition in a similar manner. The contradictory of SaP would be: "Either some late-comers are not fined; or there are no late-comers." Moreover, inasmuch as the existence of P is not implied in SeP, but is implied in PeS, the conversion of E, as usually stated, would be invalidated; therefore also the contraposition of A as well as inversion. Similarly, Camenes and its weakened form; for a negative premise with S as predicate would not imply the existence of S.

What would be the consequences if we construe propositions as not necessarily implying existence of their subjects or of their predicates? Should S then not exist, we could not declare false either the statement that "no trespassers will go unpunished" (those both S and non-P constitute a null-class: $SP' = 0$, equivalent to SaP) or that, "if there are trespassers, some of them will not be punished" (SoP). Like contradictories, so the contraries A and E, as usually stated, might be consistent. Invalid would be: the conversion of A and E; the contraposition of E and O; inversion; and all the moods of the categorical syllogism in the third and fourth figures, except Camenes and its weakened

form. For, if the conclusion is to imply that if there is any S there is some P (or non-P, if the conclusion is negative), the premises must imply the same.

More commonly, however, particular propositions are interpreted as existential and universal propositions as non-existential. Consider: "saints are perfect." SaP would deny the existence of such as are both saints and imperfect: $SP' = 0$. The contradictory would be $SP' \neq 0$: there are such as are both saints and imperfect. So SeP would deny the existence of such as are both saints and perfect: $SP = 0$. Its contradictory would be $SP \neq 0$: there are such as are both saints and perfect.

On this interpretation, if S does not exist, then A and E, as usually stated, would be consistent; I and O would both be false; and neither would I follow from A, nor O from E. Invalid too would be: the conversion of A; the contraposition of E; inversion; and the syllogistic moods with two universal premises and a particular conclusion (in other words, the weakened syllogisms: AAI1, EAO1, EAO2, AEO2, AEO4; and the strengthened syllogisms: Darapti, Felapton, Bramantip, Fesapo). This system would, however, leave unaffected: contradiction; simple conversion; simple contraposition; and the syllogistic moods in which a universal conclusion is inferred from universal premises and the moods in which a particular conclusion is inferred from a universal and a particular premise.

Note that none of these systems renders invalid the process of obversion. Nor does any of them propose novel definitions for the fundamental logical relations between propositions. But how shall our definitions of these relations be applied to the traditional A, E, I, O schedule when we have explicit regard to the existential import of such

propositions? So stated, the issue depends upon the interpretation of existential import.

DRILL

Examine typical propositions which suggest questions about their existential import. (See Exercises XII, 56-60.)

52. ORDER AND LOGIC

A brief survey of some salient topics may bring out the role of order in logic. The specific illustration chosen is here intended to serve this purpose only.

Writing on the increase of "wealth and want," Henry George used as a motto for his *Progress and Poverty* a passage from Marcus Aurelius which opens: "Make for thyself a definition or description of the thing which is presented to thee." If "the meaning of the terms" wages and capital, wealth and value, etc., was to be made clear, the terms had not only to be defined, but had also to be ordered in pertinent affirmations and denials. So George, in contending that the source of wages lies in continuing productivity, denied that wages depend directly upon a fixed fund or that nature is niggardly towards the earth's population. Capital, he maintained, is related to wealth as species to genus; as wealth is in turn similarly related to value. In particular, capital-value differs from land-value, according to George, in depending largely upon factors of production or reciprocation, instead of solely upon social pressure.

As one major premise in the cumulative argument, George took the classic division of the channels of distribution into rent, wages, and interest (corresponding to land, labor, and capital as factors of production). With this he combined the minor premise of the prior payment of rent; the margin of the basic element, land, being fixed. So he

tried to ground the conclusion that wages and interest, which vary inversely with rent, must likewise be set by the margin of cultivation. Dependence is thus a transitive relation, holding not only for rent, but also for interest and wages, as related to the productive powers of capital and to those of nature.

The law for rent seemed axiomatic in this reasoning. So, too, among the first principles taken as basic in this version of a classic economic system, we find the familiar axiom that "men seek to gratify their desire with the least exertion." Induction supplemented this deductive procedure; especially in the attempt to verify the explanation of increasing rent as an effect of increase in population, in social organization, and in the production of wealth. Ethically, George suggested, a proposed remedy must be justified by man's natural right to himself and to the results of his activity; for nature and man are interrelated. By way of solution of his problem, Henry George advocated the socializing of rent, along with the retaining of individualized uses of land.

Arguments like these present a system of inferences from observed facts. Here it is to the point simply to note that order pervades our inferences in the measure in which we strive to clarify our terms, to render consistent our affirming and denying, to support our conclusions, to demonstrate our theorems, to verify our hypotheses, to weigh our values, and to apply intelligence to the solving of our problems.

DRILL

Illustrate order in: inference and discourse; terms; affirmation and denial; syllogism; first principles; verification; evaluation; the search for the solution of a problem. (See Exercises XII, 61-64.)

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Solving Our Problems

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Inquiry

53. SITUATIONS

Successful living requires, among other essentials, development of special skills. Sooner or later, most of us face the choice of a vocation. A variety of alternatives present themselves for attentive consideration. We deliberate upon them until ready to make a decision. As our chosen plan guides our actions, it becomes subject in turn to tests in the laboratory of experience.

This brief sketch brings into high relief those phases which are most conspicuous in reflective inquiry. Life's situations generate problems. The issues we ponder must be definitely stated. With a view to a solution, we tentatively entertain and formulate alternative suggestions. We compare one suggestion to another by elaborating each into its possible consequences. A difficulty is resolved when available tests eliminate some suggestions in favor of others or, ideally, of one; although a solution may occasion further problems.

There are types of thinking other than that described. Much of our time, perhaps more than we care to admit, is spent in reverie and day-dreaming. Again, we become absorbed in the contemplation of beauty, as of a glowing sunset. Or we may acquire information from newspaper headlines or some other source. If thinking, in its broadest sense, takes such varied forms, we may be said to "think"

in nearly all our waking life. How rich the experience, how high the level of living, which alone can guarantee the progressive advancement of thought and of the activities allied to it! That is why the Sermon on the Mount cautions us to guard well the gates of the imagination.

When we deliberately undertake to improve our thinking, it loses its spontaneous and uncontrolled character and assumes the form of problem-solving. Many problems must, of course, find their solution in the process of living, as we go along. No doubt, for some of our everyday activities, even the simple sketch given of reflective thought is too complex. Then there are questions, such as those relating to human destiny, which may well engage the meditations of a lifetime. But, in the large area between these extremes, inquiry directed to solutions finds abundant opportunity for effective exercise. At any rate, it is the problem-solving type of thinking which concerns us here and for the analysis of which we shall at least attempt to summarize some leading principles.

Let us to this end examine with some care a classical example of inquiry. In the *Nicomachean Ethics*, Aristotle sought to expound what it means to live well. Having discussed virtue and happiness, he goes on to explore the difference between voluntary and involuntary action.

"Since virtue is concerned with passions and actions, and on those which are voluntary praise and blame are bestowed, but on the involuntary pardon and sometimes also pity, therefore to distinguish the voluntary from the involuntary is perhaps necessary for those who reflect on virtue, and useful for legislators as respects honors and penalties.

"Involuntary are held to be the things that happen under compulsion or because of ignorance. That is compulsory whose beginning is external and such that the doer or

sufferer contributes nothing; e.g., if a wind or powerful men were to carry him off somewhere.

"But if things are done from fear of greater evils or for something noble (e.g., if a tyrant, having power over one's parents and children, were to command one to do something base and, if one did it, they would be spared but, if not, they would be put to death), it is debatable whether such actions are voluntary or involuntary. Something of the sort happens also with regard to throwing goods overboard in a storm: for, apart from special circumstances, no one would throw goods away voluntarily; but all who have reason would do so for their own safety or for that of others.

"Such actions then are mixed, though more like voluntary actions: for they are desirable at the time when they are done; and the end of an action accords with the occasion. One must be said to act voluntarily or involuntarily with regard to the time when one acts. Now one does voluntarily [the sort of thing illustrated]: for the beginning of the moving of the organic parts is, in such actions, in the doer himself; and the things whose beginning is in him are in his power to do or not to do. Such actions therefore are voluntary, though perhaps apart from special circumstances involuntary; since no one would choose any such action in itself.

"For such actions men are sometimes even praised, when they endure something base or painful in consideration of things great and noble. In the opposite case they are blamed; since to endure indignities for nothing noble, or for the commonplace, is a mark of the weakminded. On some actions praise is indeed not bestowed, but rather pity: when, under pressure which overstrains human nature and which no one can withstand, someone on that account does what he ought not to do. But some acts perhaps we cannot

be forced to do, but ought rather to face death after the most fearful suffering; for the things that forced Euripides' Alcmaeon to slay his mother seem absurd. It is sometimes difficult to determine what should be chosen for what, and more difficult still to abide by our decisions; for, as a rule, what is expected is painful, and what we are forced to do is base; whence praise and blame come to those, respectively, who are forced and those who are not.

"What sort of action then should be called compulsory? Apart from special circumstances, when the cause is in the external circumstances, and the doer contributes nothing. But the actions which in themselves are involuntary, but desirable now and for the sake of something else, and whose beginning is in the doer himself, are in themselves involuntary; but now, and for the sake of something else, voluntary. But they are more like voluntary actions; for actions are among things particular, and these particular actions are voluntary. What is desirable for what, it is not easy to state; for there are many differences in particular cases.

"But if one were to say that the pleasant and the noble are compulsory since they force us from without, all actions would be for him compulsory; for all men do all they do for the sake of those considerations. Those who act under compulsion and involuntarily, do so with pain, but those who act for the pleasant and the noble, do so with pleasure. It is absurd to blame external circumstances, and not oneself for being easily entrapped by them, and to attribute noble actions to oneself, but the base to pleasure. The compulsory then seems to be that whose beginning is external, the one who is compelled contributing nothing.

"An action because of ignorance is in each case non-voluntary. Only that is involuntary which is followed by

more pain; namely, in regret. One who has done something because of ignorance, and feels no vexation at his action, has not acted voluntarily, since he did not know what he was doing; nor yet involuntarily, since he is not pained. Of those who act because of ignorance, the one who regrets is held to have acted involuntarily. But let the one who is without regret, since he is different, be considered a non-voluntary doer; for, since he differs, it is better that he should have a name of his own.

"To act because of ignorance seems different also from acting in ignorance. For one who is drunk, or in a rage, is held to act, not because of ignorance, but because of something like what has been said; yet not knowingly, but in ignorance.

"Now every disreputable person is ignorant of what he ought to do and of what he ought to avoid, and it is because of such failure that men become unjust and in general bad. But the term 'involuntary' tends to be used, not if someone is ignorant of his own interests; for it is not ignorance in moral purpose which is the cause of the involuntary (that is the cause of disreputableness), nor ignorance of the general (for that men are blamed), but ignorance of particulars, of the circumstances and situations of the action. For in such cases there is pity and pardon; since one who is ignorant concerning one of these acts involuntarily.

"Perhaps then it would be as well to determine what and how many these items are: who does what about or in what and sometimes also with what (say a tool) and for the sake of what (say safety) and how (say gently or violently).

"Concerning all of these no one can well be ignorant save a madman. Evidently not as to who acts; for how can one be ignorant of oneself? One may be ignorant, however,

of what one is doing: e.g., those who say that things slipped out of their mouths as they were speaking, or that they did not know those things were not to be uttered (as Aeschylus said of the mysteries); or a man might say he let it go off when he merely wanted to show it, as the man did with the catapult. Or someone might think his son was an enemy (as did Merope), or that a pointed spear had a button on it, or that a stone was a pumice stone. Or one might kill another with a drink intended to save him; or, intending to grasp him as in a sparring match, might injure him.

"Since there may be ignorance concerning each of these (i.e., the circumstances of the action), one ignorant of any of them is held to have acted involuntarily; and especially if he was ignorant of the most important points; and the most important points are held to be the circumstances of the action and its end. Then too an act, to be called involuntary in accordance with such ignorance, must involve pain and regret.

"Since what is done under compulsion and in ignorance is involuntary, the voluntary would seem to be that whose beginning is in the doer himself, the doer having knowledge of the particulars in which the action is involved. For perhaps those actions are not rightly said to be involuntary which are done because of anger or desire. First, nothing would be done voluntarily by other animals or by children. Moreover, would we do nothing voluntarily because of desire and anger; or the noble voluntarily, but the base involuntarily? Would that not be absurd, since the cause of both is the same? Perhaps it would be irrational to call involuntary the things for which one ought to strive; for one ought to have an aversion from some things and a desire for others (e.g., health and learning). Also what is involuntary is held to be painful; but what is in accord with desire,

pleasant. Again, what is the difference, in respect of the involuntary, between failure in reasoning things out and in anger? For both are to be avoided. But the irrational passions are held to be not less human than reason; hence also the actions of a man include those arising from anger and desire. It would be irrational then to treat them as involuntary."¹

The inquiry begins with something actually going on. Men mete out praise or blame upon actions they consider voluntary; pardon or pity, upon actions they consider involuntary. Our lives are highly, often too highly, geared in terms of commendation and condemnation. Here we have one of countless definite factual situations which we may observe and ponder.

If an inquiry is not to roam aimlessly, the reflective thinker must describe the situation to which his reasonings refer. How else shall we know what the investigation is about? Any principles or theories which issue from the analysis must fit the observed and stated facts of the case.

The process of investigation starts from the known and ends in something unknown becoming known. If we observe that men praise, blame, pardon, and pity, and we seek to determine in a summary manner what it is that they treat in these ways, we note repeated instances of each, ever modifying our attempted explanations, until we can establish the result: namely, that the attitudes in question vary according as the actions judged are voluntary or involuntary. If, on the other hand, this is known to us, and we wish to determine with accuracy the distinction between the voluntary and the involuntary, we frame tentative statements of the difference, making additions or subtractions at the hand of observed facts, until a satisfactory result has been attained. But, whether we proceed from particular facts to

¹ Aristotle *Nicomachean Ethics* iii.1-3.1109b30-1111b3.

general principles or explanations, or from vague principles and explanations to their clarification, observation and interpretation are closely united at each stage in the progress of the inquiry.

How then shall we interpret the initial situation? Not only by stating the observed facts to be further examined and explained. A situation occurs as one of a type and in some context. Thus human praise and blame may be classified as falling within the field of ethics. Their analysis points to a context which is not only literary but extends into the life and times of those making the study and of those for whose benefit the exposition is given.

DRILL

Point out the most conspicuous phases of reflective inquiry; the main types of thinking; the sphere of the problem-solving type of thinking; the nature of a situation as calling for observation and description, explanation and clarification, classification and contextual interpretation. (See Exercises XIII, 1-11.)

54. PROBLEMS

If a situation giving rise to reflection did not contain both familiar and novel elements, there would be no problems to solve. For how could complete familiarity stimulate thought? And how could novelty help us to do more than to fumble or to "lose our heads"? Because of novel features, there are problems; because of the familiar, we can deal with our perplexities.

To distinguish voluntary and involuntary action, Aristotle informs us, is both necessary for those who reflect on virtue and useful for legislators in bestowing honors and penalties. The issue represented by the distinction is thus

at once theoretical, calling forth thought for the sake of further thought, and practical, an affair of thought designed for the guidance of conduct. In reflective inquiry, which is directed to the solution of a problem, our thinking is accordingly controlled by a purpose and involves the fixation of right beliefs. Who can intelligently praise or blame, pardon or pity, without some criterion by which to differentiate the voluntary from the involuntary? When a standard has been set for the distinction of the voluntary and involuntary, when praise and blame along with pardon and pity have been conceived in their perfected exercise, we are in a better position to understand our attitudes and acts and to improve them.

Clearly, a problem, like that of ascertaining when actions are voluntary and when involuntary, must be well stated for reflection to have any relevancy and to pursue its way towards a solution. Without a definitely formulated difficulty, we grope in the dark. Aptness in putting issues carries inquiry itself a long way forward towards its goal. On the other hand, to misconceive a problem is to frustrate and misdirect the search at the outset. What endless debates have grown out of the query: Am I fated or free? Such a question affords no clue to an answer or to a method by which an answer may be sought or found. We do well therefore by being on our guard against pseudo-problems and meaningless questions.

Although statement of a difficulty implies a confession of ignorance or doubt, no difficulty can be accurately formulated without previous experience and knowledge. Proverbially, not a little intelligence is needed to ask an intelligent question. As we analyze a situation into its settled and problematic factors, we achieve further insight into the character and pattern of that situation. Our initial facts come to be

seen in relation to facts on a higher plane. Praise and blame, as a case in point, instead of being taken as a matter of course, give rise to questions. What do such attitudes reveal about those who hold them, about those at the receiving end, or about human nature and life generally? So far from constituting isolated facts, the attitudes in question are caught up in a network of relations involving thoughts and words and complex situations.

If insight advances from the vague to the clear, that progression becomes evident in the very formulation of a problem. The better we comprehend a difficulty, where it lies, and why it is a difficulty, the more pointed our questions become. Improvement of our thinking habits depends, in the first instance, upon improvement in the nature of the questions we ask and in our manner of asking them. After all, reflective inquiries are grouped around central questions which, more than anything else, determine the outcome of our thinking. To understand our own minds and the minds of others, it is more important therefore for us to find out the kinds of problems that interest them or ourselves, and what is done with such problems, than to "get" the answers propounded.

With growing insight, our questions become not only more specific but also more general. For, as we may cultivate ability to sense and to state significant problems, so we may also train ourselves to see specific problems in their relations. Thus the distinction of voluntary and involuntary action presents difficulties more clearly discerned in the perspective of wider ones. On the other hand, by way of negative illustration, if many continue insensitive to social issues, it is in part because they have grown accustomed to dealing with concerns of an individualistic sort.

DRILL

Show how a problem involves familiar and novel elements; a purpose and a standard; meaningful statement; doubt and knowledge; and advance from the general to the specific and from the specific to the general. (See Exercises XIII, 12-22.)

55. HYPOTHESES

As praise and blame have in common the character of referring to voluntary action, pardon and pity receive an explanation in their application to involuntary action. Conversely, for the higher-level facts of voluntary and involuntary action, we find evidence, respectively, in the lower-level facts of praise or blame and of pardon or pity. Not that inquiry necessarily proceeds exclusively in one direction, rising from facts to principles or descending from principles to facts. It is a double-barreled process in which facts and principles play a mutually supporting part.

So too, particular facts and general principles clarify each other. A first statement declaring wherein the difference between voluntary and involuntary action consists, is subject to comparison with other statements in the light of the facts, and therefore to continual modification and refinement. This process is like that of constantly revising our estimates of our acquaintances or like the re-interpretation of legal regulations in judicial decisions.

For the solution of a problem, the reflective thinker entertains a variety of alternative suggestions. If an action performed under compulsion, or in ignorance of particular circumstances, is involuntary, a voluntary action would be one initiated by the doer himself with sufficient knowledge of the circumstances involved. But there are complicating factors; as when we choose the lesser of two evils, or when

we have brought our ignorance upon ourselves. Other objections include the suggestions that we may be forced from without by considerations of pleasure or of nobility; or that no one errs willingly; or that what is done because of anger or desire cannot be voluntary. Precise formulation of such suggestions is requisite for clarity concerning the relations among them: which of them are compatible; which, incompatible; etc.

A tentatively suggested solution of a problem is stated in what modern parlance calls a "hypothesis." Moreover, the status of a suggestion or hypothesis may change in the course of an inquiry. Careful writers distinguish a hypothesis, or possible solution; a theory, for which there is considerable evidence; a law, as a tested relationship from which further inferences may reliably be drawn; and an axiom, or a principle around which a large body of knowledge may be organized and from which such further knowledge may be deduced. The idea of evolution is one which has passed through all these stages.

Whence do we derive the suggestions which we formulate as hypotheses and entertain as possible solutions? Generally speaking, from anywhere. In particular, clues are found in analysis of the very trouble to which a solution may be relevant. Then, too, valuable ideas come to those who have built up related experience and knowledge. To some extent, we may draw upon analogy to the manner in which we have solved similar problems in the past. We also avail ourselves of suggestions which have been proposed by other investigators.

Aristotle was accustomed, after describing the goal of an inquiry and listing the questions demanding answers, to declare what his predecessors had had to say upon the subject. These materials he worked over so as to bring out

the best in them and utilize that, not primarily for purposes of controversy, but in promotion of his investigations. Hypotheses clearly stated and critically examined, as alternative to other hypotheses, become central in an inquiry by laying down the lines it will take. For the objective in view throughout is to establish which of the alternative hypotheses is preferable to the rest.

In order that thinking may not be led into blind alleys, any hypothesis worthy of consideration must be fruitful. What does the hypothesis mean, and how does it differ in meaning from the rest? Can it be elaborated into illuminating consequences? Does it suggest methods by which it may be verified directly; or indirectly, by verification of its consequences?

A valuable hypothesis also looks to prediction. To this end, it must have universal application to relevant instances. Its acceptance or rejection must make a specifiable difference. And there must be at least a possibility of its refutation if pros and cons are to be weighed in the balances.

Recalling that the less known is clarified by the better known, we shall also understand the requirement that a hypothesis be simple. The simplicity demanded is not to be confused with a familiarity which varies from individual to individual, nor with an over-simplification of complex facts by means of sweeping formulas (like the Freudian emphasis on sex); but is to be taken in the sense of greater generality. For apparently isolated facts come to be explained in their relation to other facts in a comprehensive system. Other things being equal, a more comprehensive hypothesis is preferable to one less comprehensive: it takes in, and accounts for, more facts.

The conditions which a satisfactory hypothesis must fulfill find exemplification in Aristotle's procedure as he takes

up various suggestions for the distinction between voluntary and involuntary action. Those suggestions call for comparison in their relation to facts and principles and to other suggestions; in their formulation as hypotheses; in their relevance to the problem and its progressive solution; in their fruitfulness; in their predictive power; and in their relative simplicity or degree of generality.

DRILL

Examine the suggestions for the distinction of voluntary and involuntary action in the light of the conditions for satisfactory hypotheses. (See Exercises XIII, 23-33.)

56. REASONING

In order to trace what happens to alternative hypotheses within an inquiry, we must endeavor to follow closely the procedures adopted in reasoning. Preliminary statement of a situation, with its problems and tentative solutions, gives way to more precise determinations in the statement of relevant facts and principles.

No suggested solution of a difficulty gains immediate acceptance by a reflective inquirer. Taking up the commonly approved judgment of the compulsory as involuntary, Aristotle re-examines it in the light of deduction and application to fresh instances: first to actions from fear; then to actions for the noble. He interprets these instances in turn to delimit the sphere of their relevance: the former lead to the recognition of "mixed" actions; the latter, to the addition of further elements in the conception of the involuntary. Principles thus won, when similarly applied to ignorance, aid in singling out, as another special case, "non-voluntary" actions, and in differentiating culpable from pardonable ignorance. In general, the reasoning not only

brings the problem of the voluntary and the involuntary closer to a solution, but also reveals and resolves new difficulties which arise in the course of the investigation.

Aristotle's scientific treatises deal dialectically with a large variety of opinions in their mutual bearings, very much as Plato had done in his dialogues; except that Aristotle's treatment is more condensed, analytical rather than dramatic, and directed to solutions rather than to a sense of inconclusiveness. The reasons given for the rejection of suggested views are still instructive in their expression of standards controlling the promotion of reflective inquiry.

Aristotle refuses to agree that the pleasant and the noble, which appear to force us from without, are therefore compulsory. For, on this basis, all actions would be compulsory; pleasure and nobility would render an action painful; and we might repudiate responsibility for shameful actions, while claiming credit for noble actions. Sound reasoning renounces arguments which are undifferentiating in the order specified; which become involved in inner contradictions; or which appeal to a double standard in the interest of absurd apologetic. As to those who identify virtue with knowledge and vice with ignorance, they have hit upon a partial truth. But their over-simplified reasoning falls into a hasty generalization manifest in neglect of the distinction between a blameworthy and an unavoidable ignorance. Similarly, the characterization of action due to anger or desire as involuntary is barren from the standpoint of responsibility in the life of impulse and passion.

Reasoning of a high order requires a wide background and may vary, not only with the end in view, but also in terms of the assumptions, explicit or implicit, which control the reasoning. In the probing of specific views, there is a forward reference to the meanings sought as well as a back-

ward reference to meanings already established. Every inquirer carries on his special analyses along the lines of what he is trying to do and of his general convictions. In consequence, the meanings traced as the inquiry advances take their place among other meanings in a system, including the meanings reached at the conclusion of the search.

What reasoning accomplishes, with respect to a given hypothesis, is to deduce its consequences; to compare it, as thus expanded, with alternative hypotheses, at the hand of the problem; and to facilitate selection of the most promising hypothesis for the desired solution. Exploring the implications of a hypothesis also increases or decreases preliminary evidence for or against the hypothesis; or at least directs the activities by which appropriate evidence is produced.

In sum, the phase of reasoning in reflective inquiry prevents unconsidered commitment to belief or action; consists in comparative dialectic survey; applies relevant standards of acceptance or rejection; and has conditions or consequences in a system of meanings as well as important relations to every other phase of inquiry.

DRILL

Trace the accomplishments of reasoning in the quotation about voluntary and involuntary action. (See Exercises XIII, 34-40.)

57. TESTS

Since right reasoning must follow the lead of its subject matter, the structural analysis of voluntary and involuntary action is tested at every step by typical facts. What then is a "fact"? Either some discriminated actuality (such as the experience that men praise and blame, pardon and pity); or an interpretation of a given actuality (this is a

voluntary action, that is an involuntary action) ; or a number of actualities together with their common characters, as discerned in significant observation and approximated in reflective interpretation; or some established principle in terms of which our knowledge of actualities and of their common characters is organized (as in the distinction between different kinds of action with respect to voluntariness and involuntariness). Having built up our knowledge from lower- to higher-level facts, we may retrace our steps in the endeavor to verify our hypotheses.

According to one theory, correspondence with facts is a test of truth. Such correspondence does not, however, provide us with an antecedent criterion. We need, and in the nature of the case can have, no test of truth other than one which facts supply. As has been noted, principles too may be described as facts in the sense that they account for facts on a level below them. Significant observation itself connects the directly given with something else (as when we say that the cargo is being cast overboard in the interest of the safety of the crew), testimony to an observed occurrence may be fraught with erroneous interpretation; the very factors noted are actively selected under the guidance of some hypothesis which directs the observation; and the observation may or may not be aided as to its accuracy, significance, and value by the conditions under which it is made.

As inquiry orders facts, so it organizes reasons in turn, in relation to one another. A structure thus becomes articulated so as to stand out clearly within instances as their structure. In our somewhat lengthy example, inquiry is a serial process in which we learn successively the respective characters common to praise and blame, to pardon and pity, and to actions that are compulsory, mixed, non-volun-

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tary, performed in ignorance of particulars, involuntary, and voluntary.

VOLUNTARY	INVOLUNTARY	Qualifications	<i>Rejected Hypotheses</i>
Begin in the doer himself	Under compulsion:	Mixed:	Pleasant and Noble
With knowledge of circumstances	Begin externally	Partly compulsory	Compulsory
	"In" ignorance of circumstances	NON-VOLUNTARY:	Vice = Ignorance
	Involve pain and regret	"Because of" ignorance,	Virtue = Knowledge
		Without vexation	Irrational Passion as Involuntary

No facts with which inquiry deals have sufficiency or completeness in themselves. Rather are they selected and described for the purposes of an inquiry in which they serve as evidence. If some theorists have found in coherence a test of truth, we cannot take this as ultimately a coherence in a set of arbitrary principles. One fact may rather challenge another fact as to its bearings upon the problem to be solved. Yet, by insisting that a theory must fit the facts, we do not set up an external standard. The facts are so related that some are discovered to constitute reasons for others.

To the extent that articulation of a structure is adequate, it expresses the common or distinctive features of the facts considered. An achieved articulation also operates, in our thinking, as a kind of legislation specifying conditions on which fresh cases are admitted into a defined realm (as of voluntary or of involuntary action). Until proved, a hypothesis is altogether tentative; and facts taken under advisement are probationary facts of the case, aspirants to facthood. A hypothesis successfully passes testing by facts when it is found to be not inconsistent with actualities and their characters and with the principles elicited from these.

However, application to instances beyond those taken into account may call for modifications in the formulations

made. One theory accordingly has it that truth is determined by the consequences of acting upon a hypothesis in a manner suggested by that hypothesis. Yet truth is hardly created, but is discovered and applied, by inquirers into the facts of nature and of life. As something upon which competent investigators will eventually agree, truth is an ideal.

When two or more hypotheses are in possession of a field, a decision may sometimes be reached between them by what scientists call a "crucial experiment." In the analysis quoted from Aristotle, we may perhaps regard as an analogue to this device the rejection of the complete identification of virtue with knowledge and of vice with ignorance. Bad character, we are told, indeed results from ignorance in moral purpose and concerning moral principles; but the kind of ignorance underlying involuntary action has to do with particular circumstances in a moral situation. A crucial experiment accordingly tests not an isolated hypothesis, but other assumptions as well, along with the rest of knowledge involved in the hypothesis. The experiment is crucial with respect to hypotheses only as other assumptions in the investigation are retained. Neither observation nor experiment nor the facts ascertained by them have significance apart from hypotheses or apart from the processes of thought generally.

Not every inquiry remains forever an inquiry. Some researches are brought to a successful conclusion. When a determinate problem has been solved, inquiry concerning it ceases. This is not to say that one day we may know everything; but that what is knowable becomes known by knowing or thinking. Yet knowledge is not therefore a rigid dogmatic system. Aristotle recognized the difficulties not resolved by his analysis. Knowledge is a flowering of life itself, a culmination in the development of nature and of the

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things nature contains, a living growth to which we can set no antecedent limits.

Among the factors by which hypotheses are tested we may then recognize: facts accessible to observation; articulation of their structure; agreements resulting from their interpretation; and the satisfaction they afford to intellectual curiosity and to its continued cultivation.

DRILL

Show what is meant by such tests of hypotheses as: fact and observation; structure and its articulation; application and competent acceptance of formulations; experiment and the solution of a problem. (See Exercises XIII, 41-55.)

Scientific Principles

58. THE DISCOVERY OF ORDER

Scientific inquiry has, in general, a twofold aim: to explain and to control. Hence the scientific enterprise presupposes an order which is intelligible, causal, and predictive; and scientific analysis issues in discovery of mechanical structures and functions. By way of example, *An Anatomical Disquisition on the Motion of the Heart and Blood in Animals*, by William Harvey, although first published in 1628, may still offer profitable guidance to fundamentals of scientific method. (See the Everyman edition, translated from the Latin by Robert Willis.)

Harvey found the task of discovering the uses of the heart full of difficulties, partly because of the rapidity of the heart's motion, partly because of the incongruous opinions of others; yet nature finally rewarded his arduous search for an order among the varied facts. Indeed who would inquire without a firm conviction that there is an intelligible order which, by diligent search, he may find out? Detailed observation and description, with special reference to the time when the pulse is felt, led Harvey to an interpretation of the heart's systolic motion as a tension in which the heart becomes erect, elongated, and hard as well as pale—phenomena clearly indicating to the persistently thoughtful, not the taking in but the expulsion of blood. Such insistence

upon facts has relevance to an adequate view of a significant order among the facts.

Upon painstakingly noting the motions of the heart, Harvey found that the pulsation of the arteries of the body depends upon the contraction of the left ventricle of the heart, as the pulmonary artery pulsates by the contraction of the right ventricle. The actions described take place together, cease together, and vary together, thus justifying the conclusion "that the pulse of the arteries is nothing more than the impulse or shock of blood in these vessels." So any search for a causal order, in looking to a full explanation of a phenomenon, takes the form of identifying some co-present, co-absent, or co-varying factor. The investigator has to eliminate any circumstance which may be absent when the effect is present, or present when the latter is absent, or unrelated to it by corresponding variations. Against the common opinion of his day, Harvey accordingly established the fact "that the diastole of the arteries corresponds with the time of the heart's systole."

Furthermore, contraction of the ventricles of the heart is preceded by contraction of its auricles. Careful consideration of this observation, however, falsified the prediction that four distinct motions would be perceived in time and place; for, "though the places be four, the times are only two." Experience in any case confirms the expectation that the findings resulting from carefully controlled experiment will in large part be duplicated in subsequent experiments guarded with equal caution. Nature does not suddenly subvert her order. What constitutes that order in detail, no one can determine without patient exploration. But, within progressively ascertainable limits, the natural order is characterized by a predictable uniformity, without which experimental control would be impossible.

Harvey's summary account "of the motion, action, and office of the heart" stresses the harmony between the motions of the auricles and ventricles. He expressly likens this harmony to that which obtains among motions in complicated machinery, in firearms, and in the swallowing of food. It is not to be wondered at that the sciences, in their progressive development, become increasingly mechanical; for to say "order" amounts to saying "mechanism." Exploration, explanation, and control of anything whatever are contingent upon recognition, articulation, and exploitation of mechanical structures and functions.

Only those who fail to understand set mechanism in diametrical opposition to value. For it is of the essence of a mechanism to be instrumental to its uses. Abolish mechanisms or orders, and what have we left upon which to rely? When Harvey's contemporaries, like Galen in antiquity, failed to discern in the aorta "the vessel which distributes the blood from the heart to the rest of the body," Harvey quoted Galen on the valves at the orifice of the aorta preventing the return of the blood into the heart: "Nature never connected them with this, the most noble viscus of the body, unless for some important end." In short, scientific research, aiming at explanation and control, presupposes an intelligible, causal, and predictive order as a mechanical basis of significant activity. And it ought to go without saying that explicit avowal of mechanism does not commit one to a crude version of it, any more than an acknowledgment of values exacts a choice of the least desirable values.

DRILL

What, in general, are the aims of "science"? How do they presuppose intelligible, causal, and predictive order?

What, in general, is meant by "mechanism"? (See Exercises XIV, 1-12.)

59. INDUCTION AND DEDUCTION

Most of the views opposed by Harvey are traced by him to a preoccupation with the lungs and with human bodies when dead. Wider and closer examination revealed striking differences in the transmission of the blood from the veins through the heart into the arteries in lungless animals and in embryos and the most circuitous course of the blood from the right to the left ventricle of the heart in larger adult animals with lungs. Yet the passage of the blood through the lungs also presented remarkable analogies to the distribution of the blood through the body. Inasmuch as especially the quantity of blood transmitted in a short time appeared to require return of the blood from the veins to the heart, the evidence suggested to Harvey his great insight concerning the circulation of the blood, including the pulmonary or lesser and the systemic or greater circulation.

We may be inclined to ask: Why, among many called to be explorers, are so few chosen to be geniuses? More to the point, however, is the interplay of induction and deduction. For the mistaken notion persists that, whereas the science of the ancients deductively inferred particular conclusions from universal premises, our science enables us to infer universal conclusions from particular premises. Such a sharp antithesis fails to do justice to the nature of inference. Rather does deduction derive conclusions necessarily involved in premises.

Intuitive induction takes place whenever, in any thinking, we apprehend a universal character or relation in a particular case or event. So far from constituting an indirect

inference, this intellectual act results from a process in which we are "induced" or "led" to "see" a formal pattern in some vaguely experienced qualitative whole. Thus Harvey, by the exercise of his scientific imagination, realized that the blood had to pass from the arteries to the veins; even though, lacking a perfected microscope, he had no way of learning that this passage was accomplished through the capillaries.

Perfect or complete induction, which is based upon exhaustive enumeration, may be readily expressed as a deductive argument in the form of a syllogism: The right and left auricle and ventricle of the heart have openings; the right and left auricle and ventricle of the heart are the four compartments of the heart; hence the four compartments of the heart have openings. For the evidence to be conclusive, the minor premise must be convertible: The four compartments of the heart are a right and left auricle and ventricle. So too when, instead of singulars, kinds are enumerated: Arteries and veins have three main coats; arteries and veins are the blood-vessels; hence the blood-vessels have three main coats. What we find true in each examined case of a certain sort, we may declare true, by way of summary, in every examined case of that sort.

In imperfect or incomplete induction, we generalize from instances examined to all possible instances of the same class. Challenged to prove that "all animals which have a heart, have also auricles, or something analogous to auricles," one may appeal for evidence to observed cases; but only on the assumption that what holds for these holds for all. An inductive inference from some to all cases, when thus articulated in syllogistic form, is found to exemplify necessary reasoning, so far as the argument is at all demonstrative. The relation to deduction is essentially no different

for statistical generalizations, which refer to percentages of cases: "There are actually more animals which have no lungs than there are which be furnished with them." Also functional generalizations, of the form: A is a function of B, must conform to the requirements of valid inference: "The more frequently or forcibly the arteries pulsate, the more speedily will the body be exhausted in an hemorrhagy."

True, we cannot be expected to refrain entirely from making what has been picturesquely called the "inductive leap." How shall we learn from experience if we do not venture beyond what we know? Yet not all of our generalizations are true, or demonstrated, or even highly probable. Nor do we always know upon what premises our conclusions logically depend or what conclusions our premises logically imply. Instead, we fumble backward and forward to discover the assumptions and consequences of our affirmations and denials. Moreover, discovery of new knowledge, which cannot be reduced to rules, often involves the aid of deduction from previous knowledge. Induction and deduction do not then present an absolute contrast as forms of inference. But the distinctive inductive problem remains: how may we increase the reliability of the premises from which we may draw probable inferences and by which we may test our generalizations?

Both for inferential and testing purposes, induction rests upon instances. Given certain instances in which the heart moves in embryos, we proceed to look for further instances; so that, provided no cases to the contrary arise, we may generalize from the instances examined to still further instances of a similar type. The larger the number of instances adduced, the greater the weight of evidence; at least for a generalization relatively isolated from others,

where we lack sufficient knowledge concerning common properties to identify an instance as representative of a class.

Since a generalization claims to apply irrespective of circumstances, its presumed accordance with facts will vary not only with the number of instances presented, but also with the variety of circumstances under which examination of instances is instituted. Harvey charged those of his contemporaries who restricted themselves to dissection of the human body with basing general conclusions on one particular proposition; as if politics or agriculture could be mastered by study of a single commonwealth or of a single field. Determination of agreements calls for control by a maximum of differences in the characters of specific instances; and vice versa.

The precision with which instances are analyzed affords additional evidence for the accordance of a generalization with facts. Consider the varied and progressive knowledge, observation, questioning, discourse, measurement, experiment, elimination, comparison and contrast, and the like, requisite for tracing and setting forth the interactions going on in the motion of the heart and of the blood in animals!

What then may be concluded as to the crucial issue in inductive procedure? It comes to ascertaining whether an instance may be identified as a representative sample of a type. As our knowledge increases, we develop methods of obtaining fair samples. To the extent that we have systematic knowledge of a subject matter, we may rely upon the analogy of instances under consideration to instances of other well-established generalization. Hence, again, the importance of hypotheses fruitful in deductive development rather than capable of direct verification or refutation. So induction and deduction play into each other's hands. Yet,

given continued, fair, and random sampling from a large determinate collection, with the consequence that a certain proportion of the samples are found to have a given character, we may conclude, only with varying degrees of probability and approximation, that the same proportion of the entire collection has that character.

It is appropriate to point out in this connection that analogical reasoning from one case to another proceeds through a middle term. Obviously, we cannot cogently argue from the credibility of this, that, and the other statement of Galen to the credibility of any tradition you please. Nor would we attach a high degree of probability to the inference: Since the blood resembles the air and the rain in emulating the (supposedly) circular motion of celestial bodies, therefore the heart is the sun of microcosm as the sun is the heart of the world. Reasoning from analogy, as a kind of sampling, achieves a higher degree of probability when relevant similarities outweigh dissimilarities. For example, as a large quantity of liquid is passed off in a short time by urine, so a large quantity of blood circulates in a short time; hence, as it is possible for the former to percolate the liver, so it is possible for the latter to percolate the lungs.

In fine, there is not one logic of induction and a different logic of deduction. The same processes of inquiry are common to both of these directions we may take on the path to knowledge: from what things are to what they are found out to be; and the reverse direction in which we may retrace our steps.

DRILL

Distinguish intuitive, perfect, and imperfect induction; the number, varying circumstances, and precise analysis of

instances in the testing of generalizations; induction and deduction in sampling, analogical reasoning, and discovery. (See Exercises XIV, 13-30.)

60. PRINCIPLES OF VERIFICATION

In order to confirm his theory of the circulation of the blood, Harvey called attention to the large quantity of blood transmitted by the heart to the arteries, distributed in the body, and returned from the veins to the heart. In addition, he employed a number of miscellaneous arguments. We may extract from his contentions such principles of verification as empiricism, objectivity, parsimony, and converging evidence. What we call "science" may be said to be interested, not in every conceivable truth, but in such truth as can be verified.

The amount of blood expelled by the heart, Harvey reasoned, is greater than can possibly be accounted for by the food consumed. Facts accessible to sense-perception furnished abundant evidence for this reasoning; as when a live snake is laid open and killed by extinction from deficiency, or by suffocation from excess, of blood in the heart. By the principle of empiricism, an essential test of truth is appeal to the experienced facts to which an idea purports to apply. In this sense, "seeing is believing," even at the expense of prejudice, tradition, and convention.

Similar considerations pertain to the manner of the blood's distribution. Despite the confident use of ligatures in medical practice, few understood "the causes and rationale" of the effects produced. Nevertheless Harvey flung the challenge: "Let anyone make an experiment" with some type of ligature in order to learn about the behavior of the blood. A competent investigator submits his findings to the judgment of other investigators equally or more com-

petent, interested in the same problems, and willing to adopt appropriate methods in the quest for truth. Not an absolute test (since human beings differ in capacity), co-operative verification in the continuity of inquiry is nonetheless indispensable to a convincing substantiation of belief.

As to the return of the blood to the heart, observation and experiment, adequately interpreted, establish the function of the valves in the veins to be that of preventing contrary motions of the blood. In sum, says Harvey, "it is absolutely necessary to conclude that the blood in the animal body is impelled in a circle, and is in a state of ceaseless motion; that this is the act or function which the heart performs by means of its pulse; and that this is the sole and only end of the motion and contraction of the heart." Justification for the theory is thus found in its ability to unify the diverse items of knowledge unfolded. As between a theory leaving relevant facts to be accounted for by separate explanations (*ad hoc*), and one embracing these facts along with the rest, the principle of parsimony bids us to prefer the more comprehensive theory.

Harvey supplemented his main argument by others which converged upon similar results: probable reasons, such as the central functions of the heart; consequences, with crucial import to medicine; and the structure of the heart and other facts apparent from dissection. Often too we desire to explain a unique event under difficulties of verification which demand recourse to the principle of converging evidence. Harvey's *Anatomical Examination of the Body of Thomas Parr*, who had died at the alleged age of 152 years, will instance the point. The aged country-man had been brought to London; which led Harvey to conclude, from several independent lines of investigation conjointly brought to bear upon the case, "that had nothing

happened to interfere with the old man's habits of life, he might perhaps have escaped paying the debt due to nature for some little time longer."

As scientific method, then, besides assuming an intelligible, causal, and predictive order, utilizes induction and deduction, so it relies for verification on the principles of empiricism, objectivity, parsimony, and converging evidence. These features of science also condition its progress.

DRILL

Explain and illustrate as principles of scientific verification: empiricism; objectivity; parsimony; converging evidence. What features may be said, in sum, to constitute the structure of scientific method? (See Exercises XIV, 31-41.)

61. SCIENTIFIC PROGRESS

In presenting the arguments for his own theory of the circulation of the blood, Harvey rightly deemed it necessary to eliminate numerous hypotheses passionately held by others on the authority of tradition. The same tests by which he confirmed his own experimental findings also served to refute opposing conventional views.

Do lungs, arteries, and heart perform the single function of respiration? The assumption of spirits, air, and vapors in the arteries was, by the principle of empiricism, discontinuous with facts amenable to observation, experimental verification, and scientific explanation. Objectively considered, whether by a conventional, independent, or sentimental thinker, the arteries may be found to contain blood only, the same as in the veins, even if it be fondly called "spirituosus." Nor does the principle of parsimony countenance invention of explanations contrary to experi-

enced and understood fact: experience attests that, when the pulse is quick, respiration is slow. Different structures and movements presumably point to different functions.

On the other hand, since both ventricles of the heart are nearly alike, presumptive evidence is against differences in their functions. Does the heart manufacture spirits to be transmitted by the arteries, and nutriment to be transmitted by the veins, to the different parts of the body? Not only is it incongruous to attribute to the larger pulmonary veins such varied uses as passage of air, giving off of vapors, and refreshment of the lungs by "spiritualized" blood. Experiment also refutes the notion that the pulmonary vein, which has the structure of a blood-vessel, or the left ventricle of the heart resembles the lungs in containing air. If it be arbitrarily supposed that the septum of the heart is riddled with holes, like a fine sieve, through which the blood percolates, the very existence of such pores may be flatly denied.

Project Harvey's methods of verification and refutation against a background in which many likened the movement of the blood to the rise and fall of the tides, and who would fail to see that the principles underlying those methods have an essential bearing on scientific progress? This does not mean that even beliefs found erroneous may not have contributed to more refined and adequate explanations of the facts. Fruitful scientific advance is necessarily a co-operative enterprise, in which each investigator builds upon the achievements of his predecessors. Especially does development of quantitative techniques and invention of tools and instruments hasten progressive control of nature and understanding of her laws.

If Harvey had to struggle in his day against such obstacles as passionate, unyielding, and weak traditionalism,

the time has not yet come when we can cease to insist upon scientific procedures because some of us have learned to take them for granted. For, although such procedures have long since commended themselves for their impersonal, flexible, and effective character in the promotion of explanation and control, progressive science and intelligence still meet opposition from appeals to rival methods. One way, however, in which we may cultivate an improved comprehension, without over- or under-estimation, of the inherently progressive nature of scientific intelligence, is to trace it in specific instances to its conditions and consequences in experience; as we may trace dyes to the coal-tar products out of which they come. Can we appraise mechanism if we ignore machines or forget that we live only so long as we are not too much out of order?

DRILL

Upon what factors does scientific progress, in general, depend? (See Exercises XIV, 42-51.)

Rival Methods

62. COMMON SENSE AND FAITH

It would be far from easy for any of us to tell in detail how he has come by his convictions. Nevertheless, if we mean by a "method" a consciously chosen source of knowledge, opinion, or experience, together with a deliberately adopted attitude towards that source,¹ a variety of methods may be discriminated in the attempts of men to construe their life in the world. Thus Spinoza called attention, in his *Treatise on the Improvement of the Understanding*, to four kinds of knowledge, including hearsay (as to the day of one's birth and the like), vague experience (concerning man's rationality and mortality, obvious properties of oil and water, etc.), inferential reasoning (about the dependence of sensing upon body and mind, about the small appearance of large objects, etc.), and insight into the nature of things (such as the union of mind and body). Moreover, Spinoza declared that often the same idea may be held on all these levels; like the idea that two is to four as three is to six.

That rudimentary form of intelligence known as "common sense," which arises in everyday experience, lies at the foundation even of highly developed knowledge. Yet the usual acquisition of universally shared beliefs hardly proceeds by a well-considered method, but "grows" like Topsy.

¹ J. H. Randall, Jr. and J. Buchler *Philosophy: An Introduction* (New York: Barnes & Noble, Inc. 1942), p. 45.

The dominant concerns of common sense appear to be with the immediate uses and enjoyments of things and with group opinion and approval. Both as to contents and procedures, however, common sense varies with social changes. In our industrial society, a certain amount of continuity obtains between ordinary-reflective and technical-scientific inquiry or information; but there are also serious "cultural lags."

Confronted with variations in intellectual climate, many follow a natural tendency to maintain, instead of revising, their habits of thinking. The method of "faith," or steadfastness of belief, becomes a method of "tenacity," in an emphatic sense, whenever someone refuses to accept evidence unfavorable to his acquired views. But the security sought in this way turns out to be illusory in a conflict which cannot be settled by an appeal of each partisan to the method of tenacity. Even a believer's own social sense seeks to transcend the purely personal, obstinate, and complacent, no less than the provincial and narrowly utilitarian, in intellectual outlook. As for the fear that unhappiness will result from an altered point of view, actual trial often proves such misgivings groundless. "If hopes are dupes, fears may be liars."

In cases when there is no evidence for or against some persuasion, many exercise what William James called "the will to believe" or "the right to believe." Unfortunately, those who defend their inclinations in this regard do not always clearly differentiate the discipline or value of reason from other considerations.

Faith manifests itself in ways other than tenacity and the right to believe. For there is no lack of occasions when experience gives grounds for a reasonable expectation. In this sense, we may or may not have faith in ourselves, in a

friend, in a physician, in some official, in the democratic way of life, in scientific method. Here we build our reliance on some source rather than on an attitude; though not without possible reservations.

DRILL

Discuss common sense and three forms of faith. (See Exercises XV, 1-12.)

63. DOUBT AND AUTHORITY

No one can literally and at once doubt everything. Witness Descartes, who practiced this method in a provisional way, but soon came to see that, being sure of his doubting, he was also sure of his thinking and therefore of his existence. Every significant doubt has pertinence to some specific issue and itself rests upon assumptions. There are, accordingly, many kinds of skepticism. A constructive and practical type of skepticism, exemplified in the Platonic dialogues, in the Kantian philosophy, and in the sciences, inheres in persistent critical examination of claims to knowledge. Frequently, however, those cultivating a pose of incredulity undertake to set antecedent limits to knowledge; whether in defense of some faith (fideism) or by way of reducing science to what is, at least in principle, directly observable (positivism). Then, skepticism of an extremely negative sort despairs of the possibility of knowledge. One popular argument, that scientific theories continually replace one another, ignores the cumulative character of scientific discovery; a second contention, that no method available can guarantee the validity of any method, overlooks the certification of a method in its use. In general, failure of the quest for infallibility constitutes an invitation to conceive knowledge and its methods in consonance with

the actual course of inquiry. Has someone become disillusioned by misplaced expectations in the quest of knowledge? Experience warns us that he is a likely victim of some self-appointed authority.

Like tenacity, so authoritarianism opposes the spirit of tentativeness in progressive inquiry. Authoritarianism also fails to resolve a clash of tenets where the question arises: Which institution, which tradition, which expert shall have the last word? Not only is there a multiplicity of authorities; but the appeal to any one among them may be disputed by any other. Not only may any tribunal be in error; but abdication to an *ipse dixit* perpetuates the error. If private judgment is indeed not always right, history has nonetheless vindicated the individual's independence in the case of a Socrates against the state, of a Galileo against the church, of a Harvey and a Pasteur against the pressure of professional opinion in their day. Whatever the alleged authority to which we may be urged to transfer our own responsibility to determine what is true or false, it is always proper for us to ask: By what process has the supposed authority itself arrived at its teachings?

Deference to one's betters is in certain respects justified; but never as a "method" which grudges to tolerate alternative opinion or inquiry. Children need authoritative guidance by their parents; all of us must look for information of one kind or another to more competent witnesses; and the exigencies of associated living demand that we conform, in our conduct, to some customs and to some practical decisions made by those in positions of influence and power. For the evils of permanent anarchy outweigh those of accommodation to conventions. Yet, as signed blank checks differ from credit notes redeemable on demand, so the dis-

tion remains between a dogmatic and a reasonable use of the principle of authority.

DRILL

Discuss various forms of the methods of skepticism and of authority. (See Exercises XV, 13-27.)

64. EXPERIENCE AND REASON

Modern science and humanitarianism achieved their emancipation from authoritarian views of life by invoking experience and reason. Soon these allied courts of appeal in the warfare against cruel superstitions became rivals; with the result that thinkers were divided into various camps of empiricists and rationalists, according as they interpreted the roles of experience (or observation or the senses) and of reason (or thought) in human knowledge. All the more welcome are the voices now becoming heard in increasing number to the effect that, so far from being incompatible, experience and reason are mutually complementary in inquiry. The supposed rivalry then becomes reduced to the distinction between an empiricistic temper, or emphasis on facts, and a rationalistic temper, or emphasis on principles or theories.

The term "experience" has many meanings. Experience in a profession consists of knowledge, information, or skill accumulated. In such expressions as "having a thrilling experience," we have reference to the quality of some sensation or emotion. What is said to fall "within" or "outside of our experience," may include not only "firsthand experience," but anything of which we may be conscious. To "take recourse to experience" may mean to employ controlled observation; or, to appeal to some objective reality uninfluenced by the special constitution of some individual. In

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opposition to restricted reduction of "experience" to passive "sensation," it is possible to define "experience" comprehensively as a relation or process of interaction between an organism and its environment, including thought as an integral part of experience.

The term "reason" may designate an exercise of intellectual activity, as over against passive acceptance of dogma on faith or authority. The more freely and judiciously intelligence comes to be applied in inquiry, the more does life become "reasonable"; i.e., liberated from hindering influences. We may doubt that there is an absolutely certain a priori knowledge of general principles, gained by intuition and deduction, independently of observation. But when experience has become wisely organized, this state of affairs may be fittingly characterized by "reason" and cognate terms.

Few if any thinkers have advocated, without qualification, either a pure empiricism or a pure rationalism. What indeed does an experience amount to without reflective interpretation; or reason without an empirical foothold? A worm has experience. And the spinning of dreams affords little or no escape from a trap in which we may have confined ourselves.

DRILL

Distinguish several meanings of "experience" and of "reason" as methods. (See Exercises XV, 28-34.)

65. INTUITION AND WORKING MEANING

Methods of intuition have in common the taking of what seems clear and self-evident as therefore true. But maxims obvious to one individual or group may not carry conviction to another. Intuitionists have differed concerning

the principles supposed to guarantee their own truth: $2 + 2 = 4$; let your conscience be your guide; practice tact; the baby "knows" its mother's breast; the wicked cannot enjoy good things; look before you leap; he who hesitates is lost; this soup is hot; the earth is flat; planetary orbits are circular; A is A; there is being. Since the feeling of certainty, besides being private, furnishes neither rational demonstration nor empirical verification, we must, in arbitrating between opposed intuitions, repair to some method other than intuition itself. For everyone having the "intuition" of free will, another has the "intuition" of servitude to necessity.

On what grounds do intuitionists claim superiority, if not exclusive validity, for their method of "knowing"? The analyzing intelligence, so an argument runs, deals from the outside with an object or event, in some relative aspect or element, by means of abstract and static symbols or concepts; only intellectual sympathy gives a knowledge at once direct, absolute, complete, original, indivisible, dynamic, and certain. For mystics, ordinary experience, knowledge, and language cannot attain to the timeless and ineffable reality revealed in high moments of personal union and celebrated in exercises of eloquent devotion. The romantic movement has done much to foster distrust of reflective inquiry and to promote the search for a royal road to knowledge. As Goethe has Mephistopheles put it: "Gray is all theory, and green the golden tree of life!"

On the other hand, thinkers like Spinoza have used the term "intuition" to represent a culmination rather than a rejection of ordinary experience and arduous reasoning. By the same token, when a scientist or other careful inquirer has gained a valuable insight, or deliberately invented a hypothesis, he does not take for granted the final reliability

of his supposition; he proceeds to articulate and refine the meanings of his idea and to submit his formulations to tests acceptable to the scientific or reflective community. It has been well said that "knowledge, as an abstract term, is a name for the product of competent inquiries. Apart from this relation, its meaning is so empty that any content or filling may be arbitrarily poured in."²

Pragmatism, as "a new name for some old ways of thinking," has counseled us to find the meanings of ideas in the procedures in which they are employed. We think in terms of what we do. As our concepts and assertions appear in discourse and experiment, they have a working meaning, traced out by the manner in which they operate in some context. They lead us on to consider with definiteness their implications and the concrete performances with which they are connected. Often confusion of thought arises out of transfer of a meaning from one context to another. Endeavors to fix beliefs apart from solution of problems in experienced situations terminate, not in genuine knowledge, but in fruitless controversy. Before a truth-claim can be satisfactorily assessed, pragmatic analysis must render intelligible the terms used in discourse and experiment.

The operational character of concepts has received a notable exposition in *The Logic of Modern Physics*, by P. W. Bridgman. Thus, in contrast to the Newtonian notion of absolute space, we are advised to examine how we measure varying sorts of "length" with measuring rods or with beams of light; so that we may have different kinds of space such as the "tactual" and the "optical." True, "the distance of a mirror may be found equally well by measuring it with meter sticks, or by determining the time required by a light-signal to travel there and back." Yet, "when we come to astronomical phenomena, the physical

² John Dewey *Logic*, p. 8.

operation with meter sticks can no longer be carried out, and it is meaningless to ascribe to beams of light on an astronomical scale the same geometrical properties that we do on a small scale."³ Similarly, we may suggest, misapprehensions and controversies follow when we either divorce or merge the more familiar solid table on which we write and Eddington's famous but less familiar "scientific" table composed, let us say for short, of molecules in motion.⁴

A corollary of the operational clarification of meanings is the need of insistence upon evidence. In this connection, empirical verifiability, however, calls for broad interpretation; namely, in consonance with different types of inquiry. Not everyone finds himself able or willing to go as far as William James, who argued on behalf of free will and the like from the influence beliefs may have on individual emotion and conduct. Nevertheless, as verification in the laboratory sense may be direct or indirect, so verification in a general sense may extend as far as what may be called "public experience." Knowing is something we do; and the kind of knowledge we achieve varies in part with the manner in which we seek it. In any case, inquiry may be said to yield "knowledge" or "belief" in the form of "warranted assertibility."⁵

DRILL

State what various methods of intuition have in common and wherein they differ. Illustrate what is meant by pragmatic analysis or the operational character of concepts, together with empirical verifiability in a broad sense. (See Exercises XV, 35-45.)

³ P. W. Bridgman *The Logic of Modern Physics* (New York: The Macmillan Co. 1927 [Copyright 1927 by The Macmillan Company used by permission of The Macmillan Company, publishers]), p. 68.

⁴ F. J. E. Woodbridge *Nature and Mind* (New York: Columbia University Press 1937), pp. 215-29: "The Promise of Pragmatism."

⁵ Dewey *op. cit.*, p. 7.

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six

Verifying Our Hypotheses

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Causal Hypotheses

66. THE METHOD OF AGREEMENT

A classical example of experimental investigation may be gleaned from the life of Pasteur.¹ For centuries, men had discussed "spontaneous generation" or "heterogenesis," the belief in the birth of living beings without antecedents similar to themselves. Pasteur had become interested in the question through his researches in fermentation. So he performed some crucial experiments to test the hypothesis in dispute: Can living beings come into the world without having been preceded by living beings similar to themselves? Can minute organisms, disclosed by the microscope in previously sterile liquids, have come in from the air?

After drawing air through a cotton filter, Pasteur found deposited, on the latter, germs and spores like those under consideration. Lest it be supposed that the cotton, itself an "organic substance," had given rise to the organic particles, Pasteur substituted a mineral filter, asbestos, with the same result. The one circumstance common to all the experiments was the presence of dustladen atmosphere carrying micro-organisms which in turn bred further micro-organisms. Thus the experiments confirmed belief in "biogenesis," that living beings proceed from previously existing living beings; at least in the instances where Pasteur's opponents had advanced the theory of "spontaneous generation." The

¹ L. Descour *Pasteur and His Work*, tr. A. F. and B. H. Wedd ([London: T. F. Unwin Ltd.;] New York: F. A. Stokes [1922]), chap. viii.

question of primary interest here to us concerns the logic underlying the experiments.

Inquiry into causal connection progresses by elimination of some or all of a number of alternative hypotheses: Either A or B or C . . . or Z is the cause of the phenomenon P. But A is not the cause of P. Therefore either B or C . . . or Z is the cause of P. But B is not the cause of P. And so on. Truth of the conclusions inferred depends upon truth of the premises. One principle employed in the elimination is that of agreement, by which we argue: No factor which is absent when the phenomenon is present is the cause of that phenomenon; but A is such a factor; therefore A is not the cause of P. In accordance with this principle, all factors are set aside as causal save those in which alone all the instances of the phenomenon agree.

John Stuart Mill, who gave the classic formulation of "the methods of experimental inquiry,"² declared in his first canon: "If two or more instances of the phenomenon under investigation have only one circumstance in common, the circumstance in which alone all the instances agree, is the cause (or effect) of the given phenomenon." Thus bodies assuming a crystalline structure have in common as antecedent the process of solidification from a fluid state. However, the interpretation of "Mill's methods" as achieving discovery and proof claims too much. Rather do the methods serve in suggesting lines of causal inquiry, in corroborating hypotheses of causal connection, and especially in eliminating irrelevant circumstances. Hence the method of agreement is best stated negatively: No factor which is not a common circumstance in all the instances of a phenomenon is the cause of that phenomenon.

Since we know the canons formulated by Mill, but do not know the cause of scarlet fever, of cancer, of baldness,

² John Stuart Mill *A System of Logic* (New York, London: Longmans 1911), Book III, chap. viii.

of many plant diseases, and of innumerable other phenomena, the methods can hardly guarantee discovery. Which of countless factors shall we, say in trying to find the cause of war, select as a single circumstance; as expressing relevant relations; and as representing adequate analysis? Again, from the standpoint of proof, a factor recognized as causal must be invariably connected with all instances of a phenomenon, observed and unobserved; must be a true, not a false, cause; and must represent not only a necessary but a sufficient condition for the occurrence of the phenomenon. Upon learning that a habitual inebriate had been drinking whiskey and soda, brandy and soda, rum and soda, and gin and soda, may we attribute his intoxication to the soda? Not on the interpretation of the method of agreement as a method of elimination. For if we do not include a false cause in our alternative hypotheses, it can neither be rejected nor survive the process of rejection in causal inquiry.

The difficulty of complexity or plurality of points of agreement may be illustrated by the onetime supposition that malaria was caused by swamp or night air. This came to be explained later by the fact that anopheles mosquitoes, whose bite is a real cause, usually shun light and come forth at night from adjoining woods and swamps to prey upon people. Not the night air itself, but a hidden factor present along with it, was the source of the trouble.

DRILL

Describe in syllogistic form the elimination of false causal hypotheses. State the method of agreement, and show its significance and difficulties. (See Exercises XVI, 1-9.)

67. THE METHOD OF DIFFERENCE

Although the method of agreement failed to remove

the uncertainty whether the phenomenon investigated has but one cause, Mill held the method useful in cases where we are unable to alter the circumstances; hence as a method of observation rather than of experiment. Both of these faults were overcome, according to Mill, in the method of difference.

Consider a few simple illustrations. Whereas a glass of plain water has no taste, a little lemon juice mixed with it will produce a sour taste. One who suspects that carbohydrates cause his stoutness may remove them from his diet in the hope of reducing his weight. Pasteur applied the method of difference when he boiled liquid in flasks with elongated necks, some of which he had left open and others of which he had closed; dustladen atmosphere could enter the former, but not the latter; and it was in the former, not in the latter, that micro-organisms appeared.

The method of difference is concerned with two or more instances which resemble each other in every respect except that, in one instance, a circumstance and a phenomenon are jointly present whereas, in the other, that circumstance and the phenomenon are both absent. Applying the method, we argue: No factor which is present whether the phenomenon is present or absent is the cause of that phenomenon; A, being such a factor, is therefore not the cause of P. As the method of agreement eliminates all factors save that common to all the instances, so the method of difference eliminates all factors save that in which alone positive and negative instances of a phenomenon differ.

Method of Agreement

a b c P
c d f P
∴ c P

Method of Difference

a b c P
a b
∴ c P

Accordingly, Mill formulated his second canon of "the methods of experimental inquiry" thus: "If an instance in which the phenomenon under investigation occurs, and an instance in which it does not occur, have every circumstance in common save one, that one occurring in the former; the circumstance in which alone the two instances differ, is the effect, or the cause, or an indispensable part of the cause, of the phenomenon." So if a man in the fulness of life is fatally shot through the heart, the wound is the cause of his death. For reasons similar to those previously stated, the method of difference is, however, a means of elimination rather than of discovery or proof. It may therefore be stated negatively: No factor in the presence or absence of which a positive and a negative instance of a phenomenon do not differ is the cause of that phenomenon.

A complex factor is mistaken for a single one when we attribute recovery from maladjustment and from uncomfortable dreams to the free expression of hidden desires in the presence of a psychoanalyst, instead of to the finding of a sympathetic listener or perhaps to the cessation of some organic disturbances. Is the pouring of hot water into a glass a sufficient condition of the glass cracking? Glass of a certain thickness will expand on the inner surface before the outer surface is heated; unlike thinner glass in which no crack results. Pasteur's experiments, as over against earlier experiments with micro-organisms, also bring out the difficulty of plurality of points of difference. Whereas earlier investigations had encouraged the belief that small organisms inhabited the atmosphere generally, Pasteur showed that these were carried about on the dust of the air. Hence the caution to the experimenter: Vary one factor at a time.

DRILL

State the method of difference, and show its significance and difficulties. (See Exercises XVI, 10-24.)

68. AGREEMENT AND DIFFERENCE

The weaknesses of the methods of agreement and of difference, interpreted as supplying discovery and proof, are combined in Mill's joint method of agreement and of difference. Nevertheless the joint method has some value in the comparison of large groups: the method of agreement is applied to a positive set of instances; and the method of difference, to a comparison of the positive with a negative set of instances. Thus examination of a large number of married couples, some divorced, others not, may reveal some relation between frequency of divorce and such factors as differences in age, education, health, etc.

a b c P	a b g
b c d P	b d h
b c f P	m f j
∴ b c P	∴ c P

Mill's third canon reads: "If two or more instances in which the phenomenon occurs have only one circumstance in common, while two or more instances in which it does not occur have nothing in common save the absence of that circumstance; the circumstance in which alone the two sets of instances differ, is the effect, or the cause, or an indispensable part of the cause, of the phenomenon." A typhoid epidemic may, with some probability, be attributed to the eating of certain oysters if other factors, such as the drinking of milk obtained from a certain dairy, or bathing in a certain stream, have been present in cases where typhoid did not result.

In a broad sense, every experiment involves, in some form, a joint method of agreement and of difference. Selecting a relevant agreement appears to require instances differing as far as possible in other characters. Instances in which a relevant difference is found appear to have evidential value in proportion to their agreement in other characters. Especially the joint method points, accordingly, to the need of precise and comprehensive analytical comparison — a need which is best met in the application of quantitative considerations.

DRILL

Explain the joint method of agreement and of difference. (See Exercises XVI, 25-29.)

69. THE METHOD OF CONCOMITANT VARIATIONS

It is often impossible to remove a phenomenon or a supposed cause by the method of agreement or of difference. Thus we cannot exclude tidal behavior from bodies of water; or a common circumstance, like the fixed stars; or a supposed cause, such as the action of the sun and moon. Instead, we observe or introduce variations in the effect or in the supposed cause; we try to establish a correlation of variations, technically known as a "functional" relation; our method becomes quantitative.

Pasteur took some flasks of liquid to different parts of Paris: only one changed in the cellar of the Observatory; all showed alteration in the courtyard. Other specimens were taken into the country, to a hill, and to a mountain: in the first environment, only eight out of twenty underwent alteration; in the second, five out of eight; in the third, only one out of twenty. He therefore concluded: "If one

brings together all the results which up till now I have obtained, one can affirm, so it appears to me, that the dust suspended in the air is the only origin and the first and essential condition of life in infusions."

Mill stated the method of concomitant variations in his fifth canon: "Whatever phenomenon varies in any manner whenever another phenomenon varies in some particular manner, is either a cause or an effect of that phenomenon, or is connected with it through some fact of causation." More properly, the function of this as of the other experimental methods is eliminative. C and E are shown to have no causal connection if they do not vary concomitantly. No factor which varies when the phenomenon does not, or which fails to vary in a manner corresponding to the phenomenon, is a cause of that phenomenon. If: a b c P, and: a b c c P P, then: c P.

A concomitant variation may be direct, as when a family's standard of living rises and falls together with its income; or indirect, as when surface tension of liquids decreases with rising temperature. Moreover, an accidental or temporary concomitance may easily be mistaken for a more dependable functional relation: death rates from some causes fall from childhood to middle age, but rise from middle life to old age. Even high correlations do not necessarily signify causal connection; as when investigation revealed a correlation of 85 per cent between the death rate in Hyderabad from 1911 to 1919 and the number of members in the machinists' union in New York City from 1912 to 1920.

DRILL

State the method of concomitant variations, and show its significance and difficulties. (See Exercises XVI, 30-39.)

70. THE METHOD OF RESIDUES

That the methods of causal inquiry have eliminative significance, appears most clearly from the method of residues. Mill formulated this method in his fourth canon: "Subduct from any phenomenon such part as is known by previous inductions to be the effect of certain antecedents, and the residue of the phenomenon is the effect of the remaining antecedents." Symbolically, if: $a\ b\ c\ d\ P\ R$, and: $a\ b\ P$, then: $c\ d\ R$. The method is as simple, in principle, as determining the weight of a load by subtracting the known weight of a cart from the total weight of the cart and the load: $c + l = t$; $\therefore l = t - c$.

Astronomers, we are told, had plotted the orbit of the planet Uranus on the assumption that the sun and the planets within the orbit of Uranus alone determined its motion, only to find discrepancies between the calculated and the observed positions of Uranus. Later observation revealed a planet, since then called Neptune, in the vicinity of the place calculated for it. Thus the masses within the orbit of Uranus were eliminated, by the method of residues, as causes of the observed discrepancies. Although the premises had, as Mill's canon suggests, been obtained by induction, and the conclusion reached was verified by observation, the form of the inference is clearly deductive.

In what sense then may causal hypotheses, according to the interpretation given, be considered "proved"? Experimental methods, by eliminating at least some false hypotheses, restrict the field within which true ones may be found. The conditions for the occurrence of a phenomenon are therefore more closely approximated, so that certain hypotheses may be taken as logically preferable to their rivals.

DRILL

State the method of residues, and show its significance.
(See Exercises XVI, 40-50.)

71. CAUSALITY

To "cause" is to "effect." A causal proposition may deal in a prospective way with means by which an end is to be achieved: to produce an explosion, apply a lighted match to dry powder; to move a billiard ball, hit it with a cue. Once a given causal relation has been established, it may be asserted with retrospective reference: this case of pneumonia was cured by the use of penicillin. Cause and effect then, so far from competing with means and ends, are at least in some respects identical with them: both of these relations have to do with antecedents and consequents, which it is the business of practical and scientific intelligence to explore.

It does not follow that causal inquiry must in every case ascertain "the" antecedent of "the" consequence. Noting injuries from an automobile accident, or other elements, and knowing the order, or conditions under which the known elements result, we may look for the driver of the automobile, or other as yet unknown elements. As to condensation of moisture and the temperature of the surface and of the surrounding air, we may seek to determine the order between the known elements. When we observe the aurora borealis or some other change, we may make inquiry into other changes connected with it. The specific factors, or the specific orders, which are the objects of our researches, vary with the problems generating our exploratory efforts. If I read a paper before a club, a social scientist may explain the reading, in some of its aspects, by my response to the expectations of the group; a psychologist,

by factors of attention, emotion, and the like; and other specialists, in terms that interest them.

In general, a causal order is to be differentiated from other types of order: from a constant conjunction of properties (in wood and steel, in water and ether, etc.); from successive events (such as day and night); from a numerical equation (as in the law of gravitation); from a comprehensive theory (like the kinetic theory of matter). Reference to causation takes objects or events out of their isolation and links them with others in spatially and temporally continuous interactions: this rod may serve as a lever with which we can move a heavy object; this homicide was committed in self-defense. Analysis into such interactions makes use of available generalizations, including equations and theories, in which traits and characters are logically related: e.g., an equilibrium is obtained when two weights vary inversely to their distances from the fulcrum; e.g., everyone naturally desires his own self-preservation. There is also the doctrine of the uniformity of nature, which states that some pairs of phenomena are invariably connected; but this general principle does not, of course, tell us which. So too Mill's methods aid in defining the principle of causality in terms of the co-presence, co-absence, and co-variation of factors.

Concerning the topic of complexity or plurality of causes, many thinkers would maintain that no event, say an actual death, has a plurality of possible causes; though we may, in doubtful cases, construct a plurality of causal hypotheses. Only as gross qualitative events do deaths have many antecedents. Like a definition of death and a classification of its kinds, so explanation of a death presents, ideally speaking, invariant formulas. Similar considerations would pertain to a complexity or plurality of effects; as

when an artist produces a poem and a musical composition. On the one hand, the notions of a complexity and of a plurality of causes and of effects hold where scientific analysis remains incomplete: if every C_1 is E , it does not follow that E is C_1 ; since also every C_2 , every C_3 , etc., may be E . On the other hand, there is a reciprocal relation between cause and effect if the characterization of both is sufficiently precise: every E_1F_1 is C_1 ; every E_1F_2 is C_2 ; etc.

Popular generalizations formulate habitual expectations; e.g., that the flavor of an apple is improved by keeping. However, common sense uses of the conception of causation set the problem of a scientific statement of conditions, thus leading to scientific generalizations: the phenomenon in question tends to take place with a gradual reduction in the apple's acidity. Further inquiries utilize such hypotheses in the guidance of observations and reasonings: reduction of the acidity proceeds at a declining rate, especially in cold storage; and apples with high acidity will soon decompose. In this manner, definite sets of interactions come to form a basis for reliable predictions: it is better to let some kinds of apples mature naturally before putting them in cold storage. In short, causal propositions of common sense become more extensive and explicit in their prospective and retrospective reference as they give rise to causal problems and hypotheses in the scientific determination of ordered sequences and coexistences.

DRILL

Explain and illustrate the concept of causality in its prospective and retrospective reference; in its varied forms in causal inquiry; in its difference from other types of order; in its relation to the complexity and plurality of causes; in its development from common sense to scientific determination. (See Exercises XVI, 51-60.)

Measurement and Statistics

72. COUNTING AND MEASUREMENT

Comparison and contrast of objects and events first yield, in common sense as in science, affirmations and denials of qualitative differences. Many of our activities and reflections depend upon our ability definitely to distinguish one quality or character from another. Living bodies and their members, preferred edibles and their preparation and use, hot and cold days and the clothing appropriate for them, a sense of direction in our movements, a conflict and its resolution, the planets and their motions—our dealings with such selected wholes and parts are permeated with qualitative discrimination.

For many purposes, we may content ourselves with what we may call "qualitative measurements," as expressed especially in the comparative degree: "You are taller than I"; "This lesson is more difficult than the last"; "He has too much money for his own good"; "This room is warm enough now"; "This musical passage should be played more slowly"; "Mercury and Venus are nearer to the sun than is the earth." Rudimentary quantifications like these consist of judgments upon differences of degree in qualities not sharply demarcated from one another.

Numerical measurements replace qualitative differences by more precise statements of the degree of such differences. For we may wish to know just how many people attended a performance; how much money we have; how far the planets are away from us; how fast an airplane is traveling; how much more unemployment there was in one year than in another; how much less of a dose we shall take of a medicine; at what rate according to the metronome we shall play some part of a musical composition; to how long a luncheon period we are entitled. By such definite quantitative discriminations, we achieve greater accuracy of statement, comprehensiveness of explanation, security of control, and the opportunity for unambiguous confirmation or for refutation.

Reducing qualitative to quantitative differences does not abolish the former. The kind of statement required depends upon the purpose it is intended to serve. Only so far as qualities are irrelevant do propositions about quantity ignore them. Indeed, pervasive qualities, as determined by problematic situations and their resolution, underlie quantitative determination. In counting as well as in the measurement of extensive and of intensive qualities, numerically definite measurement has its units fixed, respectively, by discrete objects, by extended objects, or by functional correspondence between continuous changes in quality and serial positions in a scale.

Each of these types of numerical measurement involves matching. We may count apples, coins, chairs, rooms, houses, persons, books, pages, words, letters, libraries, anything discrete, by matching them against objects of the same or of a different kind, including fingers and toes, letters of the alphabet, and numerals (or number-names). In counting, which we undertake for some end in view (like in-

dicating a significant order), we are guided in our choice of units by hypotheses relevant to a problem. The number of individuals in a collection is a constant character of that collection inasmuch as this property does not depend on who does the counting or on the order in which the objects are counted.

Fundamental measurement is applied to extensive qualities, or properties which can be added. When we measure bodies as extended in space, we enumerate equal intervals marked off in them by superimposed extended bodies; e.g., when we span a tree, pace a distance, or place a measuring-stick against some object. Certain conditions of measurement analogous to rules for counting hold for lengths, and *mutatis mutandis*, for other measurable properties. First, we must be able to construct a scale of lengths, in terms of which we can define equality and inequality in length. We must establish experimentally that two bodies equal in length to the same third body are equal in length to each other; that if A is longer than B, and B than C, then A is longer than C; and that if A is longer than B, B is not longer than A. Next, we must be able, on the basis of some operation like physical addition, to interpret the statement that one body is double another in length to mean: this body is as long as two others combined, or twice as long as either one of them. This procedure we can extend so as to define a series of standard lengths, in terms of which one body may be said to be n times, or $1/n$ th, as long as another. Then too, as equals added to equals produce equal sums, we must make sure by physical experiment that the numbers we have assigned to lengths are themselves consistent. Not only must the same number be assigned to the same body; but our measuring-rod must be straight and rigid, etc. Similar considerations obtain in general for the

measurement of weights, volumes, areas, and other extensive qualities. Two objects having an additive property can be combined by some process by which we obtain an object with an increased degree of that property.

Derived measurement places intensive qualities, or non-additive properties, and continuous changes, in a series of degree. What, in strictness, could it mean to say that a temperature of 100° is double a temperature of 50° ? We match the temperatures of bodies against standard intervals on standardized thermometers. In so doing, we assume, among other things, that if A has a greater temperature than B, B does not have a greater temperature than A; and that if A has a greater temperature than B, and B than C, then A has a greater temperature than C. But, like most of our measurements, that of changing temperatures is indirect and depends upon theoretical assumptions. We cannot show for temperature, density, pleasure, intelligence, etc., as we can for length,

of various sorts may further all kinds of social transactions, standardized scientific formulations are designed primarily to promote inquiry itself. Thus scientific measurements serve discovery of numerical laws connecting physical properties and of the ways in which numerical laws are connected with one another. Liquids, e.g., may be arranged in an order of density such that one liquid (e.g., mercury) is denser than water if some solid will float on the former but sink in water. But density is also invariably related with the ratio of the weight and volume of a liquid, as expressed in the numerical law: $W = cV$; where W is the measure of the weight, V the measure of the corresponding volume, and c has the constant value 0.75 for gasoline, 0.79 for alcohol, 1.0 for water, 1.27 for hydrochloric acid, 13.6 for mercury, etc. Moreover, from the laws that the weight of a given volume of any substance is proportional to its density, and that the density of a gas is proportional to the pressure upon it, we can deduce that the weight of a given volume of any gas is proportional to the pressure upon it; viz., for all cases in which we can establish that if A is proportional to B , and B to C , then A is proportional to C . Beginning with hazy ideas and inexact measurements, the sciences seek to attain to the highest possible degree of accuracy as determined by the ideal of scientific system.

DRILL

Explain and illustrate: qualitative differences; qualitative measurements; numerical measurements; discrete objects; extended objects; intensive qualities; standards of measurement; numerical laws. (See Exercises XVII, 1-11.)

73. STATISTICAL COMPUTATIONS

Statistical techniques arise out of the need of simplify-

ing large collections of numerical data. Instead of trying to keep in mind the separate weights of newly-born boys and girls, we economize comparison by compressing each set into one figure; say 7.6 pounds for boys, and 7.1 for girls. Again, if five measurements of the height of a hill have given the numbers of feet as 293, 301, 306, 307, 313, we may ask: Within what limits may we rely on the average of 304? Moreover, where we cannot control one factor at a time, we sometimes detect general trends; as in the price of milk in our city from time to time. Statistical methods are accordingly designed for the evaluating of group phenomena by an analysis of data based on enumeration and measurement.

One device for the simplification of numerical data is the frequency table grouping items of a series into classes and recording the number of cases that fall into each class (e.g., homes according to monthly rentals); where the number and the limits of the class are chosen in ways most convenient. In comparisons of frequency distributions (say of weekly earnings in one area with those in another), additional methods become indispensable: averages, or figures representing entire groups, to express central tendencies; measures of dispersion, to exhibit the extent of the concentration of the individual items or of their variation from the norm; coefficients of correlation, to mark degrees of connection between two sets of phenomena; etc. Averages in common use include the arithmetic mean, the weighted mean, the median, and the mode, each with advantages and disadvantages.

The arithmetic mean is obtained by adding together a set of quantities and dividing the sum by the number of quantities: $\frac{\sum X}{n}$. If loaves of bread sell in a small city for 8, 10, 11, and 12¢ each, the arithmetic mean would be

10.25¢. Besides depending upon the value of each item in the distribution, the arithmetic mean is easily understood and computed. It is also capable of algebraic manipulation in the sense that, from the average of each of two or more series, we can compute the average of a combined series (such as average net earnings of banks). But the arithmetic mean is considerably affected by extremely large or extremely small items. Although the arithmetic average income in the United States for 1918 was around \$1690, more than 70 per cent of the income earners in that year earned much less. The arithmetic mean accordingly supplies no information about the homogeneity of a group.

Averages often require weighting. Suppose five commodities bring different prices in two different years, the sum of \$3.72 in 1930, and \$4.45 in 1935. With the former as a base (of 100 per cent), the latter are 119.5 per cent (the simple aggregative index number). According to the appended table, the arithmetic mean of the given relative prices in 1935 is 120.4.

Commodity	1930 Price	1935 Price	1935 Relative Price	Weight	1930 Product	1935 Product
A	\$1.00	\$1.17	117	2	200	234
B	0.40	0.30	75	20	2000	1500
C	0.60	0.66	110	1	100	110
D	1.12	1.12	100	5	500	500
E	0.60	1.20	200	1	100	200

However, assigning to the five commodities various weights according to their importance, with a total of 29, we obtain as total of the products of the price relatives and the weights 2900 for 1930, and 2544 for 1935; and as weighted average index of the relatives $2900/29 = 100$ for 1930, and $2544/29 = 88$ for 1935. Inasmuch as relative import-

ance is non-additive, the weighted mean raises the difficult question how a system of weights is to be chosen.

The median is the middle term in a series of items arranged in order of magnitude; in an even number of cases, the median is the arithmetic mean of the two central items. Given: 1, 2, 3, 6, 7, 8, 9; $Md = 6$. Although not as familiar as the arithmetic mean, the median is simple to describe and to determine. Unlike the former, it is not greatly influenced by extreme fluctuations in a group. But it has no algebraic properties permitting the calculation of the median of a composite group from the medians of component groups; when, e.g., 95 is the median value of the IQ's in one class, and 105 in another. Allied to the median in being based upon position in a series are such measures of frequency distribution as quartiles, quintiles, deciles, and percentiles; although these terms sometimes refer to the part of the distribution in which an item falls.

The mode is that item which occurs most frequently in a group. If seven men receive daily wages of \$5, \$6, \$7, \$7, \$7, \$8, \$10, the modal wage (Mo) is \$7 per day. A stable average, the mode often successfully represents the distribution of characters within a group and forms a fair basis for a comparison of different groups. Often, however, the mode cannot be calculated with ease or accuracy; nor can the mode of a combined group be calculated from the mode of component groups. Moreover, the mode may appear at the one extreme of the distribution (as when many students fail to participate in sports); or at several "peaks" in the distribution (as in the case of modal wages for full-time and for part-time labor); or at different positions depending upon the basis of classification (as in the choice of different intervals for examination grades). The value of the mode does not depend upon the value of all the items in a

group; in fact, the mode may not even be typical when the items in the group vary considerably.

Averages constitute points of departure for further study of frequency distribution, not only with respect to central tendencies, but also with respect to variations from central tendencies. For distributions having the same average may in other ways be dissimilar. Among measures of dispersion, let us consider briefly the range, the semi-interquartile range, the average deviation, the standard deviation, and the coefficient of variation.

The range of variation is the numerical difference between the largest and the smallest item in the group. Thus, of two counties yielding an average of fifteen bushels of wheat to the acre, one may yield from ten to twenty, and the other as few as five and as many as twenty-five bushels per acre. Though the range is readily understood and easily computed from the two items on which it depends (provided these are known), the range is obviously unstable.

In order to avoid fluctuations at the extremes of frequency distributions, statisticians sometimes discard the upper and lower quarters of the items and measure the range in terms of the quartile deviation, or the semi-interquartile range: $Q = (Q_3 - Q_1)/2$. If the third quartile of ninety scores is 147.5, and the first quartile is 89, the quartile deviation is $(147.5 - 89)/2 = 58.5/2 = 29.25$. On the assumption that the distribution is exactly symmetrical, the median would be removed by 29.25 from each of these quartiles; at any rate, the scores in the second and third quarters fall within a range of 58.5.

Usually we prefer some measure of scatter based on all the items in a group. By subtracting from each measure the mean, we may calculate the deviation of the former

HOW MAN THINKS

X	Y	x	y	x^2	y^2	xy
25.9	25.4	-3.2	-3.4	10.24	11.56	10.88
32.3	32.2	+3.2	+3.4	10.24	11.56	10.88
26.4	25.5	-2.7	-3.3	7.29	10.89	8.91
32.6	31.3	+3.5	+2.5	12.25	6.25	8.75
32.7	33.5	+3.6	+4.7	12.96	22.09	16.92
24.6	25.7	-4.5	-3.1	20.25	9.61	13.95
32.4	32.3	+3.3	+3.5	10.89	12.25	11.55
25.5	25.1	-3.6	-3.7	12.96	13.69	13.32
28.6	27.7	-0.5	-1.1	0.25	1.21	0.55
30.0	29.7	+0.9	+0.9	0.81	0.81	0.81
ΣX	ΣY			Σx^2	Σy^2	$\Sigma(xy)$
$\frac{\quad}{n}$	$\frac{\quad}{n}$			$\frac{\quad}{n}$	$\frac{\quad}{n}$	$\frac{\quad}{n}$
29.1	28.8			$\sigma_x = 3.13$	$\sigma_y = 3.15$	9.652

$$\text{Consequently, } r = \frac{\Sigma(xy)}{n \sigma_x \sigma_y} = \frac{96.52}{10 \times 3.13 \times 3.15} = \frac{96.52}{98.59} = +.979.$$

from the latter: $X - M = x$. We may then calculate the average deviation: $AD = \frac{\sum |x|}{n}$; where the deviations must be added without regard to the sign, since otherwise $\sum x = 0$. Given: 2, 3, 4, 6, 8, 9, 10; $M = 6$; $\sum |x| = 4 + 3 + 2 + 0 + 2 + 3 + 4 = 18$; $AD = 18/7 = 2.57$.

In the standard deviation, which is the most commonly used measure of dispersion, the deviations are squared before they are averaged: $\sigma_x^2 = \frac{\sum x^2}{n}$. If fifteen people taking a test receive scores of 12, 21, 21, 23, 27, 28, 30, 34, 37, 39, 39, 39, 40, 49, 54, the sum of the squares of the deviations is 1769.77; the standard deviation, $\sqrt{1769.77/15} = \sqrt{117.98} = 10.9$.

Whether a dispersion is large or small, is relative to a standard. Hence a measure of dispersion is sometimes divided by the average from which the deviations are taken. For 22,948 divorces in Wisconsin from 1887 to 1906, the average duration of the marriage preceding the divorce was 10.37 years; the standard deviation, 8.39 years. For 2,651 divorces in 1929, the corresponding figures were 9.83 and 8.26. An increasing variability of marriage duration is shown by the coefficient of variation: $8.39/10.37 = 81\%$; $8.26/9.83 = 84\%$.

The object of statistical studies, as of scientific investigations generally, is discovery of relations such that, from reflection on one factor, we can estimate others. Statistical tools for the discovery and expression of quantitative relations therefore include measures of correlation. Two variables are said to be correlated if, in series of corresponding instances, an increase or decrease in one is accompanied by an increase or decrease in the other; the correlation being

positive when the changes in the variables are in the same direction, and negative when the changes are in opposite directions. For illustrative purposes, let us apply to a simple case (x , temperature of right hand; y , of left) Pearson's

coefficient of correlation: $r = \frac{\Sigma(xy)}{n \sigma_x \sigma_y}$.

The man whose right-hand temperature is 29° C. therefore "probably" has a left-hand temperature of $29 \times .979 = 28.4^\circ$ C. (See Table, page 226.)

DRILL

Explain and illustrate: statistics; frequency table; arithmetic mean; weighted mean; median; mode; range of variation; quartile deviation; average deviation; standard deviation; coefficient of variation; coefficient of correlation. (See Exercises XVII, 12-30.)

74. MISUSES OF STATISTICS

An introductory text in logic may properly list some elementary cautions regarding the interpretation and use of statistical figures. As against Everyman's hasty impressions (that he who learns quickly forgets quickly; that ability in language and philosophy is unimportant in chemistry; etc.), exact enumeration and formulation of phenomena may help answer the previous question: To what extent, if at all, do the alleged facts hold?

However, quantitative statement does not in itself guarantee accuracy or sound interpretation. There was a time when statistical records in India issued from figures put down by the village watchman to suit himself. Nor do the best-framed questions in the questionnaires of our own day always elicit satisfactory responses. Sometimes tabulations bear the stamp of inconsistency on their face; as when

a drop in the enrolment in veterinary colleges from 3,160 in 1916 to 641 in 1921 was reported as 500 per cent. Or comparison with more reliable information may expose hidden inaccuracies; as when median salaries of teachers in different years are given without reference to the hiring of new teachers at lower salaries. At their best, quantitative representations in applied mathematics give us approximations. It is, moreover, grossly misleading when propagandists pontifically turn statistical data to their own restricted uses and when popularizers naively treat statistical findings as discoveries of exact laws instead of as hypotheses to be tested by experience.

Different statistical sources, even if trustworthy, may not be comparable: coal production has customarily been given in short tons, and coal exports in long tons; Canada is said to have imported from the United States 155,000 bushels of wheat in 1927, whereas the United States is said to have exported to Canada nearly 49 million bushels (including wheat re-exported to other countries). Interpreters frequently neglect to differentiate changes in the phenomena from changes in the application of statistical methods: thus paupers may be differently classified as such in different economic situations or in different systems of administration; and vital statistics, census reports, and other records generally have developed through many vicissitudes to their present status and import. It is easy enough to determine the unit to be counted in the case of births, deaths, earthquakes, and the like; but what divergencies in meaning may be covered by an identical terminology when we speak of urban population, intelligence, heart disease, larceny, a democracy, the "real value" of money, stockholders, a large corporation, a room! Different conclusions would follow from the use of absolute numbers and of

percentages: the number of presidential vetoes has increased, but probably not in proportion to the total number of bills submitted for the presidential signature. A variety of hypotheses may account for similar facts; so that, from an increase in the number of arrests, we may infer either an increase in the number of crimes committed or a stricter enforcement of the law.

Statistical numbers summarize information about characteristics of a group of items, known as the "universe" or "population" of individuals, not about any one item in the group. To state, in a comparison of population density, that in 1800 there were 6.1 persons per square mile in the United States and in 1930 there were 41, does not mean that just so many persons occupied each square mile. Similarly, the supposition that, if a coin is fair, the results of previous throws affect the probability of the next individual throw, exemplifies, in principle, the "gambler's fallacy."

Although random sampling counteracts deliberate bias in choice of instances, the instances selected may nonetheless have some property not characteristic of the group as a whole. The statistical worker has not then avoided the fallacy of selection. Any conclusion drawn from 363 inquiries to alumni, none of the 133 replies to which reported an income below \$1500, would be based on biased sampling. When a famous forecast of the United States presidential election in 1936 fell wide of the actual election returns, the failure was in part due to the circumstance that the mailing lists used were heavily weighted with persons in the upper income brackets and therefore unrepresentative of the entire voting population.

High coefficients of correlation do not of themselves prove causal connection. What significance, other than a negative methodological one, attaches to fortuitous correla-

tion between otherwise independent samples? If the rate for broken homes among criminals is high, what about the apparent affiliation of crime with possession of religious information? Which proclaims itself preëminent rather than incidental: the correlation of a civilization with climate or with accessibility to foreign traders? Does observed frequent association, in the absence of information about negative cases, signify special connection: between full moon and fine weather, between red hair and bad temper, etc.? Indeed it is possible to calculate correlations from mathematical data alone without direct reference to the empirical phenomena. A zero correlation between intelligence quotient and number of rooms per person for each of twelve districts may yield, in a combination of individual samples, a correlation of .45; in the use of averages for the twelve districts, a correlation of .90.¹

When automobile accidents and per capita Federal income tax payments are found to go together, may not the two be indirectly connected through some other causal factor, such as high average income? So, in a correlation of stock dividends and stock prices, the choice of the independent and of the dependent variable reflects a causal hypothesis; and an interacting causal relationship may hold between the price and the production of some commodity. Then too, statistical numbers, subject as they are to fluctuations of sampling, cannot without further evidence support an inference from temporary to permanent relations or beyond the clearly conceived and stated limits of the investigations: all gases have a coefficient of expansion increasing with the temperature, except hydrogen.

Forecasting essentially brings into play reasoning by

¹ E. L. Thorndike *Human Nature and the Social Order* (New York: The Macmillan Co. 1940), pp. 981-83.

analogy from past experience: with barometers (e.g., pig iron price changes as indicators of stock-market fluctuations), with trends (on the assumption of their continuance), with deviations from the normal (e.g., heights of prosperity and depths of depression), with historical parallels (e.g., periodic fluctuations of the business cycle), with cause and effect relationships (as determined by weighted opinions of authorities). When Buckle looked forward to the possibility of exact predictions from statistical records of murders, suicides, marriages, etc., he unjustifiably assumed the presence or knowledge of precise factors and their certain continuance. Noteworthy too is the circumstance that, like the advice of fortune-tellers, economic forecasts may have the effect of achieving or defeating themselves.

How then shall we justly estimate the value of statistical methods? Specific statistical investigations are to be judged in terms of their specific purposes by the extent to which they advance our understanding. In an address on "Statistics and Government" in 1918, Professor Wesley C. Mitchell declared:

"The time we spent in framing our war organization and getting it started might have been substantially shortened had anybody in Washington been able to put before the responsible authorities promptly the data they needed concerning men and commodities, ships and factories."²

In general, statistical studies may be said to have an instrumental value in continuous analyses of total situations in the light of fundamental problems, hypotheses, principles, and events.

² Wesley C. Mitchell *The Backward Art of Spending Money* (New York: McGraw-Hill Book Co. 1937 [Permission courtesy of McGraw-Hill Book Co.]), pp. 44 f.

DRILL

Show the need of caution in the use of statistical figures, with special reference to: accuracy; comparability; changes; units; absolute numbers; individual items; fair samples; high correlation; frequent association; reference to phenomena; causal hypotheses; inference to more inclusive groups; forecasting; analysis and evaluation. (See Exercises XVII, 31-56.)

Historical Inference

75. HISTORICAL SOURCES

It is in the present that I recollect a past personal experience or action, such as having received a kindness or paid a debt. Should anyone doubt or dispute my memory-affirmation, I may support it by several kinds of evidence: recollections of related events; corroborative facts in the present; expected future consequences; the general dependability of my reconstructions of temporal sequences in my life.

So, too, do the thoughtful witness of an automobile collision, the geologist studying a mountain, the physician diagnosing an illness, verify their impressions. Their narrations and descriptions take in extensive sequences of events and sets of interacting circumstances. In order to confirm assertions about special occurrences in nature or in the life of another, we appeal to present facts, observed and interpreted, as a basis of inference concerning the past; to earlier observations recollected and experiences analyzed; to the outcome in view of which our searchings are instituted; to all these factors woven together into a continuous course of events from its inception to its close.

No different in principle are the historian's ways of establishing the reliability of one judgment or another about a continuum of events recognized to be historical. It falls

within the historian's province to explore and record what men have done. He writes in order that their deeds might be remembered and understood. What we call "history" therefore includes facts evolved and enacted as well as their narration and literary reproduction.

No one would, if he could, preserve the entire past. People choose their activities and select the achievements they want to commemorate. The historian selects his theme and the manifold details and techniques essential to its elaboration. All who utilize the products of the past for some end select what is pertinent to an eventual issue. The memories which come within the professional historian's sphere have public and enduring significance: a singular event, like the discovery of a "new world" in 1492; typical acts, like those of a general or a ruler; collective behavior, as in war and peace. The historian may, within his limitations, elect to investigate any type of human enterprise whatever: military, political, industrial, economic, religious, cultural, educational, philosophical. But always his purpose is faithfully to record and interpret, a purpose he executes with varying degrees of failure and success.

Evidently, no one can have directly before him a happening of the past or an exact duplicate of it. The writer or reader of a history of the war from 1914 to 1918 begins, literally, not in 1914, but at the time of his writing or reading. It is from materials available in the present that we infer what has taken place in the past. Where the past has left no trace or vestige whatever in the present, no such inference is possible. Extant historical "sources" comprise "remains" fitted to yield the desired information and "traditions" designed to serve the discovery and validation of historical phenomena. Both kinds of "sources" are products of past activities. Remains of bodies, garments, foods, build-

ings, weapons, implements, ornaments, manuscripts, customs; traditions oral (like songs and tales), pictorial, and written (like inscriptions, chronicles, biographies, memoirs, public documents)—these suggest the indefinite variety of historical data.

It becomes necessary therefore for the historian to exercise an uncommon critical ingenuity. But recent historiographers have developed effective techniques for the sifting of their data.¹ They also have at their disposal the methods of such auxiliary sciences as epigraphy, paleography, archaeology, numismatics, sphragistics, linguistics, diplomatics, and ethnology. Especially relevant to historical studies are the sciences and philosophies of man. In all this division of labor, part of the logician's task is to examine typical inferences employed in historical investigations.

DRILL

Distinguish types of evidence for memory-affirmations, for narrations and descriptions, and for historical inferences; types of selection, of "sources," and of auxiliary sciences in historical inquiry. (See Exercises XVIII, 1-8.)

76. AUTHENTICATION OF SOURCES AND FACTS

The historian must test for their credibility both the sources he collects as well as the facts he infers from them. Source criticism pertains, in the first instance, to remains or traditions existing contemporaneously with the historian. However, such critical examination of the sources has historical functions, consisting as it does in the weighing of data as evidence for conclusions about past events.

A given supposed source may be genuine, or a forgery, or erroneously attributed to some author, time, or place. Who was the author of a series of letters written at

¹ Ernst Bernheim *Lehrbuch der historischen Methode und der Geschichtsphilosophie* (6th ed.; Munich and Leipsic: Duncker & Humblot 1914).

Versailles and Paris in 1789-90 and published in 1894 as the *Journal d'Adrien Duquesnoy*? A noted historian interested in this question² argued as follows: Though not in the handwriting of Duquesnoy, the letters contain cross-references and similar expressions which point to a single author; the author was a member of the third estate, represented Barrois, was on familiar terms with the deputies from Nancy, held membership on the Committee on Food Supply, and addressed these letters to the people of Lorraine; all these conditions were met by Duquesnoy; therefore Duquesnoy was probably the author. On the other hand, numerous letters with the signature of Marie Antoinette present plain cases of forgery.

It is well known, too, that scholars have replaced the traditional theory of the Mosaic authorship of the first five books of the Old Testament by the theory that they are, in parts and collectively, compilations based on at least four sorts of documents. J, referring to God as Yahveh, had recast Canaanite stories to conform to Israelite ethical ideas and to account for the subjection of the Canaanites. E, referring to God as Elohim, had described Israel as properly a theocracy in a promised land. In D, a Deuteronomist compilation, the history of Israel and Judah had been told in terms of reforms, the book of Deuteronomy itself being brought into a literary unity by means of three farewell speeches ascribed to Moses, and the reforms advocated being adopted under King Josiah in 621 B.C. The interest of P, the Priestly Code, had centered in religious rites and institutions.³

The condition of a text frequently deserves careful scrutiny. Our earliest manuscripts of the New Testament

² Fred Morrow Fling *The Writing of History* (New Haven: Yale University Press 1920), pp. 62-64.

³ R. L. Ottley *A Short History of the Hebrews* (New York: The Macmillan Co.; Cambridge: The University Press 1901), Appendix I;

J. A. Bewer *The Literature of the Old Testament* (New York: Columbia University Press 1922);

R. H. Pfeiffer *Introduction to the Old Testament* (New York: Harper & Bros. 1941).

are generally dated as of the fourth or fifth centuries A.D.;⁴ of the Old Testament, as of the ninth or tenth. Running two words in the Hebrew together as one, in Amos 6:12, accounts for the reading: "Does one plough with oxen?" instead of: "Does one plough the sea with an ox?" Because of an original ambiguity in the sound *lo*, Psalm 100:3 has been made to say: "It is He that made us, and not we ourselves"; instead of: "It is He that made us, and we are His." Glosses and interpolations often find their way into a text; as when a commentator explains, in John 4:9, that "Jews have no dealings with Samaritans." On the assumption, confirmed by experience, that copies containing the same anachronisms, or other common errors, have been derived from one another or from a common source, we may attempt to arrange extant copies of ancient documents in genealogical tables so as to distinguish various strata of manuscripts, with special reference to their dependence or independence. For, if one witness is to corroborate another, their testimony must be mutually independent.

How documentary analysis contributes to an estimate of factual reliability, may be illustrated from *Lives and Opinions of Eminent Philosophers* by Diogenes Laertius. Our traditional histories of Greek philosophy have been based primarily upon this work; yet it is a gossipy compilation, made with much diligence, but with little comprehension, for a popular audience. Mention of one Saturninus seems to fix the earliest possible date of the composition (the *terminus post quem*) in the earlier decades of the third century of the Christian era. There are scarcely a dozen indications of the existence of the work from the sixth century (the *terminus ante quem*) to the period in which the oldest manuscripts and the earliest printed editions fall

⁴ E. F. Scott *The Literature of the New Testament* (New York: Columbia University Press 1932).

(the twelfth and the fifteenth centuries, respectively). Nor can the place of the writing be determined; though the earliest known references to Diogenes Laertius come from the East (Byzantium).

The book contains no less than 1,186 explicit references to 250 authors. But similar sources, such as comic poets, provide different kinds of information, biographical, chronological, literary, scientific, philosophical. Similar subject matter, like astronomy, is vouched for by different authorities, including astronomers, historians, philosophers, and poets. From the writings of philosophers which have come down to us, where we can therefore test his methods, Diogenes Laertius collected trivial items; as when he says that, according to Plato's *Symposium*, Socrates on occasion wore fine clothes. Even thinkers of the first rank, like Plato and Aristotle, were viewed by Diogenes Laertius and his audience as schoolmasters to be praised for their eminence in view of their alleged sayings, of anecdotes about them, and of their miraculous achievements.

Other secondary sources for the history of Greek philosophy may be analyzed in similar ways. As primary source material in this field we have little more than the writings of Plato and Aristotle. The conclusion seems warranted therefore that an authentic history of Greek philosophy cannot be written. Socrates and others have become, for their posterity, symbols of various attitudes. One of the arguments underlying these contentions may be formulated as follows: An uncritical and popular compiler writing some seven centuries after the event is hardly in a position to give an accurate account; Diogenes Laertius was such a compiler of items from ancient Greek philosophy; it is highly probable therefore that he distorted his subject.

DRILL

Explain what is meant by source criticism; by textual criticism; by the mutual bearings of authentication of sources and facts. (See Exercises XVIII, 9-16.)

77. HISTORICAL INTERPRETATION AND
EXPLANATION

How can a reader guard against reading into a text his own preconceptions? He must master the general sense of the language of a document; must familiarize himself with the language of an author and of the author's country and time; and must interpret the author's meaning in terms of what the latter was trying to do and in the light of the immediate and wider context in which a passage occurs. When the captive Israelites in Babylon prayed: "We have become like men unclean, our goodness is a filthy rag,"⁵ they were not laying down an orthodox ecclesiastical dogma of the total depravity of human nature. They were complaining as might a prisoner in a concentration camp in recent days: Here I am an outcast, and all the values I have cherished have become objects of contempt.

If the historian aims not only to chronicle but also to understand some portion of the past, he must be sensitive to significant meanings and connections. It needs but a brief glance into our immediate surroundings to convince us that a knowledge of the results of past ages is requisite to a right comprehension of our own times: our food, clothing, furniture; our customs and institutions; our books, words, ideas; and so on indefinitely. For events have antecedents and consequences. No beginning or end, however precise, is cut off from all connection. It is because we can trace antecedents and consequences of events that we can treas-

⁵ Isa. 64:6.

ure and profit by the experience of the race. But it is neither possible nor necessary to explain all things by some supposed absolute first or last event or by some alleged single course of evolution.

Our fathers founded the institutions under which we live because they acted in certain ways, made certain experiences, and entertained certain purposes. Such dynamic factors are basic to historical explanation. That is why the historian, emphasizing the qualitative, individual, and unique, cannot leave altogether to the sociologist an interest in the quantitative, general, and repeatable. Canons of scientific method are required at every point at which we test historical inferences. We derive the assumptions in terms of which we study past activities from activities going on in our own living present. In other words, the writing of a history is itself a historical event. If the present is illuminated by the past, it is even more profoundly true that comprehension of the past depends upon knowledge of the present.

No single theory of history can completely explain all types of events or even, in many cases, a single complicated event. Beard, in his *Economic Interpretation of the Constitution of the United States*, attempted to show that landed, manufacturing, mercantile, and moneyed interests stood to gain, from the new system, economic stability, repayment of debts, and protection of their holdings; whereas the opposition, chiefly non-slaveholding farmers and debtors, wanted freedom from taxation and generous concessions in the repayment of obligations. According to McLaughlin, however, the Constitution replaced the Articles of Confederation because of the recognized need of a central authority to regulate commerce, to obtain revenue, to make treaties, to secure national stability, to adjust state jealousies.

A variety of historical approaches is not in every instance incompatible, but often mutually supplementary. One theory may succeed, where another fails, to make clear different factors: the influence of great men; the rise and fall of states; the growth of institutions and customs; the winds of doctrine; the interplay of economic forces; the aspirations of the common man. Hence we may properly hold to a pluralistic view of history.

There are many reasons why histories need to be re-written from generation to generation. New sources may be discovered; new relationships noticed; new tests and principles applied; new aspects and interests explored. Above all, many events continue to have consequences. The past cannot be undone; but truths about it expand in significance with every fresh perspective.

"Polybius, profoundly impressed by the dominating position of Rome in the Mediterranean basin, as he saw it with his own eyes, asked himself how it had been attained. The answer to that question, given by his constructive imagination, was the most important work of historical exposition produced by antiquity. Bryce, dwelling upon facts long known and often noted, saw a vision of the *Holy Roman Empire* which gave unity to a thousand years of European history.

"Gibbon had a vision that supplemented the vision of Polybius and described *The Decline and Fall of the Roman Empire*, while Thierry, in his *Tableau de l'Empire Romain*, described as a whole the two phases of Rome's work which had been treated separately by Polybius and Gibbon."⁸

DRILL

Explain: contextual interpretation; historical continuity; historical explanation and scientific method; the

⁸ Fling *op. cit.*, pp. 162 f.

pluralism of history; the constant need of rewriting of history. (See Exercises XVIII, 17-30.)

78. MATERIAL AND RATIONAL PROGRESS

Some look to written histories for romantic stories; others, for partisan propaganda; still others, for pure historical knowledge. Historical study does enlarge our experience. But wisdom comes by learning from experience, our own and that of others. Why should it not be possible to enjoy as well as to understand and profit by life's lessons? History does not, however, of itself provide unambiguous guidance in moral perplexity. Shall we accept the standards of our first or last teacher, of the oldest or newest institution, of a majority or minority, of conservatives or reformers, of aristocrats or proletarians, of astrologers or poets, of primitives or moderns, of Orientals or Occidentals, of Greeks or Romans, etc.? Moreover, a discriminating reader will find facts mingled with speculation in Augustine's version of the conflict between the earthly city and the city of God; in Hegel's dialectic of ideas from the absolutism of the East through our classical heritage to the self-conscious freedom of Hegelian political philosophy; in Comte's three stages, the theological, metaphysical, and positivistic; in Spengler's supposition of life-processes of an "organic" society with a culture-spirit; in conceptions of history as a temporal unfolding of some unitary fixed story, past, plan, purpose, substance, force, order, law, time, trend, evolution.

Morals and moral judgments (broadly understood) form an indispensable part of the content of any adequate history. The historian cannot exclude, but must weigh the evidence relevant to, the moral questions which arise in the course of his investigations. Why did the Renaissance

humanists revolt from ecclesiastical ethics? Wherein did the policies of the early Protestant reformers differ? How could Philipp II of Spain take pride in the inquisition? To which social classes did the ideas of Locke, Montesquieu, and Rousseau appeal? Did French foreign policy, down to recent years, deviate essentially from the path marked out by Richelieu and Louis XIV? What was public sentiment under Wilson towards the League of Nations? In what terms can the ambitions of different peoples today be better understood? In short, shall the historian's criteria be openly avowed or not?

Men not only write but make history. They may participate in historical processes with a sense of devotion and obligation to their careers. Having conceived progress in terms of the utilizing of materials for chosen ends, they may undertake to estimate what progress they have made in the past. The mastery of physical resources constitutes material progress as measured by the standards of economy and efficiency. But the intelligent pursuit of ideal ends constitutes rational progress as measured by the standards of the perfected exercise of our natural impulses and reflections. It does not help us, in transforming the possible into the actual, either to view ourselves as sheer products of the past or to try to break with the past and to live wholly in the present moment. By learning what has happened in the past, we obtain a basis for our expectations of what may happen in the future, for a better appraisal of our external and inner resources, and for our hope of bequeathing to others something that may live in their imagination. For man, then, "the purpose of history is not a secret he vainly tries to find, but a kind of life his reason enables him to live."⁷

⁷ F. J. E. Woodbridge *The Purpose of History* (New York: Columbia University Press 1916), p. 89.

DRILL

Comment, in general, on the lessons of history; on moral judgment in historical inquiry; on the distinction between material and rational progress; on "the purpose of history." (See Exercises XVIII, 31-43.)

Judicial Proof

79. LOGIC AND THE LAW

A lawyer analyzes the facts of a situation in order to construct a statement of them not unfavorable to his client. He also examines records of similar cases in order to find rules for interpreting the facts. As his thinking develops, in close interrelation, the conclusion to be reached and the premises on which it is grounded, his selection of rules and his selection of facts to be stated modify each other in turn. Further qualifications are made as he studies his case from the standpoint of the opposing party.

Given the rule that a trespasser is liable in damages in dollars commensurable to the injury done to the plaintiff, and the complaint that the defendant trespassed on the plaintiff's land, the conclusion may be drawn that the plaintiff has been damaged in the sum of X dollars. Raising an issue of law, the defendant may "deny" the major premise by a "demurrer" to the effect that the complaint does not state a cause of action. Or, raising an issue of fact, the defendant may "deny" the minor premise by an "answer" that he did not do those acts with which the complaint charged him. If the defendant "confesses" the facts charged, he may still endeavor, in his "answer," to "avoid" the premises by new matter showing that, notwithstanding,

there is no liability. To the extent therefore that correct pleading accords with logical principles, it exemplifies correct reasoning which may or may not be explicitly set forth in the form of syllogisms.

Courts, in expounding the decisions they have reached, state justifying reasons. Such rational statement in judicial decisions is a safeguard against arbitrary pronouncements. It also enables the legal profession to evaluate the decision in its function as an indicator of rules for dealing with similar cases in the future. For, within reasonable limits, men need to know what legal consequences they can count on in their transactions. When rules of law are wanting, or are too rigid, practical and social instability results; and the demands of analytical thought concerning human needs fail to be met.

In reasoning about matters of law, we accordingly inquire into probable consequences; although here also necessary inference enters into determination of probable inference. General legal principles function as hypotheses to be tested in their application. Thus, in consequence of the Industrial Revolution, old laws about masters and servants had to undergo modifications; the very purpose of those laws had been the securing of orderly methods in the achievement of social welfare in existing situations. So a constant problem of the law concerns the formulation of rules adequate to existing social needs and their re-formulation as new needs arise. In the first *Ritchie* case in 1895, the Supreme Court held the eight-hour law for women unconstitutional because arbitrary and unreasonable. But, in the second *Ritchie* case in 1910, the law was held constitutional because of the evils resulting from unrestricted hours, in contrast to the social and industrial benefits attending curtailed hours.

Logic does not dictate what premises and conclusions to accept, but articulates the connection between them and the general need of learning from experience. Logic is not a substitute for all knowledge; whether of extra-legal factors in legal thought, or of the plurality of legal processes and doctrines themselves. If a prediction of a judicial decision is probable, the probability of its contradictory must also be taken into account. When law, however, becomes a subject matter in discourse, it becomes subject to the same rules of inference and of the continuity of discourse as does any other subject matter. Here too experience and reason, adequately conceived, are not incompatible but correlative. The abuse of logic in the law calls for an outcry against bad (not against all) logic and for abandonment, in theory and practice, of a worse for a better logic in the law.

DRILL

Illustrate applications of logic in a lawyer's preparation of his case; in correct pleading; in judicial decisions; with respect to legal principles; in law as a subject matter in discourse. (See Exercises XIX, 1-9.)

80. TESTIMONIAL EVIDENCE

Technical rules of evidence, as developed in the law, limit the admissibility of evidence. There are variations of degree in the directness or remoteness with which evidence may bear on the point at issue. Some evidence is excluded in the interest of economy of time; or in order to promote confidence between priest and penitent, physician and patient, man and wife. There are also various other exclusionary rules designed to protect the jury from evidence which judges feel is likely to influence them beyond its proper weight.

In general, the law distinguishes preponderance of evidence, regarded sufficient in ordinary civil cases; proof by clear and satisfactory evidence, required for reasons of policy in certain types of civil cases; and proof beyond reasonable doubt, required in criminal cases. As to the kinds of evidence offered, a basic twofold distinction is recognized between testimonial and circumstantial evidence: the former is made up of direct statements of witnesses on the stand, asserting observation of the ultimate fact; the latter consists of testimonial evidence which requires use of logic to reach the conclusion that the ultimate fact exists. As a rule, problems of judicial proof present themselves in masses of mixed evidence. The weighing of evidence, moreover, resembles procedures of historical inference: the testimony and the evidential facts proffered are of the present; the facts to be proved are of the past; and judicial inquiry has consequences for the future.

As has been pointed out, assertions taken in court as a basis of inference to the matter in hand constitute testimonial evidence. This includes, besides declarations made on the witness stand, any statements made under the exceptions to the hearsay rule. As elsewhere, so in court, a common understanding with people whose experiences differ widely from our own is seldom easily achieved. But granted that the meaning of the testimony is properly interpreted, the testimony must also be evaluated as to its trustworthiness. Hence the questions: what is the testifier's race, age, sex, mental state, character, bias, experience, and capacity for perception, recollection, and narrative reconstruction? Is the testimony which has been presented confirmed by independent evidence?¹

Testimony is notoriously fallible. For the purpose of testing the worth of accounts given by eye-witnesses, inci-

¹ John Henry Wigmore *The Principles of Judicial Proof* (Boston: Little, Brown & Co. 1913), Part II.

dents like "holdups" have been experimentally dramatized in the presence of college students; with the result that the reports of the spectators contained more inaccuracies than correct statements. A medical expert, having testified to the mental unsoundness of an interested person, could not recall, under cross-examination on the next day, a feature of the alleged paresis he had mentioned second; that feature, the cross-examiner reminded him, was "loss of memory of recent events." According to the story of Susanna, in the Apocrypha, one elder claimed to have seen her "under a mastick tree," whereas another testified to having seen her "under an holm tree"; so that Daniel, assuming the ability of the witnesses to distinguish one tree from another, and assuming that if a witness will lie about a tree he will lie about what he saw under it, rejected the testimony of both as false.

DRILL

Explain and illustrate: exclusionary rules of legal evidence; preponderance of evidence, proof by clear and satisfactory evidence, proof beyond reasonable doubt; testimonial and circumstantial evidence; judicial proof and historical inference; interpretation and evaluation of testimonial evidence. (See Exercises XIX, 10-30.)

81. CIRCUMSTANTIAL EVIDENCE

Evidentiary facts produced or cited to prove a claimed conclusion come under the head of circumstantial evidence; although most of such facts must themselves be attested by assertive evidence. Among frequently occurring matters to be proved, we are told,² are occurrences in nature, the identity of a thing or person, human traits, and the performance of some act. Evidentiary facts comprise circumstances

² *Ibid.*, Part I.

which have preceded, attended, or followed those to be proved. Often the weighing of circumstantial evidence requires expert knowledge; like knowledge of botany, to study the preparations made from juices of plants.

To give a few simple examples of types of circumstantial evidence: X died from lead-poisoning; since he had been a potter working with lead-glaze, he had probably absorbed the poison in the course of this activity. The claimant to the Tichborne estate was ignorant of many things well known to Roger Tichborne; there were no less than fifteen significant coincidences in his life with that of Arthur Overton; he was therefore probably the latter. If A boarded a railroad train, he probably had some opinion about its destination. X, having been at another place at the time of a murder, probably could not have committed the crime. Reckless extravagance of one poverty-stricken just before may throw suspicion of theft upon the alleged offender. The fact that two railroad conductors alternating on similar round trips reported receipts in different amounts does not suffice to prove that the difference had been collected and embezzled by one of them.

As a rule, no single line of argument or evidence can by itself render a judicial conclusion probable. The proof of guilt must rest upon combined evidence. The cumulative effect of the argument presented may, however, point to a defendant's innocence. For, to the reasoning that, if and probably only if X is guilty, certain phenomena should be observed, they have been observed, so that the guilt of X may be indicated, the replies may be made: Do such phenomena always ensue? Are they precisely what they seem to be? Do only liars sweat?

Consider too why we cannot conclusively argue: If X were innocent, certain consequences are to be expected;

but these have not been observed; and X is thereby proved guilty. Though formally valid, this argument offers negative evidence which may, as such, be refuted by later positive observation. On the assumption that, if the locomotive engineer had blown his whistle, it would have been heard, a plaintiff, claiming not to have heard it, may maintain that the engineer did not blow his whistle. Nevertheless witnesses for the defense may later testify to having heard the whistle.

DRILL

Explain and illustrate: types of circumstantial evidence; cumulative evidence; negative evidence. (See Exercises XIX, 31-39.)

82. SOCIETY AND THE LAW

Legal institutions serve the social function of promoting justice. Does this comment necessarily neglect the seamy side of life? Or does it call attention to deeply-felt human needs; to an ideal attainable at least in some measure; to the kind of conviction essential to social institutions which would command the lasting respect of mankind?

True, ingrained habits of individualistic thinking tend to thrust considerations of justice into the background. So does the professional's lack of experience and of knowledge concerning social problems affecting a community. But the insight has gained ground among us that application of the law involves analysis of living situations as well as comprehensive understanding of legal principles. General security, morals, and progress; conservation of resources and the maintenance of wholesome surroundings; institutional and individual life—these are among the social interests increasingly recognized by the law. There is no in-

herent contrast therefore between legal and social science or philosophy; the relevant contrast is rather between social attitudes unarticulated and social attitudes scientifically and critically articulated.

Men direct social processes. However, the social order can ill afford leaders without sound principles. And the principle of justice is extraordinarily fundamental. Says the *Institutional Treatise* published under the authority of Emperor Justinian: "All nations who are ruled by laws and customs are governed partly by their own laws and partly by those laws which are common to all mankind." What civilized society can exist without abstention from aggression; without beneficial uses of acquisitions; without practice of good faith; without due regard to the consequences of actions; without restraints within proper bounds? If the literature on justice raises difficult questions (concerning, e.g., the natural and the conventional), they pale before the havoc wrought by systematic neglect of reflection on justice and on life's ultimate values.

We hold men responsible for their conduct because they can think and progressively learn the uses of things. Scientific knowledge aids us in intelligently determining where responsibility lies and what possibilities of betterment are to be found in the discoveries we make from time to time. Civilization, with all its institutions, offers the brightest of prospects in the measure in which it ministers to more than immediately practical uses by fostering interests which may permanently satisfy the human spirit on their own account. To the extent that the law holds to ideals of justice in all their ramifications, the law is one of the vantage-points from which the various areas of life can be surveyed in a comprehensive and ennobling manner.

HOW MAN THINKS

DRILL

Comment on legal institutions from the point of view of social needs, interests, principles, and responsibilities. (See Exercises XIX, 40-44.)

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seven

Demonstrating Our Theorems

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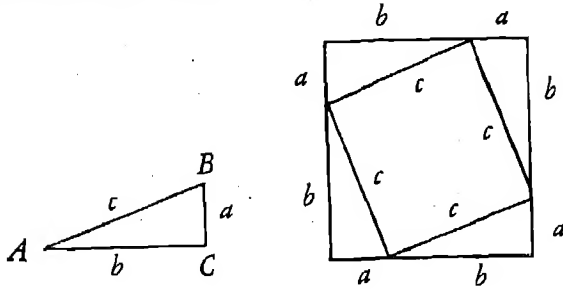
Demonstration

83. FIRST PRINCIPLES

The Pythagorean theorem states that in a right triangle the sum of the squares on the two sides equals the square on the hypotenuse. Given the right triangle ABC , construct a square of side $a + b$, and locate the four right triangles, all congruent to the given right triangle. The sides of the quadrilateral within the large square are all equal, and the angles are all right angles, since the sum of the adjacent angles of the triangles is 90° . Hence the quadrilateral is a square whose area is c^2 . But this shows that:

$$(a + b)^2 - \frac{4ab}{2} = c^2 \text{ or: } a^2 + b^2 = c^2$$

which proves the theorem.



Knowledge gained by such demonstration is reasoned, determinate, and necessary. The demonstration has shown

the reason why, in a right triangle, the sum of the squares on the two sides equals the square on the hypotenuse. It is that fact rather than some other which has been demonstrated. We know both that and why the fact under consideration cannot be other than it is.

As another simple model of demonstration, let us formulate a miniature system. Take as a base a class K of entities "a," "b," "c," etc., (such as persons or railway stations), and a relation, or rule of combination, R (such as ancestor of, successor of, west of). Whatever our subject matter, if we wish to demonstrate anything about it, we must assume that it is; that the meanings of the terms in predications about it are understood; and that reasonings about it conform to laws of thought.

Let us suppose (1) aRa is false. No element "a" has R to itself. The relation R is irreflexive. (2) For any two distinct elements "a" and "b," either aRb or bRa . Since the relation R obtains between any two elements, it possesses connexity. (3) For any three distinct elements, if aRb , and bRc , then aRc . The relation R is transitive. Such primitive propositions, which determine the nature of a deductive system, must be consistent and should preferably also be independent and, ideally speaking, complete (or sufficient but not more than sufficient).

Note a theorem which follows from the assumptions stated: (i) When aRb is true, bRa is false. In other words the relation R is asymmetrical. For if aRb and bRa were both true, then by postulate (3), aRa would be true; which contradicts (1) that aRa is false. Substitution of symbols in this procedure is an operation by means of which initial and subsequent formulations are progressively modified so as to yield an ordered series of interrelated theorems,

or derived propositions, which satisfy the demands formulated in the primitive propositions.

Modern mathematicians use more or less interchangeably such terms as "assumptions," "axioms," and "postulates." Historically, distinctions between such terms seem to have originated in the endeavor to contrast accurate scientific discourse with ordinary conversation, both of which were subjected to analysis in Aristotle's logic. Accordingly, demonstration was said to require, among first principles, definitions (of manifestations or attributes of the "genus" or subject matter) and postulates (or factual predications introduced by a special science). Postulates, when assented to, were called "hypotheses," in the sense of starting-points whose existence was given. Taken together, the postulates and definitions which were laid down formed the "theses," or special principles, of a scientific system. Distinct from these were the "axioms," which included first principles common to the sciences, along with statements containing terms of a presupposed science. Thus a typical "common notion" would be to the effect that one of two contradictions must be true.

In order to serve the purposes of adequate demonstration, first principles must satisfy a number of criteria. Aristotle pointed out¹ that the propositions from which a scientific system is deduced must be true (or asserted); must be first or primary, as means of demonstration; must be immediate or independent, not themselves subject to demonstration in that particular discourse. They must, moreover, be better known in their denotations and connotations than the propositions to be established by the reasoning; they must be prior to the derived propositions, not indeed in the order of discovery, but in systematic order of dependence; and they must state the logical grounds upon which

¹ Aristotle *Posterior Analytics* i.2-6.

the propositions deduced rest. On the conditions sketched, the "first principles" or "lordly things" (*archai*) of a science will be appropriate (*oikeiai*) to the demonstration; and the demonstration will issue in scientific knowledge. Obviously too, the fundamental principles of our knowledge and convictions ought to be as few and as certain as possible. So far from being loaded dice or a packed hat, they are discovered after much labor. By means of them, one proposition is linked with another in a common system, in an order of coherence.

Nor do we, in constructing a deductive system, face the alternative of having to demonstrate everything or nothing. In all scientific demonstration, there are propositions which are demonstrated and propositions by means of which the demonstration proceeds. True, definitions explicit in one system may be implicit in another; and axioms or postulates in one system may be treated as theorems in another. Yet each system involves the distinction between statements derived and underived. The latter are super-altern to the former: given the assumptions, the conclusions follow; but not necessarily vice versa.

If a scientific system is to demonstrate "necessary truths," more is required of its first principles and means of demonstration than of premises in the "dialectical" or conversational syllogism. It is not enough that basic truths apply to their subject matter universally, or in every instance and always, as in the ordinary subsumptive syllogism: Every S is P if every S is M and every M is P. Restricted to subsumptive deduction, we could prove for all right-angled triangles such formulas only as had been established for all triangles and for all right-angled figures.²

Nor does it suffice for basic truths to characterize a subject "as such": that is, not by way of accident or coin-

² W. E. Johnson *Logic* II.124.

cidence; but by way of definition (a point being a constituent of a line; a line, of a triangle; etc.) or by way of property (a line being either straight or curved; a number, either odd or even; etc.). This demand may also be met in the "functional syllogism,"³ in which a set of major premises (If $M = 7$ then $P = 1/49$, if $M = 11$ then $P = 1/121$, etc.) is collected into a general formula: "Acceleration varies inversely as the square of the distance"; or $P = 1/M^2$. Here P does not only hold for every instance of M , but is a function of M for all values of M : $P = f(M)$. This mathematical equation serves as a single supreme premise applicable to other values of M : if $M = 8$, $P = 1/64$; if $M = 10$, $P = 1/100$; etc. Hence, for different minor premises ($M = 7$, $M = 10$, etc.), we may (by the principle of the constructive hypothetical syllogism) draw different conclusions ($P = 1/49$, $P = 1/100$, etc.). Such functional syllogisms demonstrate factual conclusions from factual premises.

Above all, strict demonstration depends upon basic truths asserted commensurately with the subject "as a whole." For, in showing that triangles have their angles equal to two right angles, demonstration of the theorem has regard to triangles as triangles, not as instances or as kinds of triangles. To attempt the demonstration for every "sample" of a figure, would be to cover too wide a range; and to work out the demonstration "primarily" for every isosceles triangle only, would be to narrow the range quite as unduly as if it were restricted to every brass isosceles triangle only. Consider how, in "functional deduction,"⁴ formulae may be deduced from previously established formulae. Given several non-limiting equations: $ab = ba$, $(a + b) + c = a + (b + c)$, and $(a + b)c = ac + bc$, we may deduce the new formula: $(a + b)(c + d) =$

³ *Ibid.*, p. 104.

⁴ *Ibid.*, pp. 128, 131.

$ac + bc + ad + bd$. In consequence of the substitution of the compound symbol $(c + d)$ for the simple symbol c , the new formula, containing four independent variables, covers a wider range than do the premises.

Mathematical induction, so called because the conclusion generalizes from the premises, rests upon principles of deductive demonstration:⁵ If a property belongs to the number 1, and if when it belong to n it can be demonstrated to belong to $n + 1$, then it belongs to all finite integers. Thus $1 + 3 + 5 \dots + (2n-1) = n^2$. Adding $(2n + 1)$ to both sides, we get: $1 + 3 + 5 \dots + (2n - 1) + (2n + 1) = (n + 1)^2$. Hence the theorem, if true for the integer n , is true for $(n + 1)$. Now, for $n = 1$, $1 = 1^2$; therefore, for $n = 2$, $1 + 3 = 2^2$; for $n = 3$, $1 + 3 + 5 = 3^2$; and so on for each successive integer.

A deductive system seeks to provide rigorous and fruitful demonstration of theorems from basic truths, both types of propositions including such as are "commensurately universal." Neither accidental nor popularly accepted propositions, even if they happen to be true, satisfy the conditions of systematic interrelation. For not all truth is equally fundamental or appropriate to a determinate subject matter; and not all that goes by the name of knowledge is reasoned or scientific knowledge. In short, without fundamental facts and principles, there can be no system, whether of mathematics, of politics, of education, or of anything else.

DRILL

What is required for knowledge to be reasoned, determinate, and necessary? What is required of implicit and explicit assumptions and of theorems in a system? Distinguish: types and criteria of "first principles"; demonstrated and undemonstrated propositions in a system; subsumptive

⁵ *Ibid.*, pp. 133-35.

sylogisms, functional syllogisms, functional deduction, mathematical induction; rigor and fruitfulness of demonstration. (See Exercises XX, 1-24.)

84. SCIENTIFIC SYSTEMS

Elaborate deductive systems may be devised for many sorts of subject matter. Euclid, dealing with theorems in geometry, exhibited them as truths systematically connected on the basis of definitions, postulates, and common notions. Spinoza, in studying what others had to say about human loves and hates, found a limited number of definitions and axioms by which a large number of propositions could be linked with each other in a system. Both Euclid's *Elements* and Spinoza's *Ethics* may fittingly be characterized as "scientific" with far greater justification than restricted "investigations" which do not advance understanding. What is to be said therefore about the role of special and common considerations, of facts and relations, and of the inferior and superior, in demonstration?⁹

Geometry cannot, of course, prove whether medicine deals with both sickness and health; whether $aa = a^2$ or a ; whether the straight line is the most beautiful of lines or is the contrary of the circle. Principles and deductions of each science must be peculiar to the kind of subject matter with which that science is concerned. If a demonstration passes from one to an altogether different set of considerations, it would at best apply incidentally; just as, in general, we cannot by sheer reasoning establish any contingent fact we please. Where, on the other hand, propositions of one science are drawn upon for demonstrations in another, the sciences in question are not unrelated, but one is subordinate to the other.

⁹ Aristotle *op. cit.* i.7-34.

So too, principles common to all the sciences, like laws of logic, do not suffice alone to demonstrate truths belonging to a special field such as astronomy. Do we not, however, in numerous branches of knowledge, appeal to the law that "taking equals from equals leaves equals"? Yes, indeed; but only by way of analogy and only insofar as such common principles are relevant to a particular science. For the purposes of any special science, its subject matter, definitions, and the like must be worked out in the terms proper to that science itself. Hence also it is not pertinent to object that a geometrical figure cannot be drawn to the perfection requisite for mathematical demonstration: the demonstration pertains to what a diagram, as an illustration, may reveal for the purpose in hand. Nor would it be reasonable to demand that a law of logic employed be expressly stated on every possible occasion.

All this implies that no one science is under obligation to answer every question. A question or an argument is just as "unscientific" when it falls outside the universe of discourse of a given science as when it constitutes poor science. In the first sense, musical questions are "ungeometrical"; in the second sense, the notion that the sum of the angles of a plane triangle is 100° . There are good reasons for shorthand expressions like: "Science teaches us thus and so." But whoever wishes to avoid superstitions about "science" will do well by remembering that there is a large variety of special sciences. Though a blanket term like "sports" does not specify baseball or football, etc., it has no meaning apart from some such reference.

Like the special and the common, so the factual and the relational play distinct but conjoint roles in demonstration. There are related theoretical and applied sciences such that the latter deal with facts for which the former may

furnish reasons. An extreme example would be an attempt to determine, with the aid of mathematics, why round wounds heal more slowly than do others. Within a single science too, we may have knowledge of a fact rather than of a reason when our premises, though more familiar, are less illuminating than our conclusions; as when we argue from the failure of planets to twinkle that they are near, instead of vice versa. Similarly, our premises may state a reason too remote ("the Scythians have no flute-players because they have no vines"); so that we have to interpolate a series of middle terms ("having no grapes, the Scythians do not indulge in wine"). A search for middle terms to be thus interpolated may carry us from a least to a most inclusive class after the manner of the tree of Porphyry and with due regard to the predicables and to the categories. Successful organization of knowledge therefore demands possession of facts along with ability to reason.

Not only may one demonstration exhibit superiority over another; as when we demonstrate for all triangles, not for isosceles triangles, alone, that their angle-sum is equal to two right angles. There is justification also for comparing one science to another in point of exactness and logical superiority. Preference in these respects may be ascribed to a science relying more on arithmetic demonstration than on factual observation (as in observational astronomy); or than on predication of attributes (as in harmonics); or than on a larger number of assumptions (as in geometry). To the extent that their principles are not derived from the same source or from one another, two sciences may be said to differ generically. Not all reasoning to one conclusion from different premises is, however, thereby ruled out; provided the demonstration establish the necessary or the typical, not the fortuitous. To feel pleasure, for example, is

either as an experience, or as a form of relaxation, an affair of undergoing change. Nor is demonstration wholly isolated, even though distinct, from sense-perception. In short, apart from demonstration as such or in general, first principles vary from one science to another, from one category to another, and from one to another set of conclusions.

What is more, as we differentiate true and false opinion concerning the same subject, so we may conclude, from the fact of demonstration generally, the superiority of knowledge to opinion even when these pertain to the same subject. The "opinion" that man is an animal suggests that man may be other than animal; whereas "knowledge" to the same effect implies that man cannot be other than animal. Yet a theory of demonstration cannot provide a substitute for the sagacity with which we may hit upon appropriate middle terms. It is a matter of discovery, for which no logical rules can be given, that two individuals may be friends because they have a common enemy.

DRILL

Distinguish special and common considerations, facts and relations, and standards of preference, and show the role they play, in demonstration. (See Exercises XX, 25-31.)

85. EFFECTS OF DEMONSTRATION

Demonstration may be said to establish definitions on the basis of middle terms in previously established theorems. Moreover, a definition included among assumptions pervades systematic demonstration itself. In knowledge deductively organized, the significance of a definition is therefore determined operationally in close relation with the theorems in the demonstration and with the explanation which results.⁷

⁷ *Ibid.* ii.1-18.

Not inappropriately do we describe systematic inquiry as a search for middle terms or means of demonstration; viz., inasmuch as systematic inquiry aims at discovery of an order among facts. Thus, in a deliberation, desirable or undesirable consequences of a decision are brought to light through a series of intermediate "linking" ideas. In general, when we predicate one thing of another (S is P), we may inquire whether we are stating a fact (e.g., that the sun is eclipsed), and why it is a fact; or whether the thing spoken of is, and why it is. Syllogistically, the answer (*aition*) to any of these questions would be the middle term (M). We are asking, accordingly, either whether there is a middle term or what the middle term is. Just so, if we seek to know what something is (e.g., money) we must also know why it is.

Would taking a definition as demonstrative render demonstration useless or beg the question? A "nominal" definition, or a resolution to use a term in accordance with a stated meaning, may be included among the undemonstrated propositions of a deductive system. Or a definition may be a condensed syllogistic demonstration containing a genuinely explanatory middle term: "A lunar eclipse is the interposition of the earth between the moon and the sun." Then too a definition may declare what a thing is as if in the conclusion of a syllogism whose premises have not been expressly stated. Whereas definition is therefore not identical with demonstration, neither is a real definition obtained altogether independently of demonstration.

It is possible then for a definition to be explanatory. Like demonstration, an explanatory definition enhances understanding. Satisfaction thus afforded to intellectual curiosity varies, of course, with the type of question answered by the middle term. What is thunder? According to

its intelligible constitution, noise in the clouds. Defensive warfare? In its origin, resistance to aggression. Light? In its components, corpuscles or vibrations; with reference to its end-result, a means of illumination. To be sure, certain procedures like division and classification, of service in obtaining and ordering the elements of a definition, do not suffice alone for demonstration; but formulation of a definition does not therefore call for exhaustive knowledge of everything. Sometimes analysis leads to irreducible analogies: highmindedness, in some, may mean intolerance of insult; in others, indifference to good or to bad fortune. Then again, patently analogous structures may lack a common name; like a squid's pounce, a fish's spine, and an animal's bone.

In explanatory definition, a variety of phenomena may be discovered to be inter-connected: the echo, reflection, and the rainbow are alike cases of repercussion. Or one middle term may be subordinate to another; as in the connection of the rise of water in a suction-pump with the general law of gravitation, of which atmospheric pressure is a special case. Save for reciprocal relations, there may be more than one explanation of a single fact. However, science aims, in its demonstrations, at definitions as commensurately universal propositions, like those in mathematics. Vines and figs shed their leaves not so much because they are broad-leaved; their deciduous character is rather to be defined by coagulation of sap as an explanatory middle term. So, to show how the gamut of human emotions may be understood in terms of joy, sorrow, and desire, Spinoza summarized his demonstration in a "general definition of the affects":

Affect, which is called *animi pathema*, is a confused idea by which the mind affirms of its body, or any part of

it, a greater or less power of existence than before; and this increase of power being given, the mind itself is determined to one particular thought rather than to another.⁸

DRILL

Indicate the relation of definition to assumptions, to theorems, to demonstration, to explanation, and to enhanced understanding, in systematic knowledge. (See Exercises XX, 32-41.)

86. VALIDITY OF DEMONSTRATION

Inasmuch as demonstration lays claim to knowledge, the question becomes crucial how we acquire a knowledge of first principles. Aristotle answers:

"All animals can naturally discriminate, by what we call sensing. But, although sensing is native to them, what they sense comes to be retained by some animals, and not by others. So the latter (in whose cases retention does not occur) get to know, outside of perceiving, either nothing at all or nothing concerning the things they do not retain. The others, however, can not only sense things, but also keep them in mind. As this process becomes numerous, a further distinction arises between those animals which, from such retentions, develop the ability to relate them and those which do not.

"So out of sensing arises what we call memory and, out of frequently repeated memories of the same thing, experience; for it is of many memories that experience as a unity consists. From experience in turn—when the universal has in each case become fixed in the mind, as the one alongside the many, which is one and the same in all cases of the many—originate art and science. If it is about origination, it is an art; if about what is, a science.

⁸ Benedictus de Spinoza *Ethics* (at the end of Book III).

"Neither then are these habitudes innate in their determinate form; nor do they arise from other and superior intellectual habitudes, but from sensing. It is like one man after another in a fleeing army, and still another, making a stand until the original formation has been restored; for this process can also be gone through mentally. . . . Clearly, therefore, we must get to know the things that are 'first' by 'induction'; for even sensing establishes the universal in this way. . . . 'Intuition' is accordingly the beginning of science."⁹

According to this famous account, we find the universals in experience, on the basis of sense-perception and memory. Universals are in particulars, but do not become plain until repeated. Yet we do not find from experience that they are valid. By induction, a law is revealed rather than proved. The validity of science, as demonstration by means of first principles, is established by insight, which is more certain than experience. The process of "intuitive induction" had been described by Plato in terms of "reminiscence"; presumably because, when we observe a particular, we remember a universal. Later on, the Scholastics called this process "abstraction." Aristotle's own stress is on intellectual discernment of the universal exemplified in the particular.

In another passage, Aristotle shows how knowledge advances from a universal suggested at first glance towards its clearer articulation. The two passages are not contradictory; for, in the gaining of knowledge, a uniformity must be discerned in experience and that uniformity must be further clarified by application to specific instances.

"The natural way is to start from what is more knowable and obvious to us, and to proceed to what is more obvious and knowable by nature; for the same things are

⁹ Aristotle *op. cit.* ii.19.

not knowable to us and knowable without qualification. So we must advance in this manner, from what is less obvious by nature but more obvious to us, towards what is more obvious and knowable by nature.

"Now what is to us plain and obvious at first is rather confused masses, the elements and principles of which become known to us later by analysis. Thus we must proceed from generalities to particulars: for it is a whole that is best known to sense-perception; and a generality is a kind of whole, comprehending many things within it, like parts.

"The same thing happens in much the same way in the relation of names to the *logos* [the articulated structure]: for they signify some whole vaguely, like a circle; its definition analyzes it down to the particulars. So children at first call all men father and all women mother, but later on distinguish each of them."¹⁰

What is, in Aristotle's language, prior or more knowable to us, is some situation to be analyzed. One aspect of the analysis, as described in the first passage, consists in selection of common characters. The other aspect, as described in the second passage, consists in progressive clarification and modification of a universal by testing of its applicability to particular instances. In a classical illustration, an object seen at a distance may be recognized first as an animal, then as a man, then as Socrates. In fine, demonstration is an endless process from facts to principles and from principles to facts.

DRILL

Discuss the interrelation of facts and principles in demonstration. (See Exercises XX, 42-48.)

¹⁰ Aristotle *Physics* i.184 a16-b14.

Symbolic Logic

87. THE CALCULUS OF CLASSES

One of the purposes of modern systems of symbolic logic is to develop the general principles of deduction so as to exhibit their interconnection. We may, accordingly, from a calculus of propositions, derive a calculus of propositional functions; and, from the latter in turn, a calculus of classes and a calculus of relations.

Historically, however, the calculus of classes was the first to be developed. We shall here follow the historical order in this respect and shall confine our elementary presentation to some of the more important principles and formulas of symbolic logic. (For some concrete examples, see the Exercises.)

Let us understand by a "class" a set of individuals each having certain properties as a member of the class. Thus Socrates and others form the class "men"; chess and the like, the class "games"; etc. To represent such classes algebraically, we employ the letters, a , b , c , etc. Unique classes in the algebra of logic are: 1, the domain of possible classes, or the universe of discourse; and 0, the null class, which has no members ("round squares," etc.).

Three basic operations, partly analogous to algebraic operations, but non-quantitative, may be performed upon classes in the algebra of logic: negation, addition, and

multiplication. Exclude from the universe of "books" the class "fiction" (a), and we obtain the negative of the latter: "non-fiction" (a'). A selected class and its negative, besides being mutually exclusive, together exhaust the universe. The unique classes 1 and 0 may be defined as negatives (or complementaries) of each other.

By logically adding to a class a ("baseball players") the class b ("football players"), we obtain their logical sum: $a + b$; i.e., a or b , the joint class of "either a or b " (or both). The alternation "either-or" is to be interpreted as not strictly disjunctive, as not excluding both.

Logical multiplication of a ("women") and b ("famous") yields their logical product ("famous women"), the conjunct or common class of both " a and b "; symbolized: $a \times b$, or: ab . Note that "either a or b " has for its negative: "neither a nor b ," the common class $a'b'$. Note also that a common class is a class and that it may turn out to have no members, hence to constitute a null class; e.g., "mermaids" as "creatures with women's heads" and "creatures with fishes' tails."

There are special rules for the calculus of 0 and 1. A class is not changed by addition of 0: a or $0 = a$. Nor is a class changed when multiplied by 1: $a \times 1 = a$. But the product of a class by 0 is 0: $a \times 0 = 0$. And the sum of a class with 1 is 1: a or $1 = 1$.

As operations upon classes yield classes, so assertions of relations between classes constitute propositions. Thus one class ("persons born in the United States") may be said to be included in another ("American citizens") if every member of the former is also a member of the latter. We shall here employ the notation: a in b ; where "in" is to be understood analogously to the algebraic "less than or equal to."

The equality of two classes, both having the same members, may be defined in terms of mutual inclusion: $(a \equiv b)$ is equivalent to $(a \text{ in } b) (b \text{ in } a)$. Thus equilateral and equiangular triangles are coextensive. Similarly, inclusion of a in b may be defined as equivalent to $ab = a$, or to $ab' = 0$. Thus the class of "mortal men" is identical with the class "men" if and only if all men are mortal.

The algebra of logic considers class-terms only with respect to their extension. To illustrate this point by the null class: However the terms "sea-serpent" and "unicorn" differ in intension, the class of things designated by both terms is the same class of nothing existent. In extension, there is but one null class. Moreover, any class a no members of which are b' , and no members of which are b , is a class without members. Hence, if a in b , and a in b' , then $a = 0$.

The nature of the operations and relations discussed will become more apparent from the statement of a number of principles in the calculus of classes. We shall not, however, use the somewhat more complicated symbols in current systems.

Identity: a in a . Replacing b by a in the definition of equality, we obtain: $a = a$. Similarly, 0 in 0 , $0 = 0$, 1 in 1 , $1 = 1$.

Contradiction: $aa' = 0$.

Excluded Middle: $a \text{ or } a' = 1$.

Double Negation: $(a')' = a$.

Commutation: $ab = ba$. $a \text{ or } b = b \text{ or } a$.

Association: $(ab)c = a(bc)$. $(a \text{ or } b) \text{ or } c = a \text{ or } (b \text{ or } c)$.

Distribution: $a(b \text{ or } c) = ab \text{ or } ac$. $ab \text{ or } c = (a \text{ or } c)(b \text{ or } c)$. The second expression does not hold for the ordinary quantitative algebra.

Tautology: $aa = a$. a or $a = a$. Neither of these expressions holds for the usual numerical algebra. Let us demonstrate the two forms of the principle of tautology.

Probandum: $aa = a$.

$$a \times 1 = a.$$

$$a(a \text{ or } a') = a. \quad (\text{Excl. M.})$$

$$aa \text{ or } aa' = a. \quad (\text{Distr.})$$

$$aa \text{ or } 0 = a. \quad (\text{Contrad.})$$

$$aa \text{ or } 0 = aa. \quad (a \text{ or } 0 = a.)$$

$$\therefore aa = a. \quad (\text{Q.E.D.})$$

Probandum: $a \text{ or } a = a$.

$$a \text{ or } 0 = a.$$

$$a \text{ or } aa' = a. \quad (\text{Contrad.})$$

$$(a \text{ or } a)(a \text{ or } a') = a. \quad (\text{Distr.})$$

$$(a \text{ or } a)1 = a. \quad (\text{Excl. M.})$$

$$a \times 1 = a.$$

$$\therefore a \text{ or } a = a. \quad (\text{Q.E.D.})$$

Simplification: ab in a . Since $a \times 0 = 0$, 0 in a . a in $(a \text{ or } b)$. Since $a \text{ or } 1 = 1$, a in 1 .

Absorption: $a \text{ or } ab = a$. $a(a \text{ or } b) = a$.

Composition: If a in b , and c in d , then ac in bd . If a in b , and c in d , then $(a \text{ or } c)$ in $(b \text{ or } d)$.

Transposition: If a in b , then b' in a' .

Syllogism: If a in b , and b in c , then a in c .

Sorites: If a in b , and b in c , and c in d , then a in d .

De Morgan's Theorem: $(a \text{ or } b)' = a'b'$. $(ab)' = a' \text{ or } b'$. The negative of a sum is the product of the negatives of its terms. The negative of a product is the sum of the negatives of its terms.

Probandum: $(a \text{ or } b)' = a'b'$.

$$a \text{ or } a' = 1. \quad b \text{ or } b' = 1. \quad (\text{Excl. M.})$$

$$1 \times 1 = (a \text{ or } a')(b \text{ or } b') = 1. \quad (\text{Simpl.})$$

$$ab \text{ or } ab' \text{ or } a'b \text{ or } a'b' = 1. \quad (\text{Distr.})$$

$$\begin{aligned}
 (ab \text{ or } ab' \text{ or } a'b) \text{ or } a'b' &= 1. && (\text{Assoc.}) \\
 ab \text{ or } ab' \text{ or } a'b &= (a'b')' && (\text{Excl. M.; Contrad.}) \\
 &= ab \text{ or } ab' \text{ or } a'b \text{ or } ab && (\text{Tautol.}) \\
 &= a(b \text{ or } b') \text{ or } b(a' \text{ or } a) && (\text{Distr.}) \\
 &= a \text{ or } b. && (\text{Excl. M.}) \\
 a \text{ or } b &= (a'b')'. && (\text{From line 5}) \\
 \therefore (a \text{ or } b)' &= a'b'. && (\text{Q.E.D.})
 \end{aligned}$$

Probandum: $(ab)' = a' \text{ or } b'$.

$$\begin{aligned}
 (ab) \text{ or } ab' \text{ or } a'b \text{ or } a'b' &= 1. && (\text{From above; Assoc.}) \\
 (ab)' &= ab' \text{ or } a'b \text{ or } a'b' && (\text{Excl. M.; Contrad.}) \\
 &= ab' \text{ or } a'b \text{ or } a'b' \text{ or } a'b' && (\text{Tautol.}) \\
 &= (ab' \text{ or } a'b') \text{ or } (a'b \text{ or } a'b') && (\text{Assoc.}) \\
 &= b' (a \text{ or } a') \text{ or } a' (b \text{ or } b') && (\text{Distr.}) \\
 &= b' \text{ or } a' = a' \text{ or } b' && (\text{Excl. M.; Commut.}) \\
 \therefore (ab)' &= a' \text{ or } b' && (\text{Q.E.D.})
 \end{aligned}$$

Duality: There is a correspondence between principles in terms of multiplication and principles in terms of addition. We can pass from the one to the other by interchanging the signs of addition and multiplication, the terms 0 and 1, and the two members of an inclusion. Thus, since $aa' = 0$, therefore $(aa')' = (0)'$, which is equivalent to: $a' \text{ or } a = 1$. The dual of: $ab \text{ in } a$, is: $a \text{ in } (a \text{ or } b)$.

Expansion: $a = a(b \text{ or } b') = ab \text{ or } ab'$ (since $b \text{ or } b' = 1$, and $a \times 1 = a$). As applied to the universal class, $1 = a \text{ or } a'$; $1 = (a \text{ or } a') (b \text{ or } b') = ab \text{ or } ab' \text{ or } a'b \text{ or } a'b'$; $1 = (a \text{ or } a') (b \text{ or } b') (c \text{ or } c') = abc \text{ or } abc' \text{ or } ab'c \text{ or } ab'c' \text{ or } a'bc \text{ or } a'bc' \text{ or } a'b'c \text{ or } a'b'c'$; etc. The negative of a fully expanded expression is the remainder of the expansion of 1: $a \text{ or } a'b = a(b \text{ or } b') \text{ or } a'b = ab \text{ or } ab' \text{ or } a'b$; hence $(a \text{ or } a'b)' = a'b'$. To enumerate all the a 's, b 's, etc., it is sufficient to enumerate all a 's, then all b 's which are not a 's, etc.: $a \text{ or } b = a \text{ or } a'b$. For $a \text{ or } (ab \text{ or } a'b) = (a \text{ or } ab) \text{ or } a'b = a \text{ or } a'b$.

Limits of a function: a in x in b . A term x is comprised between two given terms, a and b , when it contains one and is contained in the other. A developed function is comprised between the product and the sum of its coefficients: ab in $(ax \text{ or } bx')$ in $(a \text{ or } b)$. For, on the one hand, $(ab \text{ in } a)$ in $(abx \text{ in } ax)$, and $(ab \text{ in } b)$ in $(abx' \text{ in } bx')$; therefore: $(abx \text{ or } abx')$ in $(ax \text{ or } bx')$, or: ab in $(ax \text{ or } bx')$. On the other hand, $[a \text{ in } (a \text{ or } b)]$ in $[ax \text{ in } (a \text{ or } b) x]$, and $[b \text{ in } (a \text{ or } b)]$ in $[bx' \text{ in } (a \text{ or } b) x']$; therefore: $(ax \text{ or } bx)$ in $(a \text{ or } b) (x \text{ or } x')$, or: $(ax \text{ or } bx')$ in $(a \text{ or } b)$.

Elimination of x : If $ax \text{ or } bx' = 0$, then $ab = 0$. If $ax \text{ or } bx' = 1$, then $a \text{ or } b = 1$.

Universal and Particular Propositions. (A) All a 's are b 's represented by: a in b , or by: $ab = a$, is equivalent to: $ab' = 0$. (E) No a 's are b 's, represented by: a in b' , or by: $ab' = a$, is equivalent to: $ab = 0$. (I) Some a 's are b 's which contradicts E, may be represented by: $(a \text{ in } b)'$, or by: $ab' \neq a$, which is equivalent to: $ab \neq 0$. (O) Some a 's are not b 's, which contradicts A, may be represented by: $(a \text{ in } b)'$, or by: $ab \neq a$, which is equivalent to: $ab' \neq 0$.

Equivalent Equations: $a = b$; $a' = b'$; $b = a$; $ab' \text{ or } a'b = 0$; $ab \text{ or } a'b' = 1$; $(a \text{ or } b) (a' \text{ or } b') = 0$; $(a \text{ or } b') (a' \text{ or } b) = 1$; $a \text{ or } b = ab$; etc. Each of these implies each of the following forms: $a \text{ in } b$; $b' \text{ in } a'$; $ab' = 0$; $ab = a$; $a \text{ or } b = b$; $a' \text{ or } b' = a'$; $a'b' = b'$; etc.

Equivalent Inequations: $a \neq b$; $a' \neq b'$ etc. Each of these is implied by each of the following formulas: $a'b \neq 0$; $a \text{ or } b' \neq 1$; $a \text{ or } b \neq b$; $ab \neq b$; $a'b' \neq a'$; $a' \text{ or } b' \neq b'$; etc. In general, from: if $a = b$ then $c = d$, it follows that if $c \neq d$ then $a \neq b$. Again, if $a = b$ is equivalent to $c = d$, then $a \neq b$ is equivalent to $c \neq d$. Also, from: if $a = b$ and $c = d$ then $e = f$, it follows that if $a = b$ but $e \neq f$ then $c \neq d$, or that if $c = d$ but $e \neq f$ then $a \neq b$.

Rules for Equations and Inequations: $a = b$ is equivalent to $ab' \text{ or } a'b = 0$ (multiply each side of the equation by the negative of the other, add these two products, equate to zero); hence $a \neq b$ is equivalent to $ab' \text{ or } a'b \neq 0$. In combining two equations having one member 0, always add them: $a \text{ or } b = 0$ is equivalent to the pair, $a = 0$ and $b = 0$. An equation and an inequation combined give an inequation: if $a \text{ or } b \neq 0$, and $a = 0$, then $b \neq 0$.

Syllogisms with Universal Premises. Given the premises: $ab' = 0$, and $bc = 0$; from the combination of the premises: $ab' \text{ or } bc = 0$, we obtain by elimination: $ac = 0$.

Syllogisms with One Particular Premise. Given the premises, one universal: $ab' = 0$, which is equivalent to: $ab'c \text{ or } ab'c' = 0$, and one particular: $ac \neq 0$, which is equivalent to: $abc \text{ or } ab'c \neq 0$, it follows that $ab'c = 0$, that $abc \neq 0$, and therefore that $bc \neq 0$.

The principles of the algebra do not recognize a particular conclusion as valid when drawn from universal premises alone. Hence also, the traditional square of opposition is rejected, except for the relation of contradiction. Since the rejected traditional forms depend for their validity upon the assumption that the things under discussion exist, that assumption must be made explicit; with the consequence that traditional principles become replaced by more complex deductions. Moreover, in view of the different forms of equivalent equations, given premises may yield a variety of conclusions, most of which are, however, unimportant in practice. Although certain problems concerning relations of classes are most conveniently solved by means of the class-calculus, that calculus, historically considered, depends for its intelligibility upon the syllogistic logic.

"If we were set the problem of formulating such modes of inference as will meet, in maximum degree, the criteria (1) simplicity and ease of understanding, (2) wide range of application, (3) accord with the most frequently desirable direction of thinking, we should be hard put to it to do better than the syllogism."¹

DRILL

Explain and illustrate: class; operations on classes; assertions of relations between classes; distinctive features of the calculus of classes. (See Exercises XXI, 1-42.)

88. THE CALCULUS OF PROPOSITIONS

A logic of classes in extension admits of application to propositions in extension; just as, in the classical logic, the laws of the categorical syllogism find further exemplification in the hypothetical syllogism. However, restriction of the algebra of logic to propositions requires the assumption: For every element a , either $a = 0$ or $a = 1$. We have thus a two-valued algebra (which may, however, be expanded into many-valued systems).

The letters p , q , r , etc., are propositional variables. A formula containing propositional variables is true or false when "specialized" in individual propositions: these are "special values" of propositional variables; except that formulas (like those listed below) which form the subject of logic hold for all values of the variables.

Fundamental propositional operations include negation, alternation, conjunction, implication, and equivalence. The negation or contradictory of a proposition p may be symbolized by p' ; the logical sum or alternative assertion of two propositions, by: p or q ; the logical product or conjunct assertion of two propositions, by: $p.q$, or by: pq ; p

¹ C. I. Lewis and C. H. Langford *Symbolic Logic* (New York: Appleton-Century-Crofts, Inc. 1932), p. 73.

implies q , by: $p \text{ im } q$; the equivalence of two propositions, by: $p \text{ eq } q$. The truth of a proposition may be symbolized by: $p = 1$, or by p ; the falsity of a proposition, by: $p = 0$, or by p' .

Equivalence of propositions may be defined in terms of mutual implication: $(p \text{ eq } q) \text{ eq } (p \text{ im } q) (q \text{ im } p)$. Also, by definition, $(p.q) \text{ eq } (p' \text{ or } q')'$; $(p \text{ im } q) \text{ eq } (p' \text{ or } q)$; $(p \text{ eq } q) \text{ eq } [(p.q) \text{ or } (p'.q')]$.

A formula that is true, whatever the truth-values of its constituent propositions, is an "always-true" formula (often called a "tautology"). Such formulas may be established by means of truth-tables. Thus, to establish: $(p.q)' \text{ eq } (p' \text{ or } q')$, we note that, when p is true and q is true, $p.q$ is true, $(p.q)'$ is false, p' is false, q' is false, $(p' \text{ or } q')$ is false; so that both sides of the equivalence are false, and the whole formula is true. The whole formula may be established similarly for the cases: p true q false; p false q true; p false q false.

p	p'	p	q	$p \text{ or } q$	$p.q$	$p \text{ im } q$	$p \text{ eq } q$
T	F	T	T	T	T	T	T
F	T	T	F	T	F	F	F
		F	T	T	F	T	F
		F	F	F	F	T	T

Logical formulas may also be derived as theorems from a set of initial formulas as axioms. One minimum set of axioms is given in the formulas: $(p \text{ or } p) \text{ im } p$; $p \text{ im } (p \text{ or } q)$; $(p \text{ or } q) \text{ im } (q \text{ or } p)$; $(p \text{ im } q) \text{ im } [(r \text{ or } p) \text{ im } (r \text{ or } q)]$. The following list includes some of the most important formulas in the calculus of propositions.

Identity: $p \text{ im } p$. $p \text{ eq } p$.

Contradiction: $(p.p')'$. $(p.p') = 0$.

Excluded Middle: $p \text{ or } p'$. $(p \text{ or } p') = 1$.

Double Negation: $(p')' \text{ eq } p$.

Reductio ad Absurdum: $(p \text{ im } p') \text{ eq } p'$.

Commutation: $(p.q) \text{ eq } (q.p)$. $(p \text{ or } q) \text{ eq } (q \text{ or } p)$.

Association: $[p \text{ or } (q \text{ or } r)] \text{ eq } [(p \text{ or } q) \text{ or } r] \text{ eq } (p \text{ or } q \text{ or } r)$. $[p.(q.r)] \text{ eq } [(p.q).r] \text{ eq } (p.q.r)$.

Distribution: $[p.(q \text{ or } r)] \text{ eq } [(p.q) \text{ or } (p.r)]$. $[p \text{ or } (q.r)] \text{ eq } [(p \text{ or } q).(p \text{ or } r)]$.

Tautology: $(p.p) \text{ eq } p$. $(p \text{ or } p) \text{ eq } p$.

Simplification: $(p.q) \text{ im } p$. $(p.q) \text{ im } q$. $p \text{ im } (p \text{ or } q)$. $q \text{ im } (p \text{ or } q)$.

Absorption: $(p \text{ im } q) \text{ eq } [p \text{ eq } (p.q)]$.

Composition: If $(p \text{ im } q)$ and $(r \text{ im } s)$, then $(p.r) \text{ im } (q.s)$. If $(p \text{ im } q)$ and $(r \text{ im } s)$, then $(p \text{ or } r) \text{ im } (q \text{ or } s)$.

Transposition: If $(p \text{ im } q)$, then $(q' \text{ im } p')$. $(p \text{ im } q) \text{ eq } (q' \text{ im } p')$.

Syllogism: If $(p \text{ im } q)$ and $(q \text{ im } r)$, then $(p \text{ im } r)$.

Sorites: If $(p \text{ im } q)$ and $(q \text{ im } r)$ and $(r \text{ im } s)$, then $(p \text{ im } s)$.

De Morgan's Theorem: $(p \text{ or } q)' \text{ eq } (p'.q')$. $(p.q)' \text{ eq } (p' \text{ or } q')$.

Expansion: $p \text{ eq } [(p.q) \text{ or } (p.q')]$. $p \text{ eq } [p \text{ or } (p.p')]$. $(p \text{ or } q) \text{ eq } [p \text{ or } p'.q]$.

A basic formula peculiar to the calculus of propositions is the principle of assertion: $p \text{ eq } (p = 1)$. To state a proposition is to affirm its truth. Hence $p' \text{ eq } (p' = 1) \text{ eq } (p = 0)$. The negative of a proposition is equivalent to the affirmation that this proposition is false. Since, by definition of 1, a proposition cannot be both true and false at the same time: $(p = 1) (p = 0) = 0$, and since a proposition is either true or false: $[(p = 1) \text{ or } (p = 0)] = p \text{ or } p' = 1$, it follows that the propositions $(p = 1)$ and $(p = 0)$ are contradictory: $(p \neq 1) = (p = 0)$; $(p \neq 0) = (p$

= 1). These formulas are valid for classes in the particular case when the classes can possess only the two values 0 and 1.

No less fundamental in the calculus of propositions is the formula for the dissolution of an implication: $(p \text{ im } q) \text{ eq } (p' \text{ or } q)$. By expansion in T-cases, $(p \text{ im } q)$ when p is true and q is true, or when p is false and q is true, or when p is false and q is false; but not when p is true and q is false. Hence $(p \text{ im } q)$ is also equivalent to: $(p.q')'$. In material implication, propositions are considered only with respect to their truth-values; just as and because, in the class-calculus on which, historically, the propositional calculus has been built up, classes are interpreted in extension. The material equivalence: $p \text{ eq } q$, as a reciprocal implication, shares some of the peculiarities of material implication: it fails to hold only when one of two propositions is true and the other false.

Merging of implications: $[(p \text{ im } q) \text{ or } (p \text{ im } r)] \text{ eq } [p \text{ im } (q \text{ or } r)]$. This principle would not hold for classes: it is false that, if all candidates either pass or fail, then either all candidates pass or all candidates fail. Similarly for: $[(p \text{ im } r) \text{ or } (q \text{ im } r)] \text{ eq } [(p.q) \text{ im } r]$; where each side of the equivalence is equivalent to $(p' \text{ or } q' \text{ or } r)$. Also $[(p \text{ im } q). (p \text{ im } r)] \text{ eq } [p \text{ im } (q.r)]$. Also $[(p \text{ im } r).(q \text{ im } r)] \text{ eq } [(p \text{ or } q) \text{ im } r]$.

Importation: If $p \text{ im } (q \text{ im } r)$, then $(p.q) \text{ im } r$.

Exportation: If $(p.q) \text{ im } r$, then $p \text{ im } (q \text{ im } r)$.

Addition of an Implication: $p \text{ im } (q \text{ im } p)$. $p' \text{ im } (p \text{ im } q)$.

Inferential Implication: $[p. (p \text{ im } q)] \text{ im } q$.

Rule of Substitution: In an expression true for all meanings of p, q, r , etc., any propositional expression may be substituted in the place of p , or q , or r , etc.; provided

this substitution is made in all places where the original variable occurs. Thus, from the dissolution of the implication: $(p \text{ im } q) \text{ eq } (p' \text{ or } q)$, we may, by substituting q for p' and p' for q , obtain: $(p \text{ im } q) \text{ eq } (q \text{ or } p')$, from which we may, by applying the dissolution in reverse, derive the contrapositive of the original implication: $(q' \text{ im } p')$.

Rule of Replacement: Either of two equivalent propositional expressions may be replaced by the other. Thus, from a given formula: $(p \text{ im } q). (r \text{ im } s)$, we may by replacement, derive the formula: $(p' \text{ or } q). (r' \text{ or } s)$.

Rule of Inference: If p is true, and $(p \text{ im } q)$ is true, then q may be asserted. This is the *modus ponendo ponens*. Other rules of inference would include the *modus tollendo tollens*, the *modus ponendo tollens*, and the *modus tollendo ponens* (see Ch. XI).

Reversibility of Transformation: Derivation from an expression transformed by equivalences is reversible. Example: $(p \text{ im } q) \text{ eq } (p' \text{ or } q) \text{ eq } (q \text{ or } p') \text{ eq } (q' \text{ im } p); (q' \text{ im } p') \text{ eq } (q \text{ or } p') \text{ eq } (p' \text{ or } q) \text{ eq } (p \text{ im } q)$.

Derivation by Onesided Implication: When a formula is derived as a consequence, by onesided implications, the derivation is irreversible. Example: $(p \text{ or } q)' \text{ eq } (p'.q')$, which, by simplification, implies p' .

As Couturat has pointed out, "we can say that the algebra of logic is a *mathematical* logic by its form and by its method, but it must not be mistaken for the logic of *mathematics*." "The true logic of mathematics is the logic of relations."²

DRILL

Explain and illustrate: propositional variables; propositional operations; distinctive features of the calculus of propositions. (See Exercises XXI, 43-75.)

² L. Couturat *The Algebra of Logic*, tr. L. G. Robinson (Chicago, London: The Open Court Publishing Co. 1914), p. 93.

89. INTERRELATIONS OF SYMBOLIC CALCULI

Formulas in the propositional calculus state laws for all propositions, without regard to the terms and relations of terms into which propositions may be analyzed. When we consider, e.g., the formula: $(p.q)$ im p , we take the propositions p and q as wholes, without regard to their respective logical forms. Formulas in the calculus of propositional functions, however, state laws based on an analysis of the inner structure of propositions.

A propositional function exhibits the logical structure of a proposition. Each of the expressions: " x is a man," " x is mortal," " x thinks," may be "specialized" by appropriate terms or values put for the variable x ; and the resulting propositions will have the same form. A propositional function, then, is an expression which contains one or more variables and expresses a proposition when values are given to the variables.

Examples of two-place functions are given in expressions like: " x hates y ," " x differs from y ," " x is wiser than y ," " x is similar to y ," " x is a cousin of y ." If we represent relations in propositions by the variable R , we have a propositional function of the form: xRy .

Consider next the general proposition: "All copper conducts electricity." In this general statement, we affirm a general implication: "For all instances or values of x , (x is copper) implies (x conducts electricity)." In this form, the variable x is said to be "bound" by an "all-operator." Similarly, "no crows are white," is equivalent to: "For all x 's, (x is a crow) implies that (x is white) is false."

On the other hand, we may, instead of: "Some physicians are competent," write: "There is an x such that (x is a physician) and (x is competent)." In this form, the variable x is said to be "bound" by an "existential operator."

Similarly, "some octogenarians are not inactive" is equivalent to: "There is an x such that (x is an octogenarian) and (x is inactive) is false."

The formulations exemplified render evident distinctions like that between propositions asserting class-membership and propositions asserting some other relation as well as like that between universal and particular propositions. Moreover, these formulations express a different analysis of propositions from the method of analyzing a proposition into a subject, a predicate, and a copula. The different methods of analysis are not therefore mutually exclusive; but each is more appropriate to some propositions than to others.

Now a calculus of propositional functions may be derived from the calculus of propositions. For, if we represent the different values of x , in a function of x , or in the "functional": $f(x)$, by p, q, r , etc., then, for a given meaning of f , $f(p), f(q), f(r)$, etc., are propositions, just as are p, q, r , etc., in the calculus of propositions; so that all the formulas of that calculus will hold of such propositions. Accordingly, " $f(p). f(q). f(r) \dots$ " means that $f(x)$ is true for all values of x ; in symbols, $(x)f(x)$. Similarly, " $f(p)$ or $f(q)$ or $f(r) \dots$ " means that $f(x)$ is true for some value of x ; in symbols, $(\epsilon x) f(x)$.

The close connection between the calculus of propositions and the calculus of propositional functions may be illustrated by analogues of De Morgan's rules. The truth of a function of x for all values of x (for p and for q and for r , etc.) has for its negative the falsity of that function of x for some values of x (e.g., p or q or r , etc.). And the truth of a function of x for some values of x (e.g., p or q or r) has for its negative the falsity of that function of x for all values of x (for p and for q and for r).

A special principle in the calculus of propositional functions states that what is true of all is true of any. When, e.g., all who practice love find love, we may say: "Anyone who practices love finds love." However, not always does a statement about "any," $f(x)$, have the same meaning as a corresponding statement about "all," $(x)f(x)$. For, if anything has the property f , then there is a thing which (but not: then everything) has this property. So, when we say: "It is false that anybody is persecuting you," $\neg f(x)$, we do not mean that not everyone is doing so, $\neg(x)f(x)$, but that there is no such person, $(x)\neg f(x)$, or $\neg(\exists x)f(x)$. "Any" differs from "all" inasmuch as the scope of "any," when expressing generality, is the whole formula (e.g., "If anyone will do His will, he will know of the doctrine"); whereas the generality expressed by "all" can be restricted to parts of a formula (e.g., "If all others forsake Thee, I shall not"). The statement: "If anybody has a good story, Montfort has heard it," may be symbolized: $(x)[f(x) \text{ im } p]$; which means, not: $(x)f(x) \text{ im } p$, but: $(\exists x)f(x) \text{ im } p$.

The main formulas in the calculus of propositional functions concern the scope and the order of operators. Thus, "for all x , if x is a prudent man, then x is free from poverty," does not have the same meaning as the implication that, "if all men were prudent, then all men would be free from poverty": the former, but not the latter, implies that, "if some men are prudent, then some men are free from poverty." Also "for every believer, there is some church of which he is a member," differs from: "There is a church such that every believer is a member of it." Moreover, it has been shown that all always-true formulas in functions can be derived from the formula for the transition from a bound to a free variable: $(y)f(y) \text{ im } f(x)$, the formula for the reverse transition: $f(x) \text{ im } (\exists y)f(y)$, and the

four axioms to which always-true formulas in propositions may be reduced; and this derivation would require general rules for substitution, for inference, and for the transition from $f(x)$ for a non-specialized x to $(x)f(x)$.⁸

There is also a close analogy between the calculus of propositional functions and the calculus of classes. For the class a may be determined by a propositional function for all values of x such that x belongs to a ; the class b , by a propositional function for all values of x such that x belongs to b ; etc. The calculus of classes may then be derived from the calculus of propositional functions of one variable.

So we may, from the calculus of two-place functions, derive the calculus of relations. For we may regard a function for the values of x and y as asserting some relation of x to y ; a relation R in extension, as a class of ordered couples (x, y) such that $f(x, y)$, or xRy , is true; a relation S , as a class of ordered couples (x, y) such that $g(x, y)$, or xSy , is true; etc.

We may therefore have: the negation of relation, not- R ; the logical product of relations, R and S ; and the logical sum of relations, R or S . Also, relations such as implication and equivalence may be asserted of relations; thus, two relations R and S are equivalent in extension when, for all values of x and y , xRy implies xSy and xSy implies xRy (e.g., the fractional exponent $1/2$ is equivalent to the principal square root).

Relations are, as relations, also susceptible of special operations such as conversion and relational multiplication. The converse of R results when the order of the terms related is reversed: e.g., if x is a parent of y , then y is a child of x . The relational product of two relations, $R*S$, is itself a relation of x to z when x has the relation R to y and y has the relation S to z ; e.g., the brother of a parent is an uncle.

⁸ D. Hilbert and W. Ackermann *Grundzüge der theoretischen Logik*, referred to in H. Reichenbach *Elements of Symbolic Logic* (New York: The Macmillan Co. 1947), p. 153.

A special example is the relational product of a relation with itself, $R * R = R^2$; e.g., the parent of a parent is a grandparent.

For the relation of identity, we may write: xIy ; e.g., $\sin \theta \equiv 1/\csc \theta$. For the relation of diversity, we may write: xDy ; e.g., "the abscissa is other than the ordinate."

The class of terms having the relation R to something or other is the domain of R (e.g., parents); the class of terms to which something or other has the relation R , the converse domain of R (e.g., children); the sum of the domain and the converse domain, the field. In a series, the domain will be all terms except the last (if any); the converse domain, all terms except the first (if any).

Attention has previously been called (in sec. 48) to such properties of relations as transitivity, symmetry, correlation, and connexity. Relations may also be divided into kinds in point of reflexivity: a reflexive relation obtains between any term and itself (e.g., equal to); an irreflexive relation is one for which xRx is always false (e.g., taller than); a nonreflexive relation is one sometimes reflexive, sometimes not (e.g., admires).

A relation is also said to be uniform when to each antecedent there corresponds a single consequent (e.g., fire burns); co-uniform, when to each consequent there corresponds a single antecedent (e.g., children have been conceived); bi-uniform, when it is at once uniform and co-uniform (e.g., equilateral triangles are equiangular).⁴ If R is a uniform relation, the relative product of R and its converse is a symmetrical and transitive relation. By the "principle of abstraction," we then group objects having a common property as members of a class of things connected by a symmetrical relation, with reference to a selected object: things having the same color as a sample; lines par-

⁴ L. Couturat "Principles of Logic" *Encyclopedia of the Philosophical Sciences*, tr. B. E. Meyer (London: The Macmillan Co. 1913) I.176-78.

allel to a given line as identical in direction; bodies having the same weight as a certain standard; etc.

Finally, propositional functions of different orders form a hierarchy of propositional functions. For propositional functions of the first order may be represented by variables in propositional functions of the second order; as when we assert that "intelligence is an admirable property," "punctuality is an admirable property," etc. In symbols, $f_2(f_1)$. So we may speak of a "class" (e.g., Americans) as a member of a "higher class" (e.g., the United Nations). On this approach, recourse has been taken to the theory of types of functions for the resolution of certain paradoxes: a class cannot contain itself as a member; a proposition cannot meaningfully declare itself false; the rule that all rules have exceptions cannot have reference to itself; etc. Or, according to the "vicious-circle principle," "whatever involves *all* of a collection must not be one of the collection."

Among the advances made in logical theory by symbolic logic may be mentioned especially its systematic method of obtaining propositions which may be asserted on logical grounds alone. This extension of the classical logic has undergone, and is still undergoing, continuous improvement in method and increasingly extensive application to a variety of subject matter.

More controversial are the tendencies to "nominal positivism"; as if, e.g., the meaning of a proposition were identical with its verifiable consequences. In general, it would seem difficult to see how we can altogether separate forms either from objects and events or from values; rather must we have regard to all these factors in their distinctness and in their togetherness. Having, then, stressed the important part played by inference in discourse and in factual,

⁵ Lewis and Langford *op. cit.*, chap. xiii: "The Logical Paradoxes."

historical, and formal inquiry, we turn at this point to more explicit consideration of evaluative thought.

DRILL

Explain and illustrate: propositional functions; distinctive features of the calculus of propositional functions; logical properties of relations; the interrelation of propositions, propositional functions, classes, and relations; the approach of "symbolic logic." (See Exercises XXI, 76-98.)

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Weighing Our Values

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Evaluation

90. FACT AND VALUE

Of the two cities Pittsburgh and Chicago, which has the larger population? Which manufactures more steel? In which city is it better to live? Everyone senses at once the shift in atmosphere from the first two of these queries to the last. Here is a plain case of the important distinction between fact and value. Moreover, the feeling of this difference is more important than any terminology chosen for its expression. For the customary words are easily manipulated so as to confuse: "Every fact is a value, and every value is a fact." This verbal jugglery does not obliterate the distinction, which confronts us still.

Thinking in the realm of facts has led to such large measure of agreement on pertinent standards that we call our methods and results in this area "objective." But it is of the highest moment to us to think as best we can also in the realm of values, where fundamental disagreements about criteria present special logical difficulties. At least one basic reason for the possibility of objectivity in science is the consensus in purposes presiding over the activities of scientists as such. But how can men working at cross-purposes hope to see eye to eye in what they say or do? They may claim to pursue "happiness" or to want "liberty"; but with considerable variety in concrete content! Small wonder that reason-

ings diverge with instability in meanings rooted in diversity of background, temper, interests, and social loyalties; not to elaborate on the inevitable difficulties in foretelling how our choices will turn out when unexpected factors modify our experience. Many of our problems must await their solution in the process of living; the lessons of the past afford some guidance, but break down at the point where a particular situation is quite unlike any other we have ever faced; and we cannot experiment upon human beings with the freedom with which a scientist may control the conditions he manipulates in his laboratory.

It is within the limits of possibility, however, for us to improve our understanding of the process of evaluation and therefore to reduce the dissensions among men. Recognition of the distinction between fact and value, between knowledge and opinion, between issues about the way things are or ought to be, seems itself a step in this direction. Another step forward consists in contrasting dogmatic and reflective methods of evaluation. I may openly acknowledge my prejudices, my likes and dislikes, my hopes and fears, my interests and limitations; it makes a difference whether I do so with good humor or in a contentious spirit. But, when I seek reasons why it is better to think, to act, or to feel in one way than in another, I find the beginning of an answer in another question: Better for what? Then I no longer spontaneously value, but weigh what is worth valuing; I evaluate. Thus I may come to prefer one author for his clarity, another for his knowledge, a third for felicity of expression, a fourth for entertainment, etc. Technically expressed, I have turned an immediate value (something I value on its own account) into an instrumental value (something I value for the sake of something else); I compare an end that is in question with others by treating each as a

means to some further end; I continue this process until I reach an end which is acceptable as a standard. This technique helps us to understand the comment in Aristotle's *Ethics* that we deliberate not about ends but about means.

Reflection may thus play an essential role in settling or mitigating human strife. For it is in reflective evaluation that every interest at stake, so to speak, obtains a hearing. But what if controversy continues about standards and ultimate values? Men of intelligence and good will can often find ways of agreeing to disagree. There are, to be sure, occasions when men cannot do other than fight, even to the death, for what they hold dear; but to take this course is, clearly, to postpone if not to abandon appeals to reason. Threat or application of brute force, except as a last resort in extreme cases, is uncivilized. Contrast the conviction, consonant with human dignity, that the highest value to be preserved is co-operative intelligence, which embraces within itself all the values!

The logical principles we may discern in the undebatable accordingly hold, *mutatis mutandis*, also for the debatable. Because opinion is not knowledge, it does not follow that one belief is as good as another. Even where my intelligence does not depend upon the particular opinions I may hold, it does depend upon the ways in which I arrive at them, state them, trace their implications, and examine the evidence for and against them.

DRILL

Distinguish and relate: fact and value; scientific and extra-scientific problems; dogmatic and reflective evaluation; immediate and mediate values; intelligence and other values; application of logic to the undebatable and to the debatable. (See Exercises XXII, 1-24.)

91. THE MORAL SITUATION

What shall I do next, and how? What shall I make, and how? Questions of this sort recurrently exact exercise of judgment in everyday living and in every science, art, and profession. Unless we act on impulse, caprice, blind habit, or convention, we deliberate, in such situations, concerning the means whereby we may obtain our desires and may initiate action. As we weigh at every step the alternatives under consideration, interacting circumstances supply observation with evidence for inference and reasoning to probable consequences, whether like or different from those anticipated.

Though the decisions of an umpire in a baseball game pertain directly to some single play, they bring to bear upon the immediate state of affairs a background of estimates concerning observed facts and applicable rules; and they affect performances impending and the remainder of the course of the game. The rules applied constitute tools of analysis. Because the directions which these formulas imply serve the activities of the game itself, not vice versa, the rules are subject to change in order that they might meet new needs. So regarded, the rules do not answer questions, so much as they raise questions or suggest the kinds of questions relevant to effective settlement of an issue. Hence the importance of right formulation of a system of questions and rules, as itself a matter of judgment, and in the interest of expediting judgment concerning best possible methods of procedure.

How then shall we differentiate moral from practical judgment? One, like the other, resolves a predicament as to alternative courses of action. To the extent that the options posed have far-reaching bearings upon the conduct of life,

for ourselves or for others, the problem confronting us may be said to be a moral one. The term "conduct" suggests a continuity in the manner of managing one's affairs; in "bringing together," as it were, maxims for the guidance of behavior; in carrying on a way of life. It follows that *every* act has moral import *potentially*; but that circumstances vary in which it becomes relevant or irrelevant to challenge the moral rightness or wrongness of some practice.

As a eulogistic term, "moral" is opposed to "immoral," in a vituperative sense. But the "moral" has also a wider and neutral meaning; namely, when used in contrast to the "non-moral." A geographical situation as such is non-moral; whether I ought to locate there, may become a moral question, any answer to which is moral in the descriptive sense. To characterize my decision as moral or immoral, is to pronounce a judgment of praise or blame, a judgment which falls within the sphere of the moral-not-the-non-moral. These two meanings of the term "moral" focus in one instance the distinction between fact and value; as does any descriptive or analytical use of a word taken in conjunction with or in contradistinction from its honorific or pejorative use. Thus I may describe history, or laud peace, or condemn war, each as a human "achievement."

Assuredly, what is and what ought to be are not necessarily identical; but they have a mutual relevancy. People respect human life by a natural preference. If some do not, the unqualified statement just made calls for correction. Whether they ought to, cannot be pre-judged by the fact that some do not. Yet, even if all men do, it is open to us to ask why they should and to discover good reasons for so doing. Before long, we are led to discriminate between the conditions under which disregard for human life, because of temporary gains or pleasures, marks moral folly and un-

der which well-considered sacrifice of life on behalf of a noble cause elicits our approval. So moral intelligence includes a constant balancing of the actual and of the ideal. What we ought to do is somehow what we do or would do if we are or were wise in our choices of ends and means and know or knew in advance the consequences of our actions.

The moral maxims which have come down to us as part of our social inheritance have a status in reflective morality as essential tools of analysis. The Golden Rule calls to mind crucial factors which, without this impressive injunction, we might overlook. No doubt, there are some moral imperatives and customary observances which some individuals prefer not to violate under any circumstances. More notably, there are superior and inferior laws and conventions which it is the part of wisdom to obey in view of the social necessity of harmonious living. But the correlative rights and obligations of dispassionate moral inquiry afford the most stable basis for conduct, for social accord, for human progress, and for the realization of our individual capacities as well as of our common human nature.

DRILL

Discuss practical and moral situations, judgments, reflections, and maxims. (See Exercises XXII, 25-35.)

92. SOCIAL PROBLEMS

A student of the social sciences needs, above all, ever to keep before him the distinction between fact and value. He will, in both respects, bring both individual judgment and co-operative intelligence to bear upon the changing social scene. As to the literature in the field, he will find basic such works as Aristotle's *Ethics* and *Politics*; the primary sources dealing with the formation of the Constitution

of the United States; and the recent *Encyclopedia of the Social Sciences*. May we agree that available knowledge exceeds readiness to put it into effect? May we also agree that social thought may profit from due regard to logical principles of necessary and probable inference; of terms, truth-claims, and grounded conclusions; of demonstration and proof; of inquiry and evaluation?

What kind of social investigation can one undertake who approaches human phenomena primarily in terms of white and black with no shades of grey? Can anyone analyze a social situation without explicit formulation of conditions, obstructions, resources, procedures, and goals? Do social facts and problems present themselves readymade for collection and eventual generalization; or do we select and order them at the hand of stated generalizations which are to be probed and proved? If a social explorer does not derive his proposed solution from existing tensions, to what methods will he repair? How can social thought avoid narrowness and rigidity without deliberate attention to a plurality of alternative hypotheses? Is it conducive to comprehensive, flexible, and fruitful thinking when conceptions in vogue remain unquestioned; when easy dichotomies are set up between conservatives and progressives, etc.; when specialists fail to draw upon fields related to their own; when improvised policies and fixed dogmas play into each other's hands, in opposition to continuous functional analysis? Inasmuch as social phenomena, like events generally, have bearings upon one another, do not understanding and interpretation of the significance of facts depend upon tracing these with the aid of intellectual frames of reference and as parts working together in patterns of interactions? Can prejudice be eliminated or minimized in social theory and practice without open acknowledgment of ends in view: not as

handy importations *ab extra*; but as possible outcomes of actual processes, as guiding hypotheses, as correlative to available means, as consequences to which associated activities may tend or be directed?

DRILL

Discuss social reflection, with special reference to social situations, problems, hypotheses, reasonings, solutions, and goals. (See Exercises XXII, 36-51.)

93. IDEALS

Santayana has said: "In Aristotle the conception of human nature is perfectly sound; everything ideal has a natural basis and everything natural an ideal development."¹ First, then, an ideal is significant for human life as it rounds out, fulfills, and completes some natural activity of man: an exercise of common sense; a social impulse; a quest for enlightened religion, for liberal art, for objective science. Ideals framed in disregard of natural functions are illusory. Secondly, the natural does not by itself, apart from the ideal, afford a justification of its existence or a standard of conduct. Scientific discoveries present themselves as gains in positive knowledge of something actual. They also pose a problem of what we shall do with them. To learn the possibilities of life we turn to the ideals in the light of which men may live, as shown especially by outstanding persons of history.

Science itself cannot, in its inquiries into things and their relations, dispense with "neglective fictions" or "ideal limits." When I call Herbert my brother, I "abstract" one "aspect" from everything he is. Because I regard two objects having certain common properties as "tables," it does

¹ George Santayana *The Life of Reason*, Vol. I. Introduction, in *Works* (Triton ed.; Charles Scribner's Sons) III.28.

HOW MAN THINKS

not follow that there are no other tables or that each may not have other, perhaps unique, properties as well. To classify governments, with Aristotle, as monarchies, aristocracies, democracies, and their respective perversions, helps us to recognize the mixed forms of actually existing governments. So we distinguish economic motives, economic laws, and repeatable patterns or types of situations generally, though they do not exist in isolation. How could we accurately determine certain geographical relations without some such device as lines of latitude or longitude? Degrees of rigidity, friction, homogeneity, etc., are ordered in series on such principles of order as perfectly rigid bodies, frictionless pulleys, perfect distribution, etc.

Not only then are actualities real, but possibilities are real also. Without a standard of perfection, there is no way of discriminating degrees of imperfection. At the same time harmonious social relations depend upon a sense of human limitations and therefore upon the practice of toleration. A reasonable person keeps his balance between his circumstances and his aspirations.

DRILL

State with care the interrelation of the natural and the ideal; of concrete actualities and neglective fictions or ideal limits; of circumstances and aspirations. (See Exercises XXII, 52-63.)

94. APPROACHES TO RELIGION

Man owns a kinship with other living creatures and with the rest of nature. When he takes himself in hand as a person contemplating his destiny, he naturally seeks kinship with the divine. Despite varieties of expression and of institutional embodiment, religions present common features

in their concepts, motivations, sentiments, and ceremonial forms.

Paul's epistles reflect, along with his conversion, a rich background in the Gentile culture and the Hebrew heritage of his day. By way of sharing his faith with his fellow-believers in Rome, whom he had planned to visit, Paul summarized, in his letter to them, his religious convictions. Neither knowledge nor a code, he maintained in harmony with complaints by leading moralists of the time, can guarantee achievement of the highest standards. Man must accordingly base his adjustment on a fundamental confidence, which Paul expounded in terms of a Christian mysticism (or spiritual union and absorption).²

It was against misunderstandings of this doctrine of justification by faith that the Epistle of James insisted, in the spirit of some of the ancient Hebrew prophets, on a faith active in endurance of trials, in deeds of love, and in willingness to learn, to practice humility, and to help those in trouble. But there are other approaches to religion besides the mystical, intellectual, ethical, and social; among them the aesthetic. To many, the affinities of religion appear to lie in the direction, not of science, or of reform, but of literature, especially of poetry, or of the arts generally.

For progressive formulation of criteria relevant to religion, we may draw upon the religious experience of the race. Would it be fair to say that a religion adequate to our needs would have to unite the best men can know and do and feel?

DRILL

Indicate various factors in "religion" and sources of significant religious reflection. (See Exercises XXII, 64-68.)

² Rom. 1-3; Gal. 2:20.

Art and Criticism

95. NATURE AND ART

Perhaps the most satisfactory approach to art consists in defining art as anything that is not nature. Because art deliberately supplements nature, we can have recourse to medicine and to education. Nevertheless there are analogies between nature and art inasmuch as both are productive processes. Let us then briefly sketch the kinds of factors involved in understanding a natural process, like the generation of animals, and an artificial production, like the construction of a house.¹

If a house is to be built, there must be materials, such as brick and mortar; and a foundation must be laid. From these alone, however, the completion of the house does not necessarily follow. The material and the order of development are "necessary" only on condition of the product or end to be achieved. It is the kind of dwelling to be erected that determines what is to be done in house-building, rather than the reverse. So the stages in the development of an animal become intelligible by basic reference to the kind of animal under consideration. In natural generation, no less than in artificial production, interpretation must begin, not with that which is, but with what is to be. "Chance things do not come into existence from each seed but rather this

¹ H. D. Hantz *The Biological Motivation in Aristotle*, chap. ii.

thing from this seed, nor a chance seed from a chance body."²

Again, an end-product has some form. Thus a house is not certain materials simply, but a form of matter, or matter in a certain form. Also animals and other natural developments are rendered intelligible in terms of the parts of which they are composed as well as by the ways in which they are characterized. Contrast a live creature and its bodily parts with a corpse or a wooden hand: unlike the latter, the former are characterized by forms of activity. A living being functions as a whole being; it is produced by living beings like itself; its body and parts are related to its activity as an instrument to its use. Form is therefore functional no less than structural. Yet a limitation must be acknowledged in the development of organisms because of useless parts, which come into existence as a consequence of the presence of other parts. The productive activity of architects and other skilled agents too issues at least partly, in unintended results.

Moreover, as attention centers upon the developing process by which a finished product comes into being, other prominent factors come into view. House-building may be traced to its origin in the transformation of materials. Similarly, the origination of a new creature is due to material and activating factors. Both artificial and natural production also exhibit regularity in the course of the development. For the parts do not come into existence simultaneously. Thus the development of the heart precedes that of the lung. In short, "an animal comes into being in the same way as the knitting of a net."³ Production is accordingly a serial phenomenon involving transformation, succession, and order.

² Aristotle *De Partibus Animalium* 641b27-29.

³ Orpheus, quoted in Aristotle *De Generatione Animalium* 734a19, 20.

Hence the matter necessary for production must have possibilities such as are realized in the final product. Hence also, to be transformed into its final form, the matter must be activated by something actual. In Aristotelian language, a process of natural or of artificial production is a passage from the potential to the actual. In order therefore to articulate the factors of production so as to render the process intelligible, we must discriminate in it ends and forms and materials and movers, along with potentialities (or possibilities) and actualities (or the possibilities realized).

To sum up in terms of one of Aristotle's simplest illustrations. A statue of Hermes is a realization of possibilities discerned by the sculptor in marble, bronze, etc. If the statue is to be adequately explained, four kinds of questions must receive satisfactory answers: Wherefore, wherein, wherefrom, and whereby did it come into being? These are the four traditional "causes": final, formal, material, and efficient. But the translation "cause" has misled more than one interpreter.

DRILL

Discuss the relation between nature and art; the final formal, material, and efficient factors; the passage from potentiality to actuality. (See Exercises XXIII, 1-9.)

96. EXPRESSION

Human beings desire not only to live, but also to re-live their lives as it were through its many possible interpretations and expressions: in story-telling, in the drama, in song, in painting, in sculpture, etc. We practice many arts of expression other than speech because one art may communicate what no other can say equally well. But it is only the egotist who desires to exhibit only himself to others as a

unique individual. The accomplished artist reorganizes material of universal human experience so that it may embody significant content. He actively carries forward to completion the inspiration he has received from some aspect of reality. By means of the products of his art, he shares his insights with those who come to his works for fresh revelations of the varied qualities, relations, events, and blemishes and excellences in the world about him. Artistic content is accordingly as diversified as nature or life itself.

Art does not reveal some reality behind experience. Rather does it enrich experience by distilling from it new significance. In general, both the universal and the individual enter in unique manner into the meanings set forth in an art product. Thus Michelangelo's statue of Giuliano and Lorenzo in the Medici Chapel effectively portray, in face and gesture, in symbolism and composition, the antithesis between an active "Jovial" and a contemplative "Saturnian" disposition. Plato's *Phaedo*, in dramatizing the last hours of Socrates, takes us into the presence of sheer death. Music, architecture, poetry, and other arts have a wide human appeal to the extent that this or that interpretation of grief, of exaltation, of love, etc., expresses, along with the specific, a universal grief, exaltation, love, or other trait as a type exemplified in the given and other specific instances. An artist contemplating reality must therefore achieve in his imagination a happy equilibrium somewhere between the active man's passionate participation and the scientist's dispassionate inquiry.

Each of the special arts possesses distinctive possibilities and limitations for the expression of some phase of life as actively lived. Architecture may proclaim the mutual adaptation of natural conditions and of social needs and purposes; sculpture, living forms as individualized; painting,

the natural and human scene; the dance, controlled emotions and movements; music, the dramatic changes in nature and life; literature, the multitudinous meanings with which speech has become charged. To the environment as reflected in an art object, the artist and the spectator bring their own knowledge, memory, imagination, and temperament and the culture of a people. Art therefore does not duplicate nature or create an independent world, but focuses some shareable experience in a singular perceived manifestation of it for its own sake.

As regards its effects, expressive art does not seem fraught with pure "aesthetic" or immediate pleasure, not even in the popular stories that "end well." The aesthetic may be mingled with the utilitarian, as in furniture and in works of architecture. Negative values also find expression in the arts, which suffuse them with compensating satisfactions: with nobility, in the tragedy of King Lear; with the exaltation of the sublime, in suffering Job's faith: "Though He slay me, yet will I trust in Him"; with the discovery of incongruity in the comic, of unexpected appropriateness in wit, or of friendly sympathy in humor. However, the delight which art is capable of yielding surpasses the fleeting impressions of the moment; it is an acquired pleasure with enduring worth, bought at the price of progressive cultivation; it is a factor in the general enhancement of life in the pursuit of happiness.

DRILL

Discuss artistic expression and content, with special reference to nature and life, the universal and the individual, distinctive possibilities and limitations of the special arts, and aesthetic effects and values. (See Exercises XXIII, 10-20.)

97. FORM

Reflection on what is said in expressive art is furthered by attention to how it is said. For, in aesthetic experience, content and form coalesce. In particular, if an art product is to possess formal unity, an artist must somehow reconcile the simple and the complex, the fractional and the demand for integration, the rhythmic and the arrhythmic, the conventional and the original.

A form in nature or in art presents a "unity in variety." Observing the lights of a city at night, we get a sense of multiplicity in uniformity: the elements are alike; their diversity is numerical. Elements may also be arranged so that they can be surveyed in different orders, like the maxims collected in the Book of Proverbs. But a determinate organization, such as that of birds or airplanes, suggests inevitably to the observer the scheme in which the elements have been functionally brought together.

Perfect integration, in which no part can be added or subtracted without marring the unity of the whole, is an ideal limit more readily approximated in a sonnet than in an epic, in a song than in a symphony, on a small canvas than in a large painting. Usually a composition marked by integration in some respects is fractional in others. Thus recent Negro sculpture gives in small compass the character of design built out of the head, arms and legs, and trunk of the human body; but at the sacrifice of expressed content.

Since both achievement and perception of form occur in time, the rhythm and symmetry of aesthetic structure have dynamic interrelations and are qualified by the arrhythmic and unsymmetrical. In "God Save the King," the simple rhythmic design imposed by the first six notes repeats itself, either unchanged or with slight variations; but

an indefinite number of tunes might have been based upon the initial rhythm. In "The Last Ride Together," Browning momentarily substitutes a meter other than the dominant one:

"So, one day more am I deified.

Who knows but the world may end tonight?"

Absolute conformity of design in paintings and statues to perfect geometrical patterns would be but one among a large number of values; and cathedrals essentially symmetrical are unsymmetrical in numerous ways. When Shakespeare dramatized the reflective indecision by which Hamlet's headlong emotionality was frustrated and increased, he reflected life itself, where such antitheses as the irrational and the rational occur conjointly, but without clearcut demarcations.

Convention has fixed some of the familiar compositional patterns which characterize works of art as organic unities: songs, fugues, sonatas, and symphonies; folk and social dances, and ballets; axial determinations and geometrical patterns in the plastic arts; literary forms like the lyric, epic, drama, essay, novel, etc. Formal organization also depends upon certain organized manners of treatment: music with or without words, melodic, polyphonic, or harmonic, programmatic or non-programmatic; solo, dual, or choral dancing; architecture trabeated or arcuated, cubistic or plastic; sculpture and painting linear, planear, cubistic, or plastic; the play of lights and shadows; literature lyrical, reflective, dramatic, or descriptive; etc. Musical "fugues" and some of the other units related in artistic structures also may have artistic quality and individuality on their own account. In short, an artist who wishes to be intelligible must discipline his originality by conforming in some degree to established compositional patterns and other con-

ventions. (See the chart at the end of T. M. Greene's *The Arts and the Art of Criticism*.)

DRILL

Show how achievement and perception of form in art realize typical possibilities in the blending of the simple and the complex, the fractional and the integrated, the rhythmic and the arrhythmic, the conventional and the original. (See Exercises XXIII, 21-28.)

98. MATTER

From the standpoint of means of expression, we may, with Santayana, distinguish "automatic" and "shaping" arts. Automatic arts, having for their medium the human organism itself, would include dancing and physical exercise as well as song and speech. Shaping arts depend more extensively upon materials external to the body and would therefore embrace technology, architecture and ornament, sculpture, and painting, and the like. But the invention of musical instruments and of the printing-press strikingly attests the historical development of the two broad types of art under mutual influences. Also in our own practice, automatic and shaping art blend with each other whenever spontaneity and control unite in the formation of genuinely "fine" art.

Every artist, of whatever sort, works in some medium. His medium consists, primarily, of sensuous raw material selected and organized for use as a language or vehicle of artistic expression and communication. Thus a musical composer utilizes selectively organized sounds, with their variations of pitch and timber and of intensity, duration, and tempo. These variations form the basis of tonal relations

and of musical "color" and rhythm which the musician exploits. So literature finds ready to hand verbal languages, or systems of auditory and visual symbols conveying meanings: vernaculars which the creative writer transforms into a flexible medium for his expressive purposes. As regards architecture, sculpture, and painting, or the plastic arts, controllable space and light, together with perceptual qualities of available materials, enter into their primary media. Art starts with matter, relatively formless, and realizes itself in the ideal form it gives to its material medium. "The Parthenon not in marble, the king's crown not of gold, the stars not of fire, would be feeble and prosaic things."⁴

On the other hand, the term "matter" may also designate "subject matter." This becomes an artistic medium, in a secondary sense, when chosen for interpretation in artistically expressed content and with due regard to the expressive possibilities of the primary or sensuous medium. Thus a sculptor may render massive bodies in stone and tensile grace in bronze; but not clouds, which are more suited to representation in painting. Natural objects and events, including man's individual and social activities and reflections, provide inexhaustible sources of subject matter for art to interpret. The manner of the artist's interpretation differentiates the expressed content of an art product from its antecedent subject matter. Hence the subject matter requires transformation at the artist's hand no less than does the primary medium. Both subject matter and artistic content are in turn not to be confused with a "title" which serves purposes of practical identification. Plato's *Republic*, written in Greek, derived its title from the Latin; its subject matter, from life in Athens; its content, from Plato's reflections on the theme of the perfect city.

⁴ George Santayana *The Sense of Beauty*, in *Works* (Triton ed.; Charles Scribner's Sons) I.62.

DRILL

Distinguish: matter for and in the automatic and the shaping arts; the primary and secondary artistic media. (See Exercises XXIII, 29-37.)

99. TECHNIQUE

Like every reflective thinker, the artist has some problem or other to solve. His principal problems, moreover, grow out of felt needs in man's everyday experience. Life itself demands, for the realization of its ideals, arts which shall be at once technical and liberal. In a world in which not everything is as he would desire or make it, man must act, and act with intelligence. If existence, being neither absolutely chaotic nor absolutely finished, puts obstacles and opportunities into our path, the exigencies of life call for highly developed competence in thought and action. How far could modern transportation and communication have advanced without the inventions made possible by the theories underlying them; or music, without the modern notation which in turn depends upon musical theory? Is not education distorted by those long on immediate application but short on long-time theory? All art, from cookery to systematic speculation about the universe, consists in activities deliberately ordered to some end and with a view to a product which can justify to us its own existence.

Both science and art are techniques by which man extends his everyday experience: in science, by reducing qualities to relations among equations; in art, by manifesting, intensifying, and celebrating qualities in their own sense as significant values of things. Moreover, artistic production and perception are carried forward in a manner paralleling the pattern of reflective inquiry.

One who beholds the "Avignon Pietà" in the Louvre will observe a pattern of lines and colored areas representing sorrow in the presence of death and its transfiguration by worshipful adoration. This initial total impression fuses a cumulation of sensory effects and meanings which somehow cohere and invite further consideration. Attention may then be directed to striking contrasts: among the colors, between the glaring white and the gold background; among the persons, in their faces, gestures, postures, actions, and settings; among the symbols, in the treatment of the halos; etc. Such points of tension and opposition become focal points of reflection in the reconstruction of experience to meet new demands of understanding. In the case before us, pleasure in the aesthetic surface alone would be marred by the deficiency of color harmony; as recognition of the death, grief, and adoration represented would be left unsatisfied by a certain indifference of the kneeling man in a white robe to the rest of the group.

Familiarity with Christian ideas and pictorial conventions furnishes requisite clues to the picture's meaning and significance. Thus the perceiver, assimilating the elements of an artistic composition and conserving their values, is led to consider, in their mutual bearings, alternative possible eventuations. To the extent that the means chosen by the artist are adequate, they contribute co-operatively to the final outcome of his work. Anticipation of the end-result would distract when allowed to over-determine in detail the developing process of artistic production and perception. For it is not at the first and last alone, but in the intervening stages as well, that an aesthetic experience culminates in the joys of discovery. Sense and intellect complete themselves as we trace, in the "Avignon Pietà," the

motivations for the varying degrees of realistic and unrealistic portrayal.

Instead of leading away from, analysis leads to, enriched appreciation of the human and divine, of the temporal and eternal, of the transfiguration of the real and the incarnation of the ideal, or of whatever else an artist has expressed. As anticipation and fulfilment recur throughout a work of art as experienced, so the effects of aesthetic experience carry over into renewed and sharpened sensitivity to the import of what we do and undergo from day to day.

DRILL

Distinguish and relate aspects of technique and of reflective analysis in practical intelligence, in scientific inquiry, and in artistic production and perception. (See Exercises XXIII, 38-42.)

100. CRITICISM

A sincere critic, whether amateur or professional, founds his judgment, first and last, upon the exercise and assertion of his own taste. But, dogmatism apart, he compares his own preferences with the presumably equally legitimate preferences of others. Genuine feeling, inner consistency, and outer fitness enter into the formation of adequate criticism. "A criterion of taste is, therefore, nothing but taste itself in its more deliberate and circumspect form."⁵

What is known as "impressionistic" criticism limits itself to the effects of a product of art upon a perceiver on the ground that we cannot know anything except our own impressions. But have not the impressions been made upon us by objects? And have not the objects of art been pro-

⁵ George Santayana *Reason in Art*, in *Works* (Triton ed.; Charles Scribner's Sons) IV.346.

duced by individuals with a history of their own? So-called "judicial" criticism, on the contrary, judges by fixed standards. But which of the many supposedly "absolute" standards is "the" absolute standard? And do we not rightly expect some freshness, freedom, and spontaneity in fine art? Because rules of art more or less successfully formulate past successes, they may profitably serve as tools of analysis for the guidance of new endeavor and judgment. Freedom and power come neither from neglect nor from slavish imitation, but from mastery and new uses, of the traditions.

Complete aesthetic criticism involves more than momentary effects and more than customary values. It discriminates the properties of an object which stimulate a response; it articulates the import of an aesthetic experience; it offers leading suggestions to the experiences of others; it endeavors to understand, and to help others understand, an art object with special reference to the artist's intent, available means, and actual procedures. To this end, it frankly states and progressively refines the criteria by which it evaluates artistic styles, qualities, relations, relevancies, and interpretations.

DRILL

Differentiate impressionistic, judicial, and complete aesthetic criticism. (See Exercises XXIII, 43-47.)

Philosophy and Education

101. METAPHYSICAL ANALYSIS

Tradition has divided systematic philosophy into four branches: logic, metaphysics, ethics, and aesthetics. These disciplines are manifestly interrelated. If aesthetics, e.g., be restricted (as it sometimes is) to the theory of the beautiful, there are (we must maintain) beauties elsewhere in nature than in art; and the philosophy of art embraces concepts other than that of beauty. To the extent, moreover, that an act we perform enhances our total conduct, exhibits intelligence, and may be regarded as an end in itself, that act would have at once a maximum of moral, logical, and aesthetic value. Again, nature constitutes in her own right a domain of intelligible and moral orders which may well command our admiration.

William James aptly defined metaphysics as an "unusually obstinate effort to think clearly."¹ How suggestive, in contrast to popular conceptions of something "beyond the natural," like Shakespeare's "metaphysical witches"! The peculiar term owes its origin to the circumstance that, in the Alexandrian library under Andronicus of Rhodes, certain writings of Aristotle came "after" those on "nature."

¹ William James *The Principles of Psychology* I.145.

Special sciences set forth what is a star, a plant, a rock, a bird, a man, a poem, etc. In intimate connection with such enterprises, metaphysics articulates what it means for anything to be said to be. Whatever is, may be comprehended as such; and the metaphysics of knowledge traces the development of existences into their own comprehension.

Then too, as Aristotle pointed out, when a brass ring is made, what comes into being is neither matter nor form as such, but a new combination of them. In nature as in art, matter puts on new forms in productive processes marked by originations and culminations, by actualizations of possibilities. Existence and becoming in the concrete thus arouse a wonder which finds its satisfaction in metaphysical analysis.

To be philosophically minded, we must try to conceive things as much as possible in a total way. Having compared the universe with the mechanisms and with the means and ends in it, we have gained clearer ideas about machines, about means and ends, and about the universe itself. One who has in thoroughgoing fashion delineated "what is" in terms of the individual, the continuous, the purposeful, the possible, the contingent, has become clearer-headed about the universe no less than about individuality, continuity, purpose, potentiality, chance, and whatever else has claimed his metaphysical attention.²

The chief logical method for handling the ultimate terms in which we interpret the world is an expanded kind of definition, based on what a given term denotes in its various contexts. Among the distinctive effects of metaphysical thinking, so pursued, we experience a profound sense of nature's diversities existing together in some sort of coherence; a challenging recognition of the multiplicity and integrity of knowledge; a growing mastery of language; and

² F. J. E. Woodbridge *Nature and Mind*, pp. 37-55: "The Problem of Metaphysics."

a firm conviction of our union with nature as a source of progressive understanding, achievement, and enthusiasm.

DRILL

What is metaphysical analysis? (See Exercises XXIV, 1-5.)

102. PHILOSOPHY OF CIVILIZATION

Besides its analytical task, philosophy has also historical and critical functions to perform. The history of philosophy reveals how thinkers have expressed the culture, and placed their fingers upon crucial issues, of their times: Plato and Aristotle, for the Greeks; Lucretius, Marcus Aurelius, and Plotinus, for the Romans; Augustine and Thomas Aquinas, for Western Christendom; Descartes and Spinoza, Newton and Locke, Hume and Kant, for a new scientific age; Hegel and Marx, Mill and Spencer, James and Royce, Santayana and Dewey, for more recent days. Consider a few of the suggestive ideas set forth by Dewey in his convincing essay on "Philosophy and Civilization."³

What philosophers have tried to do is to adjust persisting traditions to novel scientific discoveries and social aspirations. For, besides yielding truth and added control, our knowledge of events and existences enriches life; namely, as our imagination seeks to penetrate the significances of things and the meanings of civilization. In thought and emotion, men have ever had to reconcile their ruling interests, political and artistic in ancient Athens, economic and scientific today, with the preoccupations they have inherited. How do men then endeavor to meet the demands set by a stubborn past and by an insistent future? They may conserve selected parts of their heritage, or rebel against the status quo, or anticipate future developments. Whichever

³ John Dewey *Philosophy and Civilization* (New York: Minton, Balch & Co. 1931), pp. 3-12.

of these directions a philosophy may take or combine, it transforms human thought and action.

It follows that systems of philosophy, when marked by sincerity and vitality, must diverge in accordance with the interests they articulate. Nonetheless they do, like politics, religion, and art, tell something about existence and the actual course of events. But the scientifically verifiable constitutes but a part of their content. If, with all our energy in industry and science, we have not accomplished commensurate and adequate results, a basic reason may lie in failure to round out factual data into complete ideas, to formulate our culture with logical coherence, and to unite modesty and courage, frankness and humanity, in such deliberate and life-directing exercise of the imagination. "As long as we worship science and are afraid of philosophy we shall have no great science."

DRILL

Indicate historical, cultural, social, and critical functions of philosophy. (See Exercises XXIV, 6-8.)

103. LOGICAL EVALUATION

There are at least two senses in which we may speak of logical evaluation. First, the central core of commonly accepted logical doctrine may be set in varying contexts. Secondly, evaluation cannot be altogether excluded from any logical procedure. Comprehensive logical theories may accordingly diverge in the uses to which the rules of inference are applied as well as in emphasis on different phases of inquiry and discourse.

What we do when we perform a logical operation is accessible to inspection and analysis; no less so than what we do when we operate a machine or balance a commercial

account. But, inasmuch as science progresses in its methods, corresponding changes are introduced into logical theory. There have been times when men have been acutely conscious of progress or of their need of it. It is significant how leading thinkers have then sought to further the movements or needs of their times, especially by efforts to improve prevailing methods of thought.

Noteworthy too is the question whether logic is, or ought to be, self-applying. As a logician may insist that others state the assumptions and consequences of their affirmations and denials, so he may make the same demand upon himself. There is indeed at least an element of convention in the rules of logic as in the rules of a game or of practical transactions. Formulations in logic are not therefore wholly arbitrary. Given the purpose of building a bridge, conditions determined by local laws must be met; but also engineering conditions set by natural circumstances. Just so, logic formulates, in general terms, the means which thinking must employ if it is to accomplish certain ends. In the doctrine of the syllogism, for example, the conditions are stated under which premises may support a conclusion.

A brief word may be in order about intelligence as the highest value at stake in life. The dogmatist has definite convictions on which to act; but his mind is closed to new ideas, which are all the more eagerly welcomed by one more skeptically inclined. Now the extremes of tenacity and of indecision may well be avoided. Why not combine the advantages of practical readiness with those of intellectual tentativeness: commit ourselves to action on the strength of our best judgment; and hold our beliefs subject to correction through further reflection? Basic allegiance to the process of progressive intelligence itself may make for as much mental equilibrium as we may enjoy in a dynamic

world. Such allegiance will also contribute effectively to harmony and good will among men. The promise of this persuasion offers a more challenging appeal than does any available alternative to faith in intelligence.

DRILL

Distinguish and relate: two meanings of logical evaluation; scientific practice and logical theory; logical conditions and conventions; dogmatism, skepticism, and critical intelligence. (See Exercises XXIV, 9-13.)

104. EDUCATION

"And now . . . let me show in a figure how far our nature is enlightened or unenlightened." Thus Plato introduces his famous parable of the cave,⁴ in which he dramatically summarizes some of his educational principles. From its earliest beginnings in an ethically oriented training of mind and body, education is a lifelong process. It has for its function the progressive disciplining of the mind in imagination, intelligence, science, and philosophy.

Men dwell, as it were, in a cave with their backs to the light and to the objects the light illumines. Legs and necks of the cave-dwellers are chained so that the inhabitants of the den can only see before them the moving shadows on the inner wall. To these dancing figures the prisoners attribute even the voices whose echoes reach their ears. All the arts and languages, from the primitive to the refined, attest that man is, above all a mind. His primary activity as a mind is to exercise the powers of his imagination.

Some of the cave-dwellers become sufficiently emancipated to see around their surface impressions as they learn that what they have hitherto identified with reality are shadows of puppets manipulated by marionette players. But

⁴ Plato *Republic*, Book vii.

the arts of manipulation require practical intelligence. If the processes observed are to be profitably used for a desired end, they must be mastered and understood. It becomes imperative therefore that the imagination be disciplined.

The more enlightened of the prisoners make their way out of the den, thus advancing beyond the visible world of objects and their shadows into the intellectual world. At first, as long as they remain unaccustomed to the light of day, they behold little more than reflections in the waters. Put less figuratively, the imagination is disciplined not only by practical intelligence but also by theoretical understanding. We depend, like everything else, upon nature's laws. Hence, by way of evidence, the pervasiveness of mathematics in the sciences.

Gradually a few of the bolder adventurers dare to look directly into the light of the sun itself. They enjoy, in the intelligible world, the vision of ideas; especially of the idea of the good. They learn to discern the goals to which our activities naturally tend; controlling rational purposes by which life may be organized; ideals in which whatever is has its functional significance. In this insight, cultivated by a comprehensive mind through analysis of first principles, the processes of nature and life, and of the progressive disciplining of the imagination through education, become understood.

May we not see in Plato's parable a kind of history of man in epitome? As long as his imagination remained untrained, man chafed against the fetters which galled him only to be more firmly bound by them. But, learning from experience, man has intelligently converted his chains into weapons. His technical and scientific advances have yielded unsuspected knowledge of matter's constitution and further uses for good or ill. Who can say what triumphs shall be in

store for us when we succeed in organizing also our emotional, social, industrial, and political concerns in accord with a reasoned philosophy of ideals by which we may be guided and quickened to a richer life?

One who has been privileged to come out of his cave, and to bask in the light of the rational good, naturally feels the responsibility of sharing with his erstwhile fellow-prisoners his vision of a better and more enlightened world. An education is not complete until brought to bear upon human affairs. From the unenlightened, the enlightened may learn the problems to which intelligence is to be applied; as, from the latter, the former may learn something of the intelligence to be applied to the issues of life. Hence Plato insisted upon return into the cave for public service as a social laboratory in which the enlightened may test ideas, rise to leadership, train future leaders, and bring closer to realization the vision of a well-ordered society.

DRILL

Trace, at the hand of Plato's allegory of the cave, salient phases of educational progress. (See Exercises XXIV, 14-23.)

Exercises...

EXERCISES . . .

CHAPTER I. INFERENCE

1. Suppose a rose had been discovered on Mars. Why would scientists hail such a discovery?

2. Tell how you know: This is a book. *Connaitre* and *savoir* are French verbs. John Doe is your neighbor. Lincoln was assassinated. Twice two is four. The same sun presents itself daily in the heavens. The commensal becomes a parasite. The earth rotates on its axis. Milk differs from water. You have a brain. You have a certain ability.

3. Does an audience see a stage magician pulling a rabbit out of a hat?

4. In a country inhabited only by truthful noblemen and lying peasants, one person remarked, "I am a —." To the comment of another, "He said he is a nobleman," a third replied, "He is a peasant." How many of the three were noblemen?

5. Three passengers on a train emerging from a tunnel began laughing at one another because their faces had become dirty. Each laughed at the others only as long as he did not realize his own face was not clean. What reasoning led one of them, after a while, to stop laughing?

6. In what ways, other than feeling, can you tell whether your morning coffee is hot?

7. Give examples of inferences from the conduct of other persons.

8. What in general are the sources of our inferences?

9. What would you reply to those who say they do not know that every pair is even, but only that everything they know to be a pair is even? (Aristotle *Posterior Analytics* i.1.)

10. What use, in general, does a geologist make of fossils; a biologist, of vestigial structures; an astronomer, of perspective pictures of stars; etc.?

11. What would be the consequences of limiting a government in its powers so that it could do no harm?

12. Distinguish premises and conclusions: Mortal as thou art, nurse not wrath immortal. Mohammed was a wise lawgiver, for he studied the character of his people. "Cruelty to slaves was unusual while slavery lasted, the best proof of which is the quietness of the slaves during the Civil War when all the men and many of the boys of the South were serving in the Confederate armies." (James Bryce.)

13. State some conclusions which assume: An increase of human life is a good. Everyone's want for security has the same value. Americans are more deserving than Orientals. The President pledged himself to "preserve, protect, and defend the Constitution."

14. Can we prove that a ship wrecked in mid-ocean sinks to the bottom? (O. Blackwood *Introductory College Physics*, p. 137.)

15. Examine John Locke's jibe: "God has not been so sparing to men, to make them barely two-legged creatures, and left it to Aristotle to make them rational."

16. How would you deal with the claim that men may reason well without logic? Do similar considerations hold for other arts? (See: A. De Morgan *Formal Logic* [1926 ed.], pp. 249-58.)

17. Distinguish truth and validity: Two rectangles, equal in area and alike in one dimension, are alike in the other dimension also. Perfect men may live together without law; but no men are perfect; therefore no men can live together without law. Every month has thirty days; there-

fore April has thirty days. If I am a son of William the Conqueror, my father's name is William. Were ice heavier than water, bodies of water would freeze solidly from bottom to top.

18. Granted that valid reasoning from true premises yields a true conclusion, what kinds of conclusions may be drawn from true premises by invalid reasoning; from false premises by valid reasoning; from false premises by invalid reasoning?

19. If, in a debate, six points substantiate a seventh, how would you show that the seventh point does not hold?

20. Does one who rejects or abandons an argument thereby reject or abandon the conclusion?

21. How would you account for and correct the tendency of an advocate of a good cause to rely on poor reasons?

22. If men had counted 1, 2, 3, 4, 10, 11, 12, 13, 14, 20, would they have been free to decide how much $3 + 3$ should equal? In a system in which $7 + 5 = 12$, are we free to decide how much 7×5 shall equal?

23. Is mathematics important because or in spite of its abstract nature? (A. N. Whitehead *An Introduction to Mathematics*, chap. i.)

24. When a pair of terms satisfies a fixed relation (e.g., $y^2 = x$) and one of the terms is given (e.g., $x = 4$), is the other term always definitely determined? (*Ibid.* chap. ii.)

25. Momentum being proportional to velocity, and kinetic energy to the square of the velocity, what effect does doubling the velocity have on the momentum; on the kinetic energy?

26. Comment on the remarks made, according to De Morgan, in different parts of one conversation: sins against

the Creator are sins against society; sins against society ought to be punished by society; some opinions are sins against the Creator; it is unjust to punish anyone for his opinions.

27. According to the Nazis, most Communists in Germany were Jews; but shortly before this statement was made, at a time when there were about 600,000 Jews in the country, 6,000,000 Germans voted the Communist ticket. Comment.

28. Distinguish: This ground is rich *because* the trees on it are flourishing; the trees flourish *because* the ground is rich.

29. If you consider a painting bad because it does not bring out the voluminousness of the objects represented, what are you assuming about the art of painting?

30. Comment on these lines by Pope:

"Say first, of God above or man below,

What can we reason, but from what we know?"

31. Which statement differs from the first in form; which, in matter; which, in both? All sailors are men. All ostriches are birds. Some men are not sailors. Socrates called himself "the gadfly of Athens."

32. Show how the first clause differs from the second in each of these sentences: Smith is not robust; Mr. A is not robust. $6 - 2 = 4$; $x - y = 4$. Not all men are white; not all A is B. 3 is less than 7; x is less than 7.

33. What difficulty must a learner overcome who cannot see that, if every A is B, it does not follow that every B is A; but admits that, though every cow is an animal, not every animal is a cow?

34. How would you undertake to determine whether a conclusion can be drawn from the premises: Some A is B, and no A is C?

35. Why do we write $x + y = 1$ as $x + y - 1 = 0$? What advance is made from the latter to $ax + by - c = 0$? (Whitehead *op. cit.*, chap. v.)

36. How does meter organize articulate sounds? Identify the various "feet": "The voice of days of old and days to be"; "Hopes and fears, belief and disbelieving"; "So all day long"; "It will come, I suspect, at the end of life"; "After it, follow it, follow the gleam"; "Remember, remember." (J. F. Genung *The Working Principles of Rhetoric*, p. 196.)

37. Illustrate the role of form in human relations; in art; in science.

38. Inference is a temporal act and may be about something temporal. Is the relation between premises and conclusion temporal?

39. Supply, where appropriate, the words *implied* and *inferred*: His presence——a keen interest on his part; from which I——that he would speak to the group.

40. Do you, in an inference in the present, in any way utilize the inferences you have made in the past? Why is it advantageous to consider how an inference may be fitted to be of use in future inferences? If you take account of the criteria of valid inference generally, what value would you ascribe to logical forms?

41. Classify and discuss as probable inferences: He has lied to me; therefore I cannot trust him. No one answered the bell; the whole family must be out.

42. Draw the appropriate conclusion: Yesterday, December 31, A slipped on the sidewalk and fell. Today X, who was bitten by a dog yesterday, died in convulsions.

43. Supply appropriate premises: There is electricity in the clouds. A given tree existed a year ago. Beneath the

surface of the water is a large mass of ice. Most attorneys in Middletown have heard of Attorney Smith.

44. May we say that, because A did an act X last year, therefore he probably did the act X as now charged?

45. Does probable inference take this form: Ninety-nine Cretans in a hundred are liars; Epimenides is a Cretan; therefore Epimenides is 99 per cent a liar?

46. If "Pro" is much more probable than "Con," what would you do with the evidence for "Con"?

47. If the probability of each of the premises that A is B and that B is C is $1/2$, is the probability of the conclusion, that A is C, $1/4$?

48. Since the sick may either recover or not, is the probability of recovery $1/2$?

49. Would you charge one who suspends his judgment with lack of interest?

50. Calculate the probabilities: A letter chosen at random from the alphabet will be a vowel. A work of four volumes placed at random on a shelf will be arranged in correct order. You will get heads three times in five throws of a fair coin. Doublets will appear at least once in three throws with a pair of dice. You will get a head or a tail in one throw of a fair coin.

51. Is a hand of thirteen cards of one suit in bridge rarer than some other hand?

52. Let h = head, t = tail. Expand the binomials: $(h + t)^2$; $(h + t)^3$; $(h + t)^4$; $(h + t)^5$. What is the probability of getting three heads and two tails in tossing five coins? Explain.

53. What is meant by "favorable odds"; by "odds against"?

54. Is the calculus of probability right even when a gambler loses?

55. Why might it be desirable to set down, alongside an assertion, the numerical degree of probability with which one may be justified in making the assertion?

CHAPTER II. SOME COMMON FALLACIES

Wherever a fallacy occurs in the following exercises, locate the source of the trouble; name the fallacy; give reasons for the name.

1. The end of a thing is its perfection; life ends in death; therefore death is the perfection of life.

2. Nothing is better than honor; half a loaf is better than nothing; therefore half a loaf is better than honor.

3. I shall keep my appointment and take care of my correspondence tomorrow.

4. Sitting in a comfortable Pullman seat, the scenery flashed by.

5. "The birds filled the treetops with their morning song, making the air moist, cool, and pleasant."

6. We dream; therefore nature is a dream.

7. Various ideas and interests are represented in a political party, which cannot therefore rise above them.

8. Engineering, medicine, criminology, etc., are advancing; hence civilization is making progress.

9. We have the best football team, therefore also the best quarterback.

10. Differences in certain abilities and proclivities being due to differences in the genes, there are distinct genes or groups of genes or features of genes corresponding to the abilities and proclivities.

11. State different meanings resulting from different emphases: I saw the young man. Julius loves Julia. I hope you will come tonight. Our parents did not enjoy the last week-end at the seashore. "There is the dotage of duncedom

which cannot perceive the impudence of insignificance so presumptuous as to doubt that the elements of life and literature are indivisibly mingled one in another, and that he to whom books are less real than life will assuredly find in men and women as little reality as in his crassness he deserved to discover." (Swinburne.)

12. Show how the meaning of words may change with changing contexts: church, success, my father, freedom, argue, belief, etc.

13. "Text without context is pretext."

14. What cautions would you observe in quoting?

15. Does the adverb "very" ever qualify a verb?

16. John is not a proper name; no man is.

17. Why do we employ figures of speech: simile, analogy, metaphor, personification, allegory; trope, synecdoche, metonymy, allusion, word-coloring; exclamation, interrogation, apostrophe, vision, historical present, hyperbole, irony, litotes, innuendo? (J. F. Genung *The Working Principles of Rhetoric*, chap. iv.)

18. Credit is important in modern business; therefore credit should be extended to this applicant.

19. Like other men, this inebriate is a rational animal.

20. Because the average man reaches his top at forty, no one should be considered for a top post before that age.

21. "You know this person? Tell me all about him!"

22. He weeps; he must be a pessimist.

23. Our forefathers revolted in 1776; let us therefore maintain the *status quo*.

24. You are thinking of a man and of a horse and therefore of a centaur.

25. Freedom is ever limited; hence there is no freedom.

26. A college graduate is better fitted for American citizenship than any other.

27. What can philosophy do for me in the next half hour? Of what use is it?

28. Science is dependable; therefore the reform measures proposed by an eminent scientist are to be supported.

29. In what sense are all the angles of a triangle less than, in what sense equal to, two right angles?

30. No news is good news; war is no news and is therefore good news.

31. Religion is precious; therefore "this" religion is acceptable.

32. Harm has been done in the name of religion; therefore religion is harmful.

33. What goes up, comes down; an empty elevator went up; an empty elevator will come down for us.

34. Manhood suffrage entitles a criminal to vote.

35. Atoms do not entertain purposes; therefore, human behavior cannot be understood in terms of human purposes.

36. What species survive? The fittest. And which are the fittest? Those which survive.

37. I believe what the Church believes; the Church believes what I believe.

38. "Beer is the nation's bulwark of moderation and sobriety."

39. Nonsectarian schools are irreligious: they do not teach a religious creed.

40. The accused is a radical.

41. "What kind of an artist are you?"

42. "Was the gate not locked when you arrived at the Judson estate?"

43. In this country, where there are no classes, the government ought to be run by business men.

44. Psychoanalysis is the best available tool for the study of human nature. Has it not had therapeutic effect?

45. He praised me! No wonder! He's intelligent!

46. What fallacy pervades Mark Anthony's funeral oration in Shakespeare's *Julius Caesar*?

47. This man has knowledge of the velocity of light; what he has to say about social justice is therefore undoubtedly sound.

48. You must see things our way, Senator; we represent thousands in your constituency.

49. Vivisection must be condemned; for the cutting up of living animals for experimental purposes is wrong.

50. "Try to disprove the appearance of disembodied spirits!"

51. Must I accept my opponent's first principles or supreme premises?

52. What is a "prejudice"?

53. He is a great leader, for he has leadership.

54. Does moral earnestness raise a presumption in favor of intellectual adequacy?

55. Is "criticism" necessarily adverse; necessarily a judgment of worth?

56. "Shall we continue to submit ourselves to the influence of foreign powers?"

57. You must read this book: it's a best seller!

58. This will ought to be set aside because of the mental incompetence of the testator. That the testator was incompetent, seems plain from the provisions of the will.

59. Our forefathers warned us against entangling foreign alliances.

60. How does a "revolution" differ from a "rebellion"?

61. Economic factors are both "materialistic" and "sub-moral."

62. The contentions of those who demand free speech are worthless because they intend to use such freedom to destroy our system of government.

63. That is medievalism; eighteenth-century doctrine.

64. Make a list of question-begging epithets: Victorian, revolutionary, reactionary, etc.

65. Have astronomers discovered not planets only but also their names? Is the word "iron" a heavy solid? Can things be made the same by making their names the same? Does difference of name imply difference of meaning? (James MacKaye *The Logic of Language*, chap. iv.)

66. Show how confusions underlying verbal disputes may be removed by breaking up ambiguities into multiple question and answers. (*Ibid.*, chap. v.)

67. A: "War is wrong." B: "Including resistance to invasion?" A: "But our land is not being invaded."

68. "Admit that $2 \times 2 = 4$? Not till I see what use you will make of it."

69. He doesn't like people; he's a deep thinker.

70. There is much bad metaphysics; why then bother about metaphysics?

71. Touring the atomic world, you can see electrons rotating around the nucleus of the atom which is made up of protons, mesotrons, and neutrons; having seen all these things, you know that nothing we can see is solid and that the atoms hold the secret to the remaking of the world according to our desire.

72. The Ego is the All, the All is the Ego.

73. Human behavior is a congeries of drives, of reflexes, of frustrations and aggressions.

74. Only the arrangement of the same twenty-six letters of the alphabet distinguishes Hamlet's soliloquy from a limerick.

75. Human sex relations developed from promiscuity through group marriage to historical forms of marriage.

76. We see things "only" from a human point of view.

77. Knowledge is only opinion.

78. Heretics must be oppressed; for we must love our neighbors as ourselves.

79. Must our views of human nature be either absolutely pessimistic or absolutely optimistic?

80. "The British Empire embraces 40,000,000 square kilometers; Russia, 19,000,000; America, 9,500,000; Germany, 800,000. It is ridiculous therefore to suppose that Germany expects to conquer the world."

81. Let us eat, drink, and be merry; for tomorrow we die.

82. Is a critic of a crude statement of relativism necessarily an "absolutist"?

83. Is the *tu quoque* argument ever relevant (as when relativists charge absolutists with being themselves disagreed on their standards)?

84. Does the need of relief for the poor refute an objection to some particular scheme of relief for the poor?

85. When you learned that $7 \times 8 = 56$, you desired to gain social approval; and you are unconsciously gratifying that wish when you use that part of the multiplication table.

86. Religion has become subject to psychological explanation; it can be explained away.

87. A first-rate artist will give us a first-rate account of his art.

88. He was angry at the time; therefore he did the deed.

89. Our picture of the universe differs from that of the ancients; we must accordingly live differently.

HOW MAN THINKS

90. Is the only alternative to complete personal integration a complete personal disintegration?

91. This patent medicine booklet tells of people cured by the medicine advertised; that is what I need to cure me.

92. After a healer had given newly born, blind kittens absent treatment for ten days, the kittens were found to have been restored to perfect sight.

93. Manual labor has created all capital.

94. A man steals when he has an opportunity.

95. "And all discord harmony not understood,
And all partial evil universal good."

96. Justice can be secured only under our own form of government.

97. Mother is going to the hospital and is therefore going to have a baby.

98. Pecuniary profit on a large scale is the major incentive to productive work.

99. A judge reared on a farm spoke of the employment contract as analogous to swapping two cows for a horse.

100. Since I have used a certain artificial mnemonic system, my memory has improved.

101. He will succeed because he has previously studied psychology.

102. Buy ABC toothpaste and be successful in love.

103. What I think is as clear as the noonday sun; the thoughts of others are stars scarcely visible in the night.

104. Modern ills are consequences of attempts to live by intelligence.

105. Man is what he eats.

106. What probative force for our own future has the fall of Rome?

HOW MAN THINKS

107. Supposedly necessary evils like witch-hunting and chattel slavery have been abolished; world peace may be within the realm of possibilities.

108. Children first learn to speak without rules; hence a foreign language should be taught to adults by the "direct" method.

109. The human eye is like a camera.

110. What is the value of the analogy of government to a household; to a contract; to an organism?

111. England to Australia, France to Algiers, are as parent to child.

112. Like an army, so other groups must rely on organization, allies, and equipment.

113. Cholera is due to the action of a bacillus; therefore this disease, which resembles cholera, also probably has its origin in the action of some bacillus.

114. Logic is to the art of dealing with particular concrete arguments as ethics is to casuistry.

115. One who recalls names easily must be good at remembering figures and formulas.

116. An effective salesman is no more a good psychologist than a successful farmer is a good botanist.

117. Like good wax, a retentive mind retains its impressions; but a forgetful mind is like a tablet made up of soft wax; hence our minds must have been empty at birth.

118. City dwellers live like bees in a hive and ought therefore to be divided into classes of workers, drones, and rulers.

119. List some analogies based on the number 7.

120. Read the article on "Analogy" in the *Encyclopedia Britannica*.

121. Do the views of our opponents constitute fallacies?

122. "I didn't hear you the first time."

123. We have given more to Europe than we have received from it.

124. "Kant and Carlyle, William James and Herbert Spencer, William McDougall and Henri Bergson, Arthur de Gobineau and Houston Stewart Chamberlain—all of whom would have been horrified at the completed product of Naziism—made such a philosophy not only possible but almost inevitable." (T. P. Neill, quoted by John Dewey, in *Naturalism and the Human Spirit* ed. M. H. Krikorian, p. 5.)

125. Small stockholders control our great corporations.

126. What is the advantage, in debate, of taking account of both sides of a question?

127. On whom is the burden of proof?

128. How would you reply to one who excuses a false assumption by saying it falls in a field other than the one that is his specialty?

129. List some false popular opinions: that blondes and brunettes differ in psychological traits; that wide knowledge is attended by little depth of understanding; etc.

130. If you have accurately calculated the cost of a house at so many dollars per cubic feet, with a false result, what conclusion would you draw? (A. N. Whitehead *An Introduction to Mathematics*, chap. iii.)

131. Sodium chloride is rarer than table salt.

132. The common interest will be preserved if each legislator tries to get the most for the interests he represents.

133. The loss of one hair does not constitute baldness; nor loss of another, etc.; hence it is impossible to become bald.

134. Why does many a critic represent an opponent's position by the terms the position would have if the critic held it?

135. May one consistently plead for peace and go to war; etc.?

136. How would you judge the habit of interpreting other people's behavior as a way of obtaining substitute satisfaction?

137. "Messages in the form of sense-impressions come flowing in . . . but of the nature of things-in-themselves, of what may exist at the other end of our system of telephone wires, we know nothing at all." (Karl Pearson.)

138. "Why was Esau wrong in selling his birthright?"

139. Accept the program of this man who believes in social justice!

140. Was the earthquake which destroyed Lisbon in 1755 a punishment for the presence of Catholics or of non-Catholics?

141. In what different ways is man regarded by a salesman, teacher, religionist, artist, economist, politician, zoölogist, etc.?

142. If I dissent from some particular "scientific" or "social" theory, am I therefore necessarily opposed to science or to a better society?

143. "The speed . . . of light . . . is the speed at which the mass of matter becomes infinite, lengths contract to zero, clocks stand still"

144. The extended world is necessary for knowledge; therefore the world is extension.

145. If we had perfect knowledge of man's neurones, we could read in them his abilities, wants, and ideas.

146. Literacy is indispensable to education; everything else is unnecessary.

147. "Each person's happiness is a good to that person, and the general happiness, therefore, a good to the aggregate of persons." (John Stuart Mill.)

148. A person with fine-textured skin is interested in fine details and in fine ideals.

149. This is my home; therefore the universe is such that this is my home.

150. A sense of restraint is good; therefore our natural impulses are absolutely bad.

151. One who learns is destroyed; for he ceases to be what he was (ignorant) and becomes what he is not (informed).

152. What you are able to do, you do; even when you are not singing, you are able to sing; hence, when you are not singing, you are singing.

153. Recovery of memories attests that nothing once known is ever forgotten.

154. Is the latest always the best?

155. List some "half-truths."

156. Contrast Francesco Sizzi's and Thomas Reid's analogical arguments concerning planets, quoted in L. S. Stebbing *A Modern Introduction to Logic* (1933), pp. 251 f.

157. According to Cardinal Newman: "Half the controversies which go on in the world arise from ignorance of the facts of the case." Comment.

158. Is a conclusion characterized as fallacious thereby condemned as false?

159. The lover's rapture is a glandular secretion.

160. $9 - 48 + 64 = 81 - 72 + 16$; $(3 - 8) = (9 - 4)$; therefore $-5 = 5$.

161. Law does not consist of disembodied rules, but of individual decisions.

162. Since not all philosophers have agreed on concepts of mind, ideas, and the world, my notions of these things are true.

163. We do not know whether the Man Nobody Knows joked at dinner, ever became irritable, ever punished, etc.; hence, we have a right to form our own conception of His character.

CHAPTER III. DISCOURSE

1. How do clouds and noises differ as natural phenomena from the words *cloud* and *noise* as artificial signs or symbols?

2. In what sense do the men of the Middle Ages "speak" to us in their architectural works?

3. How does the development of vocal language differ from that of expressive movements, gestures, and picture writing? (Leonard Bloomfield *An Introduction to the Study of Language*, chap. i.)

4. Do words have fixed limits? Consider: servingman, inkstand, breakfast, in regard to, so to speak, for that matter, *quae cum ita sint, qu'est ce que c'est*.

5. Analyze the ordering of speech in this paragraph from Burke: "First, the people of the colonies are descendants of Englishmen. England, Sir, is a nation which I hope still respects, and formerly adored, her freedom. The colonists emigrated from you when this part of your character was most predominant; and they took this bias and direction the moment they parted from your hands. They are therefore not only devoted to liberty, but liberty according to English ideas, and on English principles."

6. Distinguish logical from practical, poetic, expressive, and ceremonial functions of language. (A. M. Frey and A. W. Levi *Rational Belief*, chap. i.)

7. What is the difference between "prescriptive" and "descriptive" grammar? (See "Grammar" *Encyclopedia Britannica*, [14th ed.] .)

8. Distinguish and relate: eye language and ear language; individual and social phases of language; local and nonlocal varieties of language; types of linguistic changes; phases of English history reflected in language. (See "Language" *op. cit.*)

9. Why do most dictionaries arrange words in alphabetical order?

10. Is agreement or disagreement about the dictionary meaning of a word sufficient to account for common understanding or misunderstanding?

11. Hobbes: "Of the first intention are the names of things Of the second are the names of names and speeches." Discuss accordingly; man, animal, body, and substance.

12. Explain: Logic treats "of second intentions applied to first."

13. What would you say about two stars both equally as bright as another star; about Venus, which agrees with Mars in density, and Jupiter, from which Mars differs; about Mercury and Jupiter, which differ in density from Mars; about Venus and Mars, both of which differ from Jupiter?

14. Compare the three formulas: $A = B = C$, hence $A = C$. $A = B \neq C$, hence $A \neq C$. $A \neq B \neq C$.

15. To ascertain whether two sounds are in unison, need we compare them directly?

16. State the chemical law of definite proportion by weight; of multiple proportions; of combining weights. Contrast natural laws with laws in the sense of decrees.

17. Do animals reason?

18. Note kinds of effort demanded by intelligible speech.

19. Show that grammar treats "of the formal conditions of symbols having meaning"; logic, "of the formal conditions of the truth of symbols"; rhetoric, "of the formal conditions of the force of symbols, or their power of appealing to the mind." (C. S. Peirce *Collected Papers* III.88.)

20. What light is shed on the difference between ways in which men often "think," and ways in which they ought to think; by the difference between good and bad farming, and the like?

21. Wherein lies the importance of ideas, language, and tools in human learning? (I. Edman *Human Traits and Their Social Significance*, chap. i.)

22. "What our differing utterances are relevant to is more important than their relevances to one another." (F. J. E. Woodbridge *Nature and Mind*, p. 25.) Discuss.

23. "We cannot suppose, and there is no reason to suppose, that by the constitution of the mind we are obliged to think of things differently from the manner in which they are." (W. Stanley Jevons *Elementary Lessons in Logic*, [new ed.; New York: The Macmillan Co., 1920 (Copyright 1920 by The Macmillan Company used by permission of The Macmillan Company, publishers)], p. 11.)

24. What do applications of the parallelogram law imply about the universe of discourse? (A. N. Whitehead *An Introduction to Mathematics*, chap. iv.)

25. Does the term "aliens" have reference, in all its uses, to the same persons?

26. Specify the universe of discourse in which we say: Zero is less than anything. What is not less than or equal to zero is plus. He is nowhere around. He is dead. Wine, women, and song make up the universe. There is nothing

but meeting and parting in this world. All men are created free and equal. There is no disputing about tastes.

27. Within what sphere are B's and C's asserted to be identical in the form: $AB = AC$?

28. A distinction has been made between real object sentences (Mr. A visited Africa), pseudo object sentences (This book treats of Africa), and syntactical sentences (This book contains the word "Africa"); because "it is not a quality of Africa to be treated in that book," but "only a quality of the word 'Africa' to be contained in the book." (R. Carnap *Philosophy and Logical Syntax*, p. 61.) Comment on this distinction.

29. Point out a fundamental error concerning abstraction in recent popular "semantics." (Arthur E. Murphy *The Uses of Reason*, pp. 70-80.)

30. To what kind of considerations are we led by reflection on many bodies, many energies, many places, many seconds, many vernaculars, many sciences, etc.?

31. How many experiences do we need to recognize a universal?

32. Of particulars and universals, which are more knowable in the order of sense perception; which, in the order of understanding?

33. What would it mean to say that "Socrates is a man," if Socrates and man are identical; if they are diverse; if they are alike in some respects and different in others? (F. J. E. Woodbridge *The Realm of Mind*, p. 85.)

34. Does mathematics deal with symbols, quantities, or things?

35. Indicate general problems posed for logic in Exercises 36-44.

36. Analyze your thinking in solving the cryptograms:

GJSTU DPNF, GJSTU TFSWFE.

SGD DZQKX AHQC BZSBGDR SGD VNQL.

UNFGR ZNXRF JNFGR.

1452251802018152121250201815212125021142091
202018152121250201815212125190251521.

(A key for this Exercise is on page 343. Do not look at it, however, until you have finished working the cryptograms.)

37. "... wherever the road divided there were sure to be two fingerposts pointing the same way, one marked 'TO TWEEDLEDUM'S HOUSE,' and the other, 'TO THE HOUSE OF TWEEDLEDEE.'

" 'I do believe,' said Alice at last, 'that they live in the same house! I wonder I never thought of that before—' ... on turning a sharp corner, she came upon two fat little men ... they must be ——." (Lewis Carroll *Through the Looking Glass*.)

38. " 'What is a caucus-race?' said Alice 'Why,' said the Dodo, 'the best way to explain it is to do it.' " (Lewis Carroll *Alice In Wonderland*.)

39. The March Hare: "You should say what you mean." Alice: "I do; at least—I mean what I say—that's the same thing, you know."

40. Alice: "Little girls eat eggs quite as much as serpents do." Pigeon: "If they do, why, then they're a kind of serpent."

41. King: "Herald, read the accusation!" White Rabbit: "The Queen of Hearts, she made some tarts The Knave of Hearts, he stole those tarts " King (to the

jury): "Consider your verdict." Rabbit: "Not yet, not yet! There's a good deal to come before that."

42. Alice: "I didn't know that cats *could* grin." Duchess: "You don't know much; and that's a fact."

43. "Manners are not taught in lessons," said Alice. "Lessons teach you to do sums, and things of that sort."

44. How do you know that:

$$a^2 - b^2 = (a + b)(a - b)?$$

45. With a minimum of how many terms does logic deal in a doctrine of terms; of propositions; of syllogisms?

(Key to Exercise 36, III: First come, first served. The early bird catches the worm. Haste makes waste. Never trouble trouble until trouble troubles you.)

CHAPTER IV. TERMS

1. Distinguish: vocabulary, terminology.

2. Cite a few technical terms in Euclid, in Adam Smith, in Darwin, or in a field in which you have a special interest.

3. Has the power of Congress to "regulate commerce" had a fixed meaning?

4. Indicate generalizations of meaning: Edisons, Marconis, Fabian, dainty, kindred, suns, moons, butterfly, character, alcohol, "Let him be Caesar."

5. Show, in terms of formal properties of operations (addition, etc.), how the conception of "number" has been generalized from integers so as to include fractional, negative, imaginary numbers, etc. (A. N. Whitehead *An Introduction to Mathematics*, chaps. vi-viii.)

6. Indicate specializations of meaning: the Black Forest, sweets, secret, "you are a dear," bishop, volume, book, lunatic, planet, minister, pagan, heathen.

7. Indicate underlying metaphors: obvious, occur, transfer, important, invest, suggest, point out, let drop, follow out, lay down, out of the way, elbow (of a pipe), eye (of a needle), mouth (of a river), neck of the woods, anxious, tone color, ward heeler.

8. Consult a thesaurus for words connected with the general subjects of heat and light. Which words are usually used literally; which used figuratively also?

9. What shift has taken place in the meaning of "egregious"?

10. Note the allusions in: tantalize, quixotic, boycott, Hobson's choice, Mother of Parliaments, Mother of Presidents, phoenix, Quisling.

11. Name the languages from which are derived the words: Sabbath, seraph, jubilee, algebra, alkali, zenith, cipher, caravan, shawl, check, calico, chintz, punch, toddy, tea, nankeen, canoe, moccasin, sachem.

12. In what sense does the exception "prove" the rule?

13. Explain: archaism, provincialism, neologism, euphemism, cant, argot, tall talk, etc. (H. L. Mencken *The American Language* [4th ed.].)

14. Illustrate (e.g., from the Constitution of the United States) the need of a mastery of terms.

15. Study some of the ambiguous words listed in Richard Whately *Elements of Logic*, Appendix I.

16. Distinguish concrete and abstract terms: a hard stone, hard work, he turned pale, he turned round, brotherhood, mankind, society, essence, cause, act, ingratitude, black, blackness, reasonableness, homely, homeliness, the pleasant is beneficial, the man who is pleased is benefitted, mermaid.

17. From what properties of bodies does physics abstract? Geometry?

18. Seeing the spatial and the temporal, may we expect therefore also to see space and time? (F. J. E. Woodbridge *An Essay on Nature*, p. 84.)

19. Do images provide understanding of such terms as: personality, respect for law, acceleration, quanta of action?

20. In the history of philosophy, according to Windelband, "European humanity" has embodied its views of the world and of life.

21. Does loyalty require loyalty to loyalty? Must a complete map include itself? Does the barber who shaves all those and only those who do not shave themselves shave himself?

22. Ought we to flee abstractions like the plague?

23. Distinguish singular and general terms: horse, Socrates, a Diogenes, this book, William Pitt, your house, the President of the United States, the victor of Waterloo, the first Christian, a building of more than one hundred stories, an English sovereign six times married.

24. Distinguish the distributive and collective use of terms: the crowd surged forward; the crowd chewed gum; white sheep eat more than black ones; the Pyrenees; oil is lighter than water.

25. Use distributively and collectively: the student body, choir, bushel of wheat, forest, jury.

26. Distinguish positive and negative terms: anonymous, unassuming, independent, homeless, nonsmoker, the Antisaloon Party, radio, let the weak perish that the strong may survive.

27. Is victory in war to be conceived as absence of defeat or in terms of a free world? (A. MacLeish, in H. W. Weigart and V. Stefanson *Compass of the World*, chap. i.)

HOW MAN THINKS

28. Select terms opposed:

knowledge	ignorance	virtue	bad
man	horse	animal	plant
above	to cool	Socrates	Plato
to heat	cold	good	vice
black	red	yellow	white
discrete	two	sour	sweet
continuous	three	bitter	below

29. Distinguish pairs of contradictories and of contraries below:

consistent	inconsistent	beautiful	ugly
continuous	discontinuous	rich	poor
expensive	cheap	living	nonliving
heavy	light	moral	nonmoral
moral	immoral	sweet	bitter
hard	soft	smooth	rough
sharp	dull	sharp	flat

30. Is "revolutionary" the contradictory of "upholder of the divine right of kings"?

31. Select terms that admit of degrees of more or less:

skilled	like	unlike	place
double	just	to heat	to be heated
triangular	three	health	hour
equal	unequal	rectangular	white
syllable	warm	line	surface
solid	man	beautiful	unskilled

32. State the correlatives of: organism, stimulus, sound, right, duty, privilege, no-right, power, liability, immunity, disability, teacher, servant.

HOW MAN THINKS

33. Point out the correlative, contrary, contradictory, and positive and negative terms, and affirmations and denials:

odd	even	hairy	bald
double	half	he sits	he does not sit
sight	blindness	Socrates has sight	Socrates is blind
health	disease	he is well	he is not well
good	bad	he is well	he is ill
black	white	knowledge	the thing known

34. State part of the denotation of: senator, genius, angel, quadruped, continent.

35. Which term within each pair is wider in extension: animal, man; conic section, ellipse; lyric, ode; seaplane, airplane; fugue, counterpoint?

36. Distinguish universal and attribute: war is an "evil"; your plans are "evil"; Jesus was a "man"; this man is "white"; my procedure is "scientific"; critical historical inquiry is "scientific."

37. By what process do you discover the authentic connotation of: birds, security, the spirit of America, foot, justice, sanity, war, tree? (A. M. Frye and A. W. Levi *Rational Belief*, pp. 100-102.)

38. What is the logical scope of triangularity?

39. Discuss the logical uses of the terms as given: square (a plane figure having four equal sides and four equal angles); blackness (the quality common to black objects); house (a building intended to be used for human habitation).

40. According to what logical considerations is "home" viewed as "one's dwelling-place"; as having all attributes common to all homes; as calling up different ideas in different minds?

41. Does a lawyer's hunt for precedents exemplify extension or intension?

42. Does the intension of the term "infant" fluctuate with the birth rate?

43. Discuss the law of the inverse ratio of extension and intension: iron, metal, element, matter, substance; matter, organized matter, animal, man; book, printed book, dictionary, Latin dictionary.

44. Arrange in series, placing each term of greater extension before a term of less extension:

horse	mammalian
heavenly body	matter
planet	Baptist
teacher of philosophy	Christian
Alexander	ruler
quadruped	animal
being	organized substance
Jupiter	monotheist
philosopher	Protestant
emperor	Napoleon III

45. Does "disinterested" mean "uninterested"?

46. What kind of intellectual advance takes place when, instead of asking what Karl Marx "means to me," we inquire into his relation to Hegel?

47. "Under the going canons of philosophy, theology, and the rest, there is no possibility of wide agreement. Referents are too few. There is little in these studies, which A can point to B and say: 'That is what I am talking about: go and touch it. Now do you see what I mean?' (S. Chase *The Tyranny of Words*, p. 360.) Discuss.

48. List some of the more obvious differences between a tree and a cow.

49. Differentiate: assault, battery; clairvoyance, telepathy; drama, tragedy; littoral, pelagic, abyssal, freshwater, terrestrial, aerial (haunts of life); molecule, atom, element, compound; cohesion, adhesion; mixture, compound; reflection, refraction; natural equality, and social equality.

50. Is a distinction a separation? Consider the legislative, executive, and judicial branches of government.

51. Does an observer construe the ascent of an elevator as the building going down?

52. Mention important parts of each: house, eye, flower, automobile, airplane.

53. What is wrong with a book review which says: This is, in the best sense of the word, a very readable book?

54. Is the love of God with the whole heart, soul, mind, and strength a fourfold love?

55. Where do we stop in our analysis of a sentence when interested in grammar; in spelling; in the historical development of letter forms? (R. Carnap *Introduction to Semantics*, p. 4.)

56. List some obvious distinctions made by people impatient with subtler distinctions.

57. If our knowledge is not complete, is it therefore not genuine?

58. Illustrate analysis and synthesis as correlative.

59. Examine Benjamin Franklin's letter to Peter Collinson, October 19, 1752, (*Experiments and Observations on Electricity* [London: 1769]) for the clearness, distinctness, truth, and adequacy of his account of an experiment with a kite.

60. What ideas—clear, distinct, true, and adequate—did Newton most obviously have to have in mind to formulate the law of gravitation? (Whitehead *op. cit.*, p. 30.)

HOW MAN THINKS

61. Distinguish: speed, velocity, acceleration. (*Ibid.*, pp. 52 f.)

62. What is implied for discrimination by protective coloring—e.g., of a chameleon or of a flatfish—or by man-made camouflage?

63. Formulate a brief comparison and contrast of two different forms of government.

64. To which of the senses, if any, would you refer the invisible, the inaudible, etc.? Do these adjectives then have different meanings?

65. How do mechanical, social, and intellectual intelligence agree and differ? (I. Edman *Human Traits and Their Social Significance*, pp. 187 f.)

66. If you dip one hand into cold and the other into hot water, do you receive a "message" from the former that it is cold and from the latter that it is hot? If you then dip both hands into the same warm water, do you receive contradictory "messages"?

CHAPTER V. CATEGORIES

1. Place under the appropriate Aristotelian category:

body	motion	rock	atom
earth	water	air	fire
solid	liquid	gaseous	caloric
mass	energy	force	lever
osmosis	wind	velocity	humidity
ampere	variable	equation	line
co-ordinate	contact	imbalance	candlelight
clock	season	death	temperature
life	metabolism	horizontal	upside down
tropism	secretion	growth	success
plan	intelligence	personality	consciousness
memory	accuracy	inhibition	unconscious

HOW MAN THINKS

character	will	euphoria	fear
thought	sensation	image	hypotheses
habit	belief	mind	discourse
color	sound	hunger	satiated
configuration	beloved	superstition	miracle
future	society	government	pleasure
presumption	despair	prudence	peace
heavier	city	health	to be lectured at

2. State in which of the categories numerical identity ("this" man, etc.) persists through change from one contrary to another (good and bad, white and black, etc.).

3. Point out which of the following have parts with common boundaries: number, articulate sound, line, plane, solid, place, time. Which have parts spatially ordered?

4. Distinguish traits pertaining to dispositions or habits, to capacities or incapacities, to experiences undergone, and to figures or shapes: good boxer, just, irritable, square-jawed, hack writer, crimson-cheeked, expert mathematician, anemic, light-complexioned, sweet.

5. In what way is quality indicated when Socrates is declared to be a "man" or an "animal"? Compare with: "Socrates is white."

6. To what is the subject of an experiment reacting when trained to choose stimulus B which is lighter than stimulus C and than A which is still lighter than B?

7. Distinguish: origination, destruction, increase, diminution, qualitative alteration, locomotion.

8. How would you refute the statement that knowledge is memory?

9. Does an action which may be "long" or "short" fall under the category of quantity?

10. Distinguish between: Arsenic is poisonous. Arsenic is poisoning.

11. What situations are indicated by the terms "percept" and "concept"?

12. Is a person a soul using interests, habits, skills, ideas, etc.?

13. In what different sense is the first of each pair "prior" to the second: the earlier, to the later; one, to two; axioms, to theorems; character, to momentary advantage; the being of Socrates, to the statement that "Socrates is"?

14. Does evolution constitute a power; consciousness, a substance; sound, a wave length? Do body and mind run parallel courses, like railway trains? Does heat warm objects; motion, move them; energy, make them work? Are space, time, and matter the only categories?

15. Compare Aristotle's, Kant's, and later accounts of the categories. ("Categories" *Encyclopedia Britannica* [14th ed.].)

16. If Aristotle's list of categories is defective, must the enterprise of formulating a doctrine of categories therefore be given up?

17. What role does categorical explanation play in the first chapter of Genesis?

18. Discuss: To influence your fellowmen, you must reckon with their background. I misconceived the situation because I failed to realize the issues involved. When life has ceased, the individual fails to respond. Good sense organs are selective and sensitive and respond differently to different intensities and qualities of a stimulus. What is done, is done by an agent.

19. Discuss: Protoplasm contains some thirty identifiable chemical elements. A living process is concerned with matter and energy. Liquids rise in evacuated tubes because

of atmospheric pressure. Force = Mass \times Acceleration. Acceleration is change of velocity per unit-time.

20. Explain: Co-ordinate geometry identifies algebraic correlation with the geometrical locus. The connection between geometry and algebra is essential. The type of loci corresponding to $ax + by = c$ is the class of straight lines in the plane. Given an origin and two axes at right angles, we can determine the position of a point by noting its co-ordinates. The idea of the vector includes determinate direction and determinate length. The relation of co-ordinates to a vector is an instance of the parallelogram law. (See: A. N. Whitehead *An Introduction to Mathematics*, chap. ix.)

21. Give the correlative:

convex	centrifugal	constant	abscissa
finite	perfect	contingent	commensurable
one	cause	motion	beginning
parasite	systole	element	positive
analysis	hear	near	up
darkness	day	hate	take
happy	pleasure	fragile	good
persuade	approach	domination	choice

22. Name some respects in which we have a common human nature; in which we differ.

23. How do the actual and the ideal interpenetrate in our dealings with people? (Charles Beard *The Republic*, chap. i.)

24. Why do we need a balance of change and permanence in the interpretation of a document like the Federal Constitution? (*Ibid.*, chap. ii.)

25. What is neglected in excessive emphasis on our "rights"?

26. Point out dangers of falling into extremes of militarism, pacifism, etc.

27. Account for the movement of the rotary lawn sprinkler.

28. A chemical equation not balanced violates the law of conservation of matter. Explain.

29. Does one who says a motion is relative affirm or deny that motion?

30. Ideas contrasted fall within the same class, whereas analogy brings together ideas belonging to radically different categories. Illustrate.

31. What kind of categories are required if an act of writing is to be described as the making of a will?

32. What operations are indicated by the term "conic section"? (Whitehead *op. cit.*, chap. x.)

33. What is the practical significance of such adjectives as: erroneous and sound, morally right and wrong, valid and fallacious, important and unimportant, real and illusory, beautiful and ugly, etc.?

34. Interpret in terms of possible operations: sweet, sugar, table sugar, sugar of lead, red, gunpowder, water, lumber, lightning, weather. (John Dewey *Logic: The Theory of Inquiry*, pp. 127-31, 134 f.)

CHAPTER VI.

CLASSIFICATION AND DEFINITION

1. Construct a Porphyrian "tree" with the terms: organism, material thing, animal, vertebrate.

2. Indicate the kind of predicable employed: Presidents of the United States are elected officials. Squares are equilateral. Rectangles are parallelograms. Equilateral triangles are equiangular. Several Presidents of the United States have been Episcopalians. Since perception is "dis-

crimination," it may be correct or erroneous. Rubber is elastic. Acid turns litmus paper red.

3. Substitute other subjects for those given: An organism is a material body. A church is a building. A triangle is a rectilinear figure. Wealth has value.

4. Triangles having their corresponding angles equal are similar. Does this hold for other rectilineal figures? (A. N. Whitehead *An Introduction to Mathematics*, pp. 178 f.)

5. Would you prefer to say that Socrates is a "man" or an "animal"; that a spruce is a "tree" or a "plant"? Why?

6. In what sense may the genus be said to be in the species; the species, in the genus?

7. In what sense does the *infima species* not have any extension?

8. Construct sentences containing the words for the five predicables.

9. Show how comparison and contrast enter into the determination of the predicables.

10. Examine critically: Pleasure is the good. Pleasure is a motion. Mixture is a fusion. Temperance is a harmony. Anger is a kind of pain. Good is the opposite of evil. Air is breathable. A man is what Socrates is. That white object is Socrates. That which is approaching is Callias.

11. What questions about a thing spoken of are answered in terms of the categories; of the predicables?

12. How do astronomers proceed in classifying the heavenly bodies?

13. On what basis have plants been divided into trees, shrubs, and herbs; into annuals, biennials, and perennials? On what basis were they divided by Linnaeus; by men in more recent times?

14. Upon what basis is a scientific classification of animals built?

15. What can be concluded from the classification of a porpoise as a mammal?

16. What criteria determine lists of human abilities which include, respectively: absorbing, moving, choosing; manufacturing hormones, executive ability, controlling one's temper; ability to understand such words as elephant and watermelon, and such words as analysis and potentiality? (E. L. Thorndike *Human Nature and the Social Order*, p. 21.)

17. Criticize the following divisions. Literature: prose, verse, history, fiction, religious literature. Geometrical figures: plane figures, parallelograms, rectangles, polygons. Colleges: state-controlled, privately owned, small-town. Business organizations: partnerships, corporations, retailers, wholesalers. Living creatures: walking, swimming, creeping, flying.

18. Should a horse be classified as a mammalian quadruped, a means of transportation, or a kind of meat?

19. Give examples of classifications in which it is difficult, if not impossible, to adhere to the rules.

20. Laws concern persons, property, contracts, torts, crimes, remedies, and government. Laws concerning persons deal with such topics as: manufacturers, divorce, agency, powers, trusts, abstracts of title, annuities, insurance, gas, waters, bridges, animals, commerce, fish, logging, motor vehicles, newspapers, and wharves. "The vice of the whole process (of filing devices) is due to the spirit of concreteness." Comment. (Jerome Hall *Readings in Jurisprudence*, p. 628.)

21. What difference does it make if we classify as a contract or a declaration of trust the promise of a parent,

in the meantime deceased, to give his watch to a child on the latter's birthday? (Charles Morris *How Lawyers Think*, p. 67.)

22. Why do we have the single word "burglary," but no single word for "a contract made at night"? (*Ibid.*, p. 94.)

23. Point out the dichotomies by which we may reach the definition of a tuber as "a stem creeping underground, much thickened, and possessing buds in the form of eyes." (H. W. B. Joseph *An Introduction to Logic* [2nd ed.], p. 128.)

24. If you are not a Democrat, must you then be a Republican?

25. Are there two types of people in the world: those who think in terms of dichotomies and those who do not?

26. What is the significance for inference of the position of an element in a table of chemical elements?

27. In what sense is the circle a particular case of the ellipse? (Whitehead *op. cit.*, p. 120.)

28. Identify genus, species, and differentia: A farce is a light, dramatic composition of a satirical nature. Geography is the science of the earth and its life. Gravity is that force which attracts all bodies toward the center of the earth.

29. Does a dictionary give the true meaning of a disputed word?

30. What method of definition is employed in inter-language dictionaries?

31. Distinguish psychological motives and logical purposes of definitions. (M. R. Cohen and E. Nagel *An Introduction to Logic and Scientific Method*, pp. 231-33.)

32. Why is mass defined in terms of the operation of measurement?

33. What is indicated by the plus sign in the equation: $2 + 2 = 4$; in the equation: $2\text{H} + \text{O} = \text{H}_2\text{O}$?

34. How would you go about finding out what is meant by terms needing clarification in ordinary discourse: intelligence, democracy, rights, liberty, political power, constitutionalism, church, God, etc.?

35. Definition presupposes that the meanings of certain terms are already known. Illustrate.

36. Humpty Dumpty: "When *I* use a word it means just what I choose it to mean." Comment.

37. "The persistence of all matter throughout all time can, I imagine, be secured by definition." (B. Russell *Mysticism and Logic*, p. 172.) Discuss.

38. Compare definition with reduction of a fraction to its lowest terms.

39. Define "function" and "continuous function" in mathematics. (Whitehead *op. cit.*, chap. xi.)

40. Characterize the following definitions as essential, genetic, distinctive, descriptive, physical, or causal: Rent is the price paid for the better grades of durable goods and determined at the margins of intensive and extensive utilization. Economics is the study of man and his material environment with respect to gratification of desire and to human welfare. Rent is the price paid for the use of a durable good or a specified period of time. A wage is a price paid for human services. Competitive capitalism is a form of social organization characterized by freedom of individual enterprise and by governmental exercise of police power for the preservation of free markets. Economic value is the quality of importance attached to objects of choice.

41. What kinds of definition are those proposed by Thomas Aquinas for beauty—that which pleases on sight

(or insight), and the splendor of form shining on proportionate parts of matter?

42. Why would an attempt to explain the meaning of color to a blind man end in failure?

43. Why are *summa genera* and individuals not definable?

44. Examine as definitions:

Time is the moving image of eternity.

Sacerdotal means priestly.

A skyscraper is the Empire State Building.

A parallelogram is a foursided figure whose opposite sides are parallel and equal.

Elasticity is the power of bodies to recover their form after compression.

Quickness is the quality which accomplishes much in little time.

A politician is a person interested in public affairs.

Cooling is the privation of natural heat.

Logic is the Baedeker of the world of thought.

A scraper is an instrument for dipping water.

Virtue is doing good in obedience to the will of God and for the sake of everlasting happiness.

Belief is an assent to that which is credible insofar as it is credible.

Matter is the substance of which a material thing is made.

Learning is what we do in school.

A chair is to sit on.

Beauty is that which gives pleasure.

Sensations are typical constellations of centrifugal irritations of the central nervous system.

45. In what different senses may a "doctor" cure his patient, cure himself, build a house, turn grey?

46. Distinguish generic and specific properties: Parallelograms have their opposite angles equal. Rectangles have all their angles right angles.

47. In what different senses is it a property of a cow to give milk; of man, to write poetry; of the male egret, to grow a certain kind of feather used in ladies' hats; of a rectilinear triangle, to have its angles equal to two right angles? (Joseph *op. cit.*, p. 104.)

48. What kinds of predicable are included in the description of man as a mortal and rational animal walking erect and having broad nails?

49. Distinguish property and accident: Art is creative. Play is creative. A screwdriver is a hammer. Automobiles are vehicles with gasoline engines in the front. Those disinherited are bad.

50. Having thought of circles of various sizes, do you find that the idea of the circle in general is a confused assemblage of circles of which you have thought?

51. Given several ellipses, would you define the ellipse by the magnitude of the circumference, by the variable relation of the axes, by the particular eccentricity, or by two mutually dependent foci equidistant from the points of the circumference?

52. How does the distinction between the essential and the incidental bear upon periodicity in nature? (Whitehead *op. cit.*, chap. xii.)

53. What different consequences ensue if one makes freedom, or knowledge, essential to social welfare?

54. With what certainty may we distinguish *ante mortem* from *post mortem* wounds?

55. Show how distinctions of the essential and incidental may enter into disagreements between experts and the untrained.

56. Can you think of anyone to whom man's featherless and biped qualities would be important?

57. Do a philosopher's incidental opinions constitute his characteristic philosophic position?

58. In what different senses is self-love universal; commendable; reprehensible?

59. In what different senses may one desire health; medicine; sweet wine?

60. Enumerate devices useful for the clarification of terms.

CHAPTER VII. PROPOSITIONS

1. What is a sentence; a proposition?

2. Describe the way a speaker builds up a sentence in ordinary conversation.

3. How many propositions are expressed in the sentence: "The last rose of summer is gone; it is fled"?

4. Relate to the doctrine of propositions the sentences: "Fire!" "Avaunt!" "A horse! My kingdom for a horse!"

5. Is the definition of a triangle, as a plane figure bounded by three straight lines, a proposition?

6. State the various propositions contained in this sentence from Machiavelli's *The Prince*: "A prince ought to inspire fear in such a way that, if he does not win love, he avoids hatred; because he can endure very well being feared whilst he is not hated, which he will always be as long as he abstains from the property of his citizens and from their women."

7. In what order would you apply to propositions the tests of intelligibility, probability, and utility?

8. How does the statement, "this proposition is true," differ in applicability from the statement, "this proposition is either true or false"?

9. Classify grammatically the forms taken by the subjects in the following: Whales are mammals. You are right. No good will come of it. To err is human. Seeing is believing. Who steals my purse, steals trash.

10. State and illustrate kinds of expression which may serve as an object.

11. Classify grammatically the attributive adjective modifiers: good books, falling stars, the iron mask, a cross of gold, a dog's life, Isaiah the prophet, the way to proceed.

12. Distinguish the grammatical forms taken by predicates: Pippa passes. The sun has risen. I felt such a fool. He died young. He broke the ice. The years pass swiftly.

13. What parts of speech may be modified by objective and adverbial modifiers?

14. Note forms assumed by adverbials: Make haste slowly. He entered in haste. He could not hear well from where he sat. Present company excepted. I came as soon as I could.

15. Analyze the construction of the compound sentences: Art is long, and time is fleeting. Neither duty, nor honor, nor gratitude has any possible claim on him. Thou hast neither heat, affection, limb, nor beauty. Hearts, tongues, figures, scribes, bards, poets, cannot think, speak, cast, write, sing, number, his love to Antony.

16. Analyze the construction of the complex sentences: Where he is weakest is in his facts. My brother is a good businessman, which I am not. He has been a good influence wherever he has been.

17. Exhibit in a diagram: "I feel confident that if Jefferson were living in our day he would see what we see: that the individual is caught in a great confused nexus of all sorts of complicated circumstances, and that to let him alone is to leave him helpless as against the obstacles with

which he has to contend; and that, therefore, law in our day must come to assistance of the individual." (Woodrow Wilson *The New Freedom*, p. 283.)

18. Does the word *Glass* on a case constitute a kind of proposition?

19. Why may it be advisable, in preparing a speech or an article, to write down and keep before one a clearly articulated proposition?

20. Classify the propositions:

No uninitiated persons are admitted.

Many unintentional offenses are inexcusable.

Napoleon was defeated at Waterloo.

Successful individuals have large vocabularies.

All misdemeanors are crimes.

Some crimes are statutory.

No offer without consideration is a binding contract.

Some promissory notes are negotiable instruments.

Enzymes are catalysts.

Some water is not fit to drink.

Certain inconveniences are intolerable.

A certain man had two sons.

The cow is in the meadow.

The cow is a domestic animal.

This painting is a Turner.

She was a perfect lady.

Most neurotics are self-centered.

All men are animals.

All the apostles were twelve.

No blue-green algae form flagellated cells or gametes.

Few succeed.

A few succeed.

Not a few succeed.

21. Put into logical form, distinguishing logical subject, copula, and logical predicate: Fallen is Babylon the great. They little know who count me out. Sweet are the uses of adversity. They never fail who die in a good cause.

22. State carefully the meaning of "all":

All angles of a triangle are equal to two right angles.

All angles of a triangle are less than two right angles.

All diamonds are combustible.

All the apostles of Jesus were Jews.

All the fish weighed five pounds.

All mathematical operations will illustrate integrated infinity.

All's well that ends well.

All the king's horsemen couldn't put Humpty Dumpty together again.

Not all practice what they preach.

The eyes of all in the audience were filled with tears.

23. What is the meaning of "*some* number x " in the algebraic statements: $x + 2 = 2 + x$; $x + 2 = 3$; $x + 2 > 3$?

24. Which of the following formulas are satisfied by every number; by no number; by some number? $x + 1 = 4 + x$; $x^2 = 36$; $(x + 2)(x - 2) < x^2$; $x + 12 > x + 18$.

25. What would be the scientific status of the information that "some plants are built up of cells"?

26. How would a court understand "some" in the testimony of a witness that he had seen some of the prisoners at the bar near the scene of a crime?

27. Deny: A whale is not a fish. I know that a whale is not a fish.

28. Formulate two simple propositions which differ in form; in matter; in both.

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29. Only citizens have the right to vote. Does this mean that all citizens have the right to vote?

30. Restate as "A" propositions: Only the plant manufactures chlorophyll. Only light can travel with the speed of light. Only civil officers are subject to impeachment. Man alone seems to have continuous binocular and stereoscopic vision. No one ever broke with logic but logic broke him.

31. Can you restate an "exclusive" proposition in "E" form?

32. Add the word "only" in different positions: She smiled at him.

33. Restate: All plants absorb carbonic acid, except certain fungi. All our states have a county board except Georgia and Rhode Island. All but twenty of the elements are metals. Water expands when heated and contracts when cooled, except between 4°C. and the freezing point.

34. When an exception to a universal proposition is noted, is it to the point to object, "That is an extreme case"?

35. How would a statement about "all Europe except Spain and Portugal" differ from the same statement about "all the states of Europe except two"?

36. Specify accurately the distribution of subject terms: All mammals are vertebrates. No terrestrial animal is stationary. Some fungi are poisonous. Some oral contracts are not enforceable. Brutus is an honorable man.

37. Specify accurately the distribution of predicate terms: All lions are tawny. No plants with opposite leaves are orchids. Some books are uninteresting. Some things observed are not remembered. Every person has his limitations. He who does not hope is unfortunate.

38. Specify accurately the distribution of subject terms and the distribution of predicate terms: Some yeasts can

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induce alcoholic fermentation. Any two circles are similar figures. Some sharks are not dangerous. None but solid bodies are crystals. No equilateral triangle is right-angled. The capitals of all the great military powers, except Chinking, are nearer to the Arctic Circle than to the equator. A certain man went down from Jerusalem to Jericho.

39. Is the distribution of the subject term considered in connection with the quantity or with the quality of a proposition? The distribution of the predicate term?

40. Construe: All men are sometimes downcast. Gladstone is always eloquent. Browning is sometimes obscure. The sun is the center of the planetary system. A man's a man. The dodo is an extinct bird. Homer was a poet.

41. Classify the propositions:

Some books are still written in Latin, and some scholars speak it.

If the weather turns warm, the mercury rises.

Every little boy or girl is either a little Liberal, or else a little Conservative.

Traffic lights show either red or yellow or green.

Triangles are equilateral, isosceles, or scalene.

Conic sections are hyperbolas, ellipses, or parabolas.

When forces are in equilibrium, no motion is produced.

Although clay is not a metal, it is malleable.

Unless acted upon by some external, unbalanced force, a body persists in its state of rest or of uniform motion in a straight line.

42. Determine truth-conditions of constituent propositions under which respective types of compound propositions are true: I came and saw and conquered. If he persists in his extravagance, he will be ruined. Successful men

are either intelligent or lucky. Either we teach ourselves, or we remain ignorant.

43. Determine truth-conditions under which respective types of compound propositions are false: Columbus was a great sailor, and he sailed around the world. All men would be tyrants if they could. Teachers or students will receive a special discount. Either all men are responsible for all they do, or none are.

44. Show how the truth of each respective type of compound proposition determines, or fails to determine, truth or falsity of its constituent propositions: He cannot act, but he can sing. If this is carbon, it is inflammable. A speech is deliberative, judicial, or declamatory. Rectilineal figures are triangles or quadrilaterals or polygons.

45. Show how the falsity of each respective type of compound proposition determines, or fails to determine, truth or falsity of its constituent propositions: It is raining, and it is cold. If he is in doubt, he will inquire. Either he or I must go. It's a case of all or nothing.

46. Translate into categorical form: If a body of land is completely surrounded by water, then it is an island. Many fail in health because they starve their minds. Only where there are centers of organization other than political is liberty possible.

47. If I admire a person as a neighbor, must I therefore necessarily admire him also as a public leader; and vice versa? Express in the form of a conjunctive proposition the meaning intended when it is said that our national enemies are to be hated.

48. Construct a truth table with columns for: p ; q ; p and q ; not both p and q ; p or q ; p implies q (i.e., either p false or q true). Which compound propositions are affirmative, which negative, in their significance?

49. Construct a truth table with columns for: p ; q ; p and q ; (therefore) p true; (therefore) q true. Which of these inferences is valid?

50. Construct a truth table with columns for: p ; q ; p or q ; not both p and q ; either p or q but not both p and q .

51. Show by means of a truth table that p is equivalent to q if and only if p implies q and q implies p . (John C. Cooley *A Primer of Formal Logic*, p. 34.)

52. Construct a truth table with columns for: p ; q ; r ; p or q ; p and r ; if p or q then p and r . (Alfred Tarski *Introduction to Logic*, p. 41.)

CHAPTER VIII.

THE OPPOSITION OF PROPOSITIONS

1. Comment on a conversation in which A says, "I went to——this afternoon," and B replies, "No, I did not." Or A says, "I would feel guilty if I were not constant," and B replies, "I wouldn't feel guilty in the least." (F. P. Ramsey *The Foundations of Mathematics*, p. 289.)

2. Are assertions and counterassertions satisfactorily related in the usual type of popular discussion?

3. If two speakers do not say the same thing, are they always talking about different subject matters?

4. By what propositions, simplest to establish, could you refute one who asserted: all children are unselfish; no girl is indifferent to handsome men; some musicians have mechanical hobbies; some scientists are philosophically minded? Given p true, is q true, false, or undetermined? (p) all clergymen are sincere, (q) no clergymen are sincere; (p) some great men come from the North, (q) some great men come from the South; (p) this is an apple, (q) this is a fruit; (p) this is a fruit, (q) this is an apple. Given

p false in these examples, is q true, false, or undetermined? (After completing chap. viii, check your answers.)

5. Show the logical relation between each in the pairs of propositions: Every religion is good; no religion is good. He is in England; he is in Sweden. These stockings are silk; these stockings are rayon.

6. Give a contrary of each: No one knows every truth. All's well that ends well. There are ten people in this room. You always scold me. Everything he touches turns into gold. No centaurs feed on grass.

7. Does a denial warrant affirmation of the contrary? Does an affirmation warrant denial of the contrary?

8. How are universal affirmation and denial related as applied to the question whether marine vertebrates are cold-blooded? What is the function of such universal affirmations and denials?

9. Why is it necessary to stress that contraries may both be false? Consider the controversies whether everything or nothing is new; all or no institutions were consciously invented; all or no abstractions are fictions; the whole of a selected religious literature or none of it is reliable; individualism is wholly or not at all desirable; freedom is in every or no respect dangerous.

10. How does the "all or nothing" principle operate, according to Ibsen's *Brand*?

11. State which propositions exemplify subcontraries: Reason serves the emotions; reason guides the emotions. Poverty fertilizes genius; poverty kills genius. Individuals are molded by social pressure; individuals are stimulated to adapt themselves. Men are *only* dumb cogs in a machine; men can rise above their circumstances and passions. Human behavior is often predictable; human behavior is often unpredictable. Perception is a selective response; perception

is an isolating response. Some vampires rise from their graves at night; some do not.

12. Give a subcontrary of each: Much illness is due to lack of thorough mastication. Mathematical symbols make things easy. Potter quarrels with potter. Like attracts like. Poverty breeds character. Some writings are not admissible in court without production of the person who made them. Individuals differ. Personality depends upon glands. Government ought not attempt too much.

13. How are particular affirmation and denial related as applied to marine vertebrates as cold-blooded? Does the form of the propositions suffice to determine that some are and some are not (cold-blooded)? Compare the status of propositions so related to that of contraries. Do the former present final results?

14. Why is it important to distinguish subcontraries from contraries and contradictories? Consider the issues whether: tradition is to be followed or improved; a constitutional provision shall be respected or amended; wealth is a power for good or for evil; men are like other animals or superior to them; light is vibratory or discrete; Hegel is right or wrong.

15. Are public affairs, as a rule, questions of black or white?

16. Comment on the texts: "Every man shall bear his own burden." "Bear ye one another's burdens." "Cast thy burden on the Lord."

17. Draw up a list of proverbs and counterproverbs: Too many cooks spoil the broth. Many hands make light work. Etc.

18. Ought one firmly persuaded of a conviction willingly submit it to debate?

19. Distinguish superalterns and subalterns: SaP; SiP. SoP; SeP. A number of experiments have established a given result for the cases examined; the result seems to hold for all cases of that type. All devils have cloven hoofs; some devils have cloven hoofs.

20. Supply subalterns: All partnerships have two or more members. Every plant is adapted to its station in life. He is suspicious of everyone on the committee. No roses without thorns. Some officials are officious. The furniture in this room is very expensive.

21. Supply superalterns: Some men have imperfections. Many are called. Few are chosen. Not all who rave are divinely inspired. All tyrants are mortal. No students of logic can achieve mastery of their subject without perseverance. Political extremists are difficult to get along with. Isosceles triangles have three sides.

22. Discuss added determinants: Plants are bodies decomposing carbonic acids; therefore microscopic plants are microscopic bodies decomposing carbonic acid. Form constitutes an animal; a dead body has the same configuration as a living one. An elephant is an animal; therefore a small elephant is a small animal. A fly is an animal; therefore a large fly is a large animal.

23. Discuss complex conceptions. An elephant is an animal; therefore the tail of an elephant is the tail of an animal. Protestants are Christians; therefore a majority of Protestants are a majority of Christians.

24. Under what circumstances may it become pertinent to call special attention to a superaltern proposition? Consider: Some war is what Sherman said it was. Some men do not altogether lack virtue. All Protestant believers are Christian believers. School administrators are school people interested in their schools.

25. Under what circumstances may it become pertinent to call special attention to a subaltern proposition? Consider: All theaters in this town have a closing time fixed by law. All admission fees to public performances are subject to tax.

26. Point out how "some" may change in meaning in the course of an inquiry: Some of my answers to these questions seem correct. This gambler has lost some of his money. Some of your children are following in your footsteps. Some of the injured are dying.

27. Distinguish contraries, subcontraries, and contradictories: All gems are incombustible; all diamonds are combustible. Theory is futile; I have good ideas on government, trade, and the like. A pistol may carry two-hundred yards; but not mine. Governors have their interests; so have the governed.

28. Contradict: Not every tale is to be believed. Whom the gods love, die young. All corporations are immune from conviction for crime. There is nothing new under the sun. Some escaped. Only the humble can be elevated.

29. If SaP is true, SiP is true; and if SiP is true, SoP may be true. Does it follow that, if SaP is true, SoP may be true?

30. When a generalization has been negated by an exception, what positive use can be made of the contradiction?

31. How do contradictory propositions compare in determinateness with contraries and subcontraries?

32. Does the establishment of a contradiction suffice of itself to eliminate one proposition in favor of another?

33. May we assume that alternatives not equivalent are mutually exclusive? Consider: Is worry mental or physical? Are men naturally selfish or unselfish? Is personal

beauty culture natural or artificial? Do men behave as they do because of heredity or environment?

34. Examine the opposition of two indefinite propositions, one affirmative and one negative: "horses are beautiful"; "horses are not beautiful." When are such propositions contrary; when subcontrary; when contradictory?

35. Consider the relations among the propositions: He knows what is good. He knows what is not good. He does not know what is not good. He does not know what is good.

36. Show the relations among the propositions: If I were rich, I would be happy. If I were rich, I would not be happy. If I were rich, I might be happy. If I were rich, I might not be happy.

37. Distinguish the necessity we may ascribe to tomorrow's naval battle: that it will be; that it will not be; that it will either be or not be. (Aristotle *De Interpretatione*, chap. ix.)

38. Contradict: For X to be, is necessary. For X not to be, is necessary. For X to be, is possible. For X not to be, is possible.

39. State the sense in which it is or is not necessary, and the sense in which it is or is not possible, that: The sum of the angles of a triangle is two right angles. Fire gives heat. I may either walk or not. My coat may either tear or not. I can walk when I do. Given certain conditions, I could walk. (Aristotle *op. cit.*, chap. xiii.)

CHAPTER IX.

EQUIVALENT AND INDEPENDENT PROPOSITIONS

1. If all scholars are studious, does it follow that either a person is not a scholar, or he is studious? No scholars are

other than studious? All studious persons are scholars? All persons not studious are nonscholars? All nonscholars are other than studious? Since my sister is studious, she aspires to be a scholar? (After completing chap. ix check your answers.)

2. What is the point about the challenge to restate a thought in other words?

3. What happens when, after saying, "Nothing acts except what is actual," we add, "If something does not exist, it cannot do anything"? And, "No one neglects to plant seed because no rain may fall"?

4. Duchess: "... 'tis love that makes the world go round." Alice: "... it's done by everybody minding their own business." Duchess: "'Take care of the sense, and the sounds will take care of themselves.'" Comment.

5. Formulate the contrapositives of the hypothetical propositions: If a promise to sell land is enforceable, then it is in writing. If $a = 1$, then $ab = b$. If we lower the tariff, we lower our standard of living. If a triangle is equiangular, it cannot be right-angled.

6. Apply the formula for equivalent compound propositions: If she weeps the provocation was slight. If a statute deprives a man of his property without due process, the statute is unconstitutional. If loanable funds are scarce, the call rate is very high. Either eggs must be stored for winter, or they will be very dear. You cannot have your cake and eat it too. If men were wise, they would not struggle for power.

7. If only scientists and logicians care for definitions, what is to be affirmed about those who care for definitions? Apply the formula for equivalent compound propositions.

8. Does "either A or B" imply: if A then not-B; if not-B then A; if B then not-A; if not-B then A?

9. Contradict:

If a student worries about an examination, he desires to fail.

Both candidates have much to commend them.

"Unstain'd by gold or fee."

Soldiers and militiamen are not the only people who wear uniforms.

Except the wise man (said the Stoics), all men are truly fools.

All S is P , and all P is S .

We lose money, and then gain it.

You can't have your cake and eat it too.

He was poor, and if not unintelligent, was weakness itself—influenced by evil companions or an unfavorable environment.

"They have broken your doll, I know; and your tea-set blue, and your play-house too, are things of the long ago; but childish troubles will soon pass by."

10. Give equivalent compound propositions for: If p then q . If p false then q false. If p then q false. If p false then q .

11. Give the conjunctive proposition which contradicts: If p then q . If q then p . Not both p and q . Either p or q .

12. Given: If p then q , does it follow that, if q then p ; if p false then q false; if q false then p false? Try: If x is a positive number, then $2x$ is a positive number. Then replace $2x$ by x^2 .

13. Construct a truth table showing that "not both p and q " is equivalent to "either p false or q false."

14. Construct a truth table showing that the contradictory of "either p or q " is equivalent to " p false and q false" (or "neither p nor q ").

15. Why are equivalent formulas important in the field of mathematics?

16. Obvert:

All infants are responsible.

No planets are self-luminous.

Some violets are blue.

Some who are asked for advice do not answer according to their own opinion.

All ants are segmented.

No infants in arms admitted.

Every mistake is not a proof of ignorance.

Blessed are the peacemakers.

Twice two is four.

p is independent of q .

There are no values which are not functions of human interest.

Nothing happens without a cause.

17. Obvert: No gentlemen kiss and tell. Every conqueror is either a hero or a villain. Only action can bring relief from inner conflict.

18. In what kind of term do we determine what can be predicated of S if S is not P ? How does the original proposition become transformed?

19. What is the value of propositions containing as a predicate a negative term? Is it impossible to tell what another is thinking by asking him questions to be answered only by Yes or No?

20. If sight is rendered painful or is destroyed by excessive brightness or darkness, what may be affirmed about the nature of sight? Apply to sense perception generally.

21. Simplify: I would not deny that an increase in taxation is not an unwarranted and unjustified procedure.

22. Distinguish: to be able not to see; not to be able to see.

23. How is "proved to be guilty" related to "proved not to be guilty"? If "X is hated," would this be equivalent to, "X is not loved"?

24. What is the logical relation between, "All antelopes have horns," and, "It is not the case that some antelopes do not have horns"?

25. What is the equivalent of the statement that, it is false that a given proposition is false?

26. Convert:

Every conscientious objector is religious.

No part of Colorado is west of any part of Utah.

Some poets have imagination.

Beautiful things give pleasure.

The sublime is big.

All knowledge is acquired.

Any two sides of a triangle are greater than the third side of the triangle.

Pines are evergreens.

No good Moslem eats pork.

Every common nitrate is soluble.

27. If some scientists are not musicians, does it follow that some musicians are not scientists? If some apples are not sour, does it follow that some sour things are not apples?

28. Convert:

a is greater than b .

Freedom is synonymous with liberty.

Russia invaded Finland.

Intellectual discipline is a prerequisite for intelligent action.

Democritus was a younger contemporary of Socrates.

Supposition is not identical with opinion.

Supposition is not a genus of opinion.

Someone likes everyone.

29. Convert: Twice two is four. The Empire State Building is the tallest building in the world. p is not incompatible with q . p is not an alternative to q . All equilateral triangles are equiangular.

30. Convert: Every trespasser will be prosecuted. Does the converse follow?

31. Might SoP be consistent with PaS; with PeS; with PiS; with PoS?

32. Is "I see others" the converse of "Others see me"?

33. Convert: What I achieve best, I enjoy best.

34. Just what do we accomplish by obversion; by conversion; by obversion of the converse?

35. What do we accomplish by contraposition?

36. Give the partial and full contrapositives: All our employees are reliable. No unfriendly people are happy. Some reformers are not radicals.

37. Give the partial and full contrapositives of PaS.

38. If only those who love their neighbors have a sense of something beyond themselves, what is to be said about such as do not love their neighbors?

39. By what processes do we pass from, "Only those who respect others will learn from them," to, "No one learns from those he contemns"?

40. If A is B, does it follow that every not-A is not-B? By what reasoning would you show that if every not-B is not-A, then every A is B?

41. Give the partial and full inverses: Decline of ability is due to ageing. All matter is atomic. No antelopes are ungraceful.

42. As to the assertion that "all dogs bark," we may ask whether dogs exist; whether there is anything that barks;

whether there are beings other than dogs; whether there is anything that does not bark. Also we may inquire: If there are beings other than dogs, are there any of these beings that do not bark; also, if there are beings that bark and if some of these are not dogs, are there beings that are not dogs and that do not bark?

Show how the two sets of questions differ in their relation to the original proposition. What transformation of the original proposition is called for in the last question? Granting the original proposition, could you expect to find animals that do not bark among dogs? Where, if at all, would you find beings that do not bark? Does the fact that all dogs bark prove that foxes do not bark? (C. H. Rieber *Footnotes to Formal Logic*, p. 114.)

43. If no mathematicians can square the circle, then, if you persist in searching for a circle-squarer, you would find him among nonmathematicians, if there are any such. How does this statement illuminate the process of inversion? (*Ibid.*, p. 115.)

44. Note that inversion assumes that both S and P and their negatives exist in the same universe of discourse. What would be the effect on inversion as a process if we were told that neither S nor P exist; that S exists but not P, or vice versa; that S or that P has no contradictory? (*Ibid.*, p. 116.)

45. Examine:

Warmth is agreeable; therefore cold is disagreeable.

War produces evil; therefore peace produces good.

Poverty breeds character; therefore the abolition of poverty will not breed character.

The good are happy; the wicked are therefore unhappy.

All birds have feathers; nothing that is not a bird possesses any trace of them.

Those who see find the visible world to be their evident environment; those who do not see do not find the visible world to be their evident environment.

Crystalline substances melt at definite points; non-crystalline substances have no definite melting point.

All great novelists have failed to write a perfect novel; some who were not great novelists have not failed to write a perfect novel.

46. Justify the processes of obversion, conversion, contraposition, and inversion. Apply as far as possible to: All men are vertebrates. No oxygen is hydrogen. Some foreigners are unpopular. Some lectures on art are not sentimental. Those who have built their castles in the air must lay foundations under them.

47. Discuss: The theory of evolution is correct; X does not think the theory of evolution correct. Space is vast; why then be bigoted? Collectivism is inevitable; collectivism is desirable.

48. Indicate the relation within each of the following pairs of propositions:

"Let love be without dissimulation" (Rom. 12:9. A.V.);
"Let your love be a real thing" (Rom. 12:9. Moffatt tr.).

Nations can demand nothing more noble than death in battle; nations ought to settle their disputes in a World Court.

Plants have their roots in the dark soil; plants depend on rain and sunshine from above.

Intelligence is a product of evolution; intelligence cannot know life.

Every religion is an opiate; all religions are equally good.

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Mammals have lungs; human beings have lungs.

Philosophy ought to be experimental; philosophy ought to be adventurous.

God is the author of everything; God is not the author of evil.

The years pass swiftly; the years drag their weary way.

The world ought to be made safe for democracy; unions ought to be scrapped.

Men want freedom; men cannot endure freedom.

This piece of gold is yellow; all gold is yellow.

People have married voluntarily; the voluntary element plays a part in all human associations.

Teachers must subject pupils to discipline; teachers must get their pupils interested.

College graduates have become starred scientists; college graduates have become inmates of jails.

Only the right triumphs; what triumphs is always right.

Much clothing is manufactured in sweatshops; I buy ready-to-wear clothing.

Christians believe in Divine Providence; some who believe in Divine Providence are Christians.

Moderns stand in awe of the collective; moderns exaggerate the importance of the individual and personal.

You have criticized an evolutionary theory; but, surely, you must admit that evolution looks like a fact.

Nature acts by the shortest ways; nothing is or happens without a reason.

Without a cause there is no effect; everything is connected in definite ways with definite other things.

49. Define the seven possible logical relations among propositions. (C. A. Mace *The Principles of Logic*, p. 99.)

50. Point out the order of consistency of p with q in which the following formulas are arranged: p implies q , and q implies p . p implies q , but not vice versa. q implies p , but not vice versa. p is true or false, and q is true or false or undetermined. Either p or q , or both. Not both p and q . Either p true or q true but not both. (W. E. Johnson *Logic* I.49.)

CHAPTER X.

CATEGORICAL SYLLOGISMS

1. Analyze into component parts: A business affected with a public interest is subject to price regulation; this business is affected with a public interest; therefore this business is subject to price regulation. All witnesses are unreliable, and unreliable people need training; therefore some in need of training are witnesses. Shakespeare was both poet and dramatist; hence some poets are dramatists. True patriots seek the common welfare; some public leaders therefore, who do not do this, are not true patriots.

2. Distinguish valid and invalid arguments: A is taller than B; C is shorter than A; therefore C is shorter than B. The unemployed should be forced to go to work because they are idle, and idlers will work only if you force them to. Scientists are learned; Darwin was a scientist; Darwin was therefore a learned scientist. No live creature is dead; every man is a live creature; therefore no man is dead. One has a right to the fruits of one's labor; private correspondence is a fruit of the author's labor; therefore private correspondence is the property of the author.

3. Supply missing premises: A maximum absorber of radiant energy, a black body is therefore also a maximum emitter of radii. All particles are bodies; therefore all par-

ticles are extended. Triangles ABC and DEF , having their corresponding angles equal, are similar. Those who use their imagination are artists; historians are therefore artists. Who lacks gratitude, lacks understanding; you are therefore lacking in understanding. Gold is fusible.

4. Supply conclusions: Steel structures, in which are included some garages, are fireproof. Anger may turn to pity; not so hatred. There is a love of peace, but not a virtue, which conflicts with love of truth.

5. Any two circles are similar; since the ratio of the distance between any two points on the one, and the distance between the corresponding points on the other, is constant. Supply a premise. (A. N. Whitehead *An Introduction to Mathematics*, chap. xiii.)

6. An observer, having determined the directions in which two stars are seen, can measure the angle between them; when the angle between two given stars has been measured, an astronomer can determine the directions in which they are seen. Explain. (*Ibid.*, p. 175.)

7. Why do we need a rule for an inference from a case to a result?

8. Does the use of the syllogism in legal decisions imply that the rule of every case is known before the case arises?

9. Cardinal: "My first penitent was a murderer." Count (later): "I was the Cardinal's first penitent." Could the inference from both premises have been drawn from either premise alone?

10. May a syllogism demonstrate more than just one conclusion?

11. Suppose an entire regiment, in which a cousin had enlisted, has been taken prisoner. What would you conclude about your cousin? If later you learn that your cousin had

previously been transferred to another regiment, would this affect the validity of the original syllogism?

12. Is every valid argument a syllogism?

13. Comment on the argument: All syllogistic principles are false; if MaP and SaM then SaP, and this is a syllogistic principle; therefore it is false.

14. Does truth of a syllogistic conclusion demonstrate truth of the premises? Does falsity of a syllogistic conclusion demonstrate falsity of the premises? Does truth of all three syllogistic propositions guarantee validity of a syllogistic argument? Can true premises be made to yield a false syllogistic conclusion?

15. If you had seen, in Pavlov's laboratory, the flow of saliva start in a dog's mouth at the sound of a bell, how would you account for this response? What does the explanation suggest concerning the function of the middle term in a syllogism?

16. By what means would you bring together in a syllogistic conclusion the terms: this man, ailing; mathematics, worthy of study; defendant, liable for punitive damages; your course, the best policy; this day, unpleasant; platinum, combines with oxygen; potassium, sodium, lithium, etc., lighter than water?

17. Is the syllogism a substitute for experience or for scientific inquiry?

18. What bearing has knowledge of syllogistic structure on the use of enthymemes?

19. Note suggestions, in the doctrine of terms, of propositions, and of syllogisms, for the selection of premises for syllogistic conclusions.

20. Construct possible syllogisms: I am ill. Men smell only the painful or pleasant. He calls the tune. No smoking in this oil refinery. Orestes will love his mother. He has

given a promise for what he was already bound to do. Intellect is an achievement.

21. Point out the fallacies committed:

All Englishmen are British subjects; all Australians are British subjects; so all Australians are Englishmen.

All alcoholics are undernourished; but X is not an alcoholic; therefore X is not undernourished.

Whoever believes this is a heretic; so you are not a heretic, because you do not believe this.

Danes are not Hindus, and Hindus are not Lutherans; therefore Danes are Lutherans.

If neither Gauls nor Celts were Romans, how are Gauls and Celts related?

Animals, including dogs, have feeling; therefore a dead dog has feeling.

The sick have increased expenses and need a physician; therefore all who need a physician have increased expenses.

Valid syllogisms have three terms; this syllogism has three terms; hence it is valid.

Current events are interesting; traditions are not current events and therefore not interesting.

Faith, being directed, like love, to the good, is love.

22. In what respect might there be a fallacy of four terms in the fallacies of illicit major, illicit minor, or undistributed middle?

23. Do two affirmative, two negative, or two particular premises warrant affirmative, negative, and particular conclusions, respectively? (H. A. Larrabee *Reliable Knowledge*, pp. 103, 104.)

24. What rules are violated by: PaM, SaM, therefore SoP? By: MaP, MaS, therefore SeP?

25. Show that the total number of terms distributed in the premises of a valid syllogism must always be greater than the total number of terms distributed in the conclusion.

26. Discuss: Most able men have good will towards others; most able men prefer self-chosen work. Most animals are invertebrates; most animals are capable of repairing damage to themselves.

27. Analyze:

Some X is Y; all Z is X; therefore some Z is Y.

Men are plants; both require air.

Some teachers are not preachers: therefore not shepherds of men.

Since no right action is inexpedient, and this action is not wrong, this action is expedient.

A is fairer than B, but darker than C; who is darker, B or C?

Some professional men suffer from professional jealousy; some professional men slander their colleagues; therefore those who slander their colleagues suffer from professional jealousy.

This Italian must be a Fascist; because many Italians are, you know.

Communists support consumer co-operatives; the league of women shoppers supports consumer co-operatives and is therefore communistic.

Great men have risen above poverty; I have risen above poverty; I ought therefore to be elected senator.

Buddhists are pacifists; no Methodists are Buddhists; therefore no Methodists are pacifists.

No alien may vote; X is not an alien; therefore X may vote.

Gold is a mineral and is precious; therefore all minerals are precious.

All heroes deserve rewards; some soldiers are heroes; therefore all soldiers deserve rewards.

Some scientists are not philosophers; some plutocrats are scientists; so some plutocrats are not philosophers.

Food is necessary to life; corn is food; therefore corn is necessary to life.

Pisistratus, Theagenes, and others asked for a bodyguard, whereupon they established themselves as tyrants; now Dionysius, in asking for a bodyguard, aims to set up a tyranny.

28. Identify figures and moods:

No balloons are safe, and all dirigibles are balloons; then no dirigibles are safe.

Some afflictions, by prompting us to reflection, are beneficial.

Some forms of sex behavior, being responses occurring suddenly at maturity, are not due entirely to environmental influences.

The good attracts desire and is recognized by the intellect; hence some of the things recognized by the intellect attract desire.

No virtue is incompatible with love of truth, as love of peace sometimes is; therefore sometimes love of peace is incompatible with love of truth.

No cripples are strong; some strong men detest work; therefore some who detest work are not cripples.

Some criminals are not sane; for they are not free from delusions, as are all sane men.

29. Put into standard logical form: Only those who try to find out how things really are can advance science; hence no fanatics can advance science. No child is patient; impatient persons cannot sit still.

30. Indicate all the rules which render invalid: EEE2; EAE3; EIA1; IAI2; OIO2; OOO4; IIE2; AEE2; EII3; IEE3; AOO1. (C. W. Churchman.)

31. What quality and what quantity must the conclusion have in a categorical syllogism of the first figure?

32. Give the distribution of M in I; P, in II; S in III.

33. In what figure are both features present that S is subject in the conclusion and in the premises and that P is predicate in the conclusion and in the premises?

34. How does the arrangement of terms in the fourth figure of the categorical syllogism compare with that in the first?

35. Which syllogistic figure contains three terms, each once subject, once predicate? Which terms occupy this double position in the other figures?

36. Which moods are both valid in the first figure and invalid in the second, and vice versa?

37. Why is a mood valid in both the second and third figures also valid in both the first and fourth?

38. Why is IEO invalid, but EIO valid, in every figure?

39. Which propositions may be A, E, I, or O in the first figure; in the second; in the third?

40. Why cannot O be a premise in the first figure; the major in the second; the minor in the third; a premise in the fourth?

41. How does AAI4 differ from AAI1?

42. Which weakened syllogisms may also be considered strengthened?

43. In which valid syllogisms do the major premise and the conclusion agree in quantity but differ in quality?

44. If, in the second figure and in negative syllogisms of the third figure, a premise is combined with its contradic-

tory (no medicine is science; some medicine is science), what sort of conclusion will result?

45. Analyze:

No pessimist is contented; some contented persons are poor; therefore some poor persons are not pessimists.

Whatever is not metallic is not capable of powerful magnetic influence; carbon is not metallic and is therefore not capable of powerful magnetic influence.

Planets are subject to gravity; but fixed stars, not being planets, are not subject to gravity.

Many nations are capable of self-government; some nations should not be ruled by despots; therefore no nations should be ruled by despots.

Liberals hold opinions such as yours; you are therefore a liberal.

Departures from law are reprehensible; whatever happens by chance is therefore, as a departure from law, reprehensible.

46. Apply the *dictum de omni et nullo* to man who, as a physical mass, behaves in accord with gravitation. Note further illustrations from our knowledge of man. (E. L. Thorndike *Human Nature and the Social Order*, p. 5.)

47. Show how the *dictum de omni et nullo* expresses both the structure and the general rules of the categorical syllogism.

48. Reduce to the first figure by a direct method:

Educational institutions do not foster partisan propaganda; some schools foster partisan propaganda; therefore some schools are not educational institutions.

All intellectuals are feared; they are also admired; therefore some who are admired are feared.

No wildcats are pets; all wildcats are ferocious; therefore some ferocious animals are not pets .

Sages advocate renunciation; worldlings do not advocate renunciation; therefore worldlings are not sages.

All eels migrate to the mid-Atlantic to spawn; no salmon do this; therefore no salmon are eels.

No movie stars are without fame; some without fame are worthy of honor; therefore some worthy of honor are not movie stars.

49. In what respect would "s" or "p" as final letter in the name of a syllogism indicate manipulation of the new rather than of the given conclusion?

50. To what types of argument does an accused person have recourse when, in order to defend his innocence, he establishes an alibi?

51. Apply the indirect method of reduction:

The virtuous are prudent; but some zealous persons are not prudent and therefore not virtuous.

Some steel is not magnetic; all steels are metals; therefore some metals are not magnetic.

Some believers are not unintelligent; all believers yield to emotional suasion; therefore some who yield to emotional suasion are not unintelligent.

52. A trio of syllogisms may be obtained by writing Barbara (in symbolic form); by replacing the minor premise of Barbara with the contradictory of the conclusion (Baroco); and by replacing the major premise of Barbara with the contradictory of the conclusion (Bocardo). Deal similarly with AAI1, Celarent, EAO1, Darii, Ferio, Bramantip, Camenes; and arrange the original syllogisms with the resulting syllogisms in sets of trios.

53. Obvert the major premise in Barbara and the new conclusion thus obtained. Interpret the result.

54. Reduce Celarent to Barbara; Darii to Ferio; Ferio to Darii.

55. Trace the processes employed in the following reasoning, and interpret the result:

MaP	MaP	SaM
SaM	SaM	P'iS
∴ SaP	∴ SoP	∴ P'iM
	∴ P'iS	∴ MoP

56. Trace the processes employed in the following reasoning, and interpret the result:

MaP	MaP	PaS'
SiM	SiM	MaP
∴ SiP	∴ SeP	∴ MaS'
	∴ PaS'	∴ SeM

57. Analyze the form of the arguments: You say that all politicians are grafters; but Theodore Roosevelt was a politician, and he was not a grafter. It cannot be the case that all possible objects of thought are sense-given, that substance is a possible object of thought, and that substance is not sense-given. He claims to be a patriot, yet he collaborated with the enemy. Socrates was an atheist, for he introduced false divinities.

58. Take the antilogism: (*p*) MaP, (*q*) SaP, (*r*) SoP. Construct three syllogisms with a new conclusion contradicting, *r*, *q*, *p*, respectively.

59. Deal similarly with the antilogism: (*p*) PaM, (*q*) MeS, (*r*) SiP.

60. Take an antilogism *p*, *q*, *r*. If a term is undistributed in *p*, what is its distribution in the conclusion of the syllogism: if *p* and *q* then *r*'? What then is its distribution in *r* in the syllogism: if *p* and *r* then *q*'?

61. Take the premises PeM and SeM. What is the distribution of the middle term in the obverses of these premises? What follows concerning the original premises?

62. Take an antilogism p, q, r . if the premise p is negative in the syllogism: if p and r then q' , what is the quality of the premise r ? What then is the quality of the conclusion in the syllogism: if p and q then r' ?

63. Examine, in each of the four figures of the categorical syllogism, the consequences of combining affirmative premises with a negative conclusion. Examine EAO4 in the light of the possibility that three terms may coincide in extension.

64. Analyze in the light of the *dictum de omni nullo*: Lions are treacherous, for they are carnivorous. Virtuous actions, being praiseworthy, are therefore not involuntary. Faith, being impure, cannot purify the heart. "All things excellent are as difficult as they are rare." As other than aimless changes, writing, building, healing, etc., are processes in which mistakes occur; so mistakes occur also in natural processes (as, e.g., in the case of monstrosities).

65. Analyze:

The solution of this problem must satisfy the conditions specified; since this formula satisfies the conditions specified, it is the solution of the problem.

Potassium floats on water; potassium is a metal.

X is not a thinker; he is too positive in his opinions.

Since poetry gives pleasure, that with which everyone is pleased cannot be unpoetical.

Like play, so art is creative; hence art is play.

The whale suckles its young, which no fish does.

Like everything which comes into being, so mountains undergo change and pass out of being.

Ecclesiastical property is exempt from taxation; this property is not ecclesiastical property and therefore not exempt from taxation.

Lions are quadrupeds and carnivorous; therefore all quadrupeds are carnivorous.

Who but radical pessimists would despair of the only world of whose existence we have assurance? But the masses of the orthodox conceive the universe as thoroughly corrupt by nature.

A mischievous child receives attention, which he accepts as semi-envious recognition.

One who is intelligent pursues ends and chooses means for their attainment, which sticks and stones apparently do not do.

To cultivate one's reason is a form of self-love; therefore some forms of self-love are not irrational.

This dogma is beyond reason; therefore I believe it.

Faces interest me; I remember them easily; so we remember what interests us.

X knows more of Y than Y of X; superiors usually provoke an impulse to excel.

Sirius is a kind of thing in whose spectrum the hydrogen line is shifted, which is a kind of thing that moves in the line of sight.

CHAPTER XI. CONDITIONAL SYLLOGISMS

1. Analyze:

If the ignition is not on, the car won't start; but the car is starting; therefore the ignition is on.

If one pupil does well, all have been well taught; Bill has done well; therefore all have been well taught.

If we had good housing conditions, the poor would be happy; but housing conditions are bad; therefore the poor cannot be happy.

If two languages have extensive grammatical similarities, they are related; but French has discarded more Latin

verb flexions than has Italian; these two languages are therefore unrelated.

If money grew on trees, this bargain would not interest you; but money does not grow on trees; therefore this bargain will interest you.

If all the information about a candidate is favorable, he will be accepted; some of the information is not favorable; therefore he will not be accepted.

If the testator was insane, his will is invalid; the latter is the case; therefore the testator was insane.

A was seen running away from the fatal spot five minutes after the murdered victim expired; now if A had committed the murder he would have been near the spot and made his escape as soon as possible. Therefore A did commit the murder.

2. Test the validity of several hypothetical syllogisms by translation into the form of categorical syllogisms.

3. Complete and analyze:

This key is made of iron; for, when placed near a magnet, it is attracted.

If a child has no capacity, he cannot start to learn; but he does start to learn.

If a geometric construction, say the trisection of an angle, is expressible as an irreducible algebraic equation of degree greater than two, it cannot be constructed by compass and straight edge alone.

When people were ignorant, restrictions on sex relations were appropriate; but, of course, people are no longer ignorant.

Without some knowledge of truth, the will cannot be rightly directed.

If matter is atomic in structure, chemical substances should combine in definite proportions; and this they do.

4. Out of nothing nothing comes. What follows since something comes?

5. To square the circle, one must accurately determine π and by purely geometrical methods describe a straight line equal in length to the circumference; both of which are impossible. State in the form of a hypothetical syllogism. (A. N. Whitehead *An Introduction to Mathematics*, p. 187.)

6. Two angles of the triangle ABC are known; hence the third can be deduced. State in the form of a hypothetical syllogism. (*Ibid.*, p. 177.)

7. When all the angles of a triangle are known, so is its shape. May we argue vice versa? (*Ibid.*)

8. Given: if p then q . Compare the value of the inference, p therefore q , with the value of the inference, q false therefore p false. (C. S. Peirce *Collected Papers* IV. 49.)

9. What major premise is required if we are to argue validly from the truth of the consequent to the truth of the antecedent and from the falsity of the antecedent to the falsity of the consequent?

10. Analyze: When he blushes, man's cheeks are red; when ashamed of himself, man blushes; therefore, when a man is ashamed of himself, his cheeks are red. If today is a holiday, the bank is closed; if the blinds are up, the bank is not closed; therefore, if the blinds are up, this is not a holiday.

11. Complete and analyze: If a gas is heated, its temperature will rise; if its temperature rises, its elastic force will increase.

12. Construct a truth table with columns for: p ; q ; p implies q ; (but) p true; (therefore) q true. Construct a truth table with columns for: p ; q ; p implies q ; (but) p

false; (therefore) q false. Which of these inferences is valid?

13. Construct a truth table with columns for: p ; q ; p implies q ; (but) q false; (therefore) p false. Construct a truth table with columns for: p ; q ; p implies q ; (but) q true; (therefore) p true. Which of these inferences is valid?

14. Construct a truth table with columns for: p ; q ; r ; (if) p implies q ; (and if) q implies r ; (then) p implies r .

15. Analyze:

X agrees to work for Y or for someone else; he has agreed to work for Y , therefore not for someone else.

A owns a pointer or a setter; he owns a pointer; therefore he does not own a setter.

Either Newton or Leibniz invented the calculus; but Newton did; therefore Leibniz did not.

Either all our deliberate acts are fated or free; but a recent act of mine was not fated; therefore all our deliberate acts are free.

Hamlet was too timid or unbalanced to avenge his father; but he manifested timidity in several ways and was therefore not unbalanced.

16. Complete and analyze:

John Doe is either a minor or he is of age.

This business is either a partnership or a wholesale concern.

Devotion to civil liberties may be the result of geographical and economic conditions, or of democratic institutions, or of education in toleration; but it is now threatened by geographical and economic conditions.

Aviation sickness is due to being in the air, to the motion of the plane, to cold temperature, to decrease of atmospheric pressure, to the amount of available oxygen, or to the individual's condition.

17. Assuming there is no mail for you, and assuming the postman has not yet come, could you conclude there is or there is not any mail? What could you conclude if the postman has come?

18. A tooth is either an incisor, canine, bicuspid, or molar. What follows about a not-incisor tooth?

19. From what major premise may we conclude that, because A is second, A is not first?

20. What kinds of compound propositions are confused in the customary fallacies of mixed syllogisms?

21. Construct a truth table with columns for: p ; q ; p or q ; (but) p false; (therefore) q true. Construct a truth table with columns for: p ; q ; p or q ; (but) q false; (therefore) p true. Which of these inferences is valid?

22. Formulate equivalent syllogisms: If ignorance is bliss, 'tis folly to be wise; 'tis not folly to be wise; hence ignorance is not bliss. A man cannot at the same time be a profound thinker and not have questioned his first principles; this man is a profound thinker; therefore he has questioned his first principles. He is either a fool or a knave; he is not a knave; therefore he is a fool. Capri lies in latitude 40° or 35° ; it lies in latitude 40° ; hence it does not lie in latitude 35° .

23. Analyze the dilemmas:

If I play classical music, I shall starve; if I play popular music, I shall despise myself. But I shall play either classical or popular music. Therefore I shall either starve or despise myself.

If the weather report is right, it will rain; if X is right, the game will be played. But either it will not rain, or the game will not be played. Therefore either the weather report or X is not right.

If we surrender, they will hang us; and if we do not surrender, they will capture us and hang us. But either we do or do not surrender. Therefore, in either case, they will hang us.

If p then either q or r ; but neither q nor r ; therefore p false.

24. Analyze: If he has an inventive mind he'll be a good engineer; but if he likes poetry, he won't. But either he will or he won't be a good engineer. Hence he is either inventive or poetical.

25. Escape between the horns: If judges decide according to the rules, the law is arbitrary; but if they decide according to their discretion, the law is uncertain.

26. Take the dilemma by the horns: If taxable property be intangible, it is taxable at the domicile of the owner; but if tangible, at the situs.

27. Euathlus agreed to pay Protagoras, for instruction received, after Euathlus had won his first case. Protagoras sued Euathlus for the fee, arguing: If he wins, then by the terms of the contract he must pay me; if he loses, then by the judgment of the court he must pay me. Euathlus, however, argued: If I win, then by the judgment of the court I need not pay; but if I lose, then by the terms of the contract I need not pay. Discuss.

28. Construct a dilemma with the major premise: If fighting in armor is knowledge, it ought to be learned; if not, those who profess to teach it are deceivers. (Plato *Laches* 182E.) How would you deal with the dilemma?

29. If I swim across the river, I shall get wet; if I do not cross the river, I shall have to stay on the wrong side. Construct a dilemma on the basis of this premise.

30. In the equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$: if $ab - h^2$ is a positive number, the curve is an

ellipse; if $ab - h^2 = 0$, the curve is a parabola; if $ab - h^2$ is a negative number, the curve is a hyperbola. Does it follow that, since conics are either ellipses, parabolas, or hyperbolas, $ab - h^2$ is a positive or negative number or equal to zero? Give reasons for your answer. (Whitehead *op. cit.*, p. 142.)

31. Discuss: Despite his advanced age, he goes to his office daily; apparently, he derives much satisfaction from reminiscing.

32. Analyze: The object of sadness is a supposed evil; envy is a kind of sadness; hence the object of envy is a supposed evil; but the object of envy is another's good; hence another's good is sometimes a supposed evil.

33. Peter, James, John, etc., were Jews and were apostles of Jesus. What condition must be fulfilled if we are to be able to conclude that all the apostles of Jesus were Jews?

34. Peace with a powerful neighbor is desirable; a case in point would be peace between Poland and Russia; it is therefore desirable, like the peace between Canada and the United States. How does this argument differ in form from the preceding argument?

35. Analyze: The purpose of life is happiness, which has as its essential condition prudence, or doing one's duty—an action which is eminently reasonable; hence the purpose of life is to do what is eminently reasonable.

36. The benevolent are carefree; the wise, benevolent; the disillusioned, wise; the brave, disillusioned; the fearless, brave; therefore the fearless are carefree. Give the logical form of this sorites.

37. Montaigne's fox reasoned: "This stream is noisy; that which is noisy moves; that which moves is not frozen; that which is not frozen is liquid; that which is liquid gives

way under weight; therefore this river will not bear me." Give the logical form of this sorites.

38. Compare a sorites with the telescoping of connections between stimuli and responses. (E. L. Thorndike *Human Nature and the Social Order* [New York: The Macmillan Co. 1940 (Copyright 1940 by The Macmillan Company used by permission of The Macmillan Company, publishers)], p. 18.)

39. Legislation is not *res judicata*; to make a rule for the future is legislation; for an administrative board to fix a rate is to make a rule for the future; therefore the fixing of a rate is not *res judicata*. Analyze.

40. If a duty is levied on an article, the producer will have increased expense; so that he will have to raise his price. In consequence, he will sell less and therefore produce less; but only if he gets a less price. So consumers will have to pay more; namely, the less price plus the duty, including increased expenses of all kinds. This consumers will do only in case of a large demand such that a small decrease in the demand will cause a relatively large fall in price. Other things being equal, it is upon such articles therefore that duties are best levied. Analyze. (Peirce *op. cit.* IV.87.)

41. "The doctrine of reprobation, in the judgment of many very able men, follows by syllogistic necessity from the doctrine of election. Others conceive that the Antinomian heresy directly follows from the doctrine of reprobation; and it is very generally thought that licentiousness and cruelty of the worst description are likely to be the fruits, as they have often been the fruits, of Antinomian opinions." Discuss. (T. B. Macaulay "On Hallam's Constitutional History" *Essays* i.443, quoted in J. F. Genung *The Working Principles of Rhetoric*, p. 622.)

42. "Who rules east Europe commands the Heartland; who rules the Heartland commands the World-Island; who rules the World-Island commands the World." Analyze and classify this argument. (Mackinder.)

43. Given a monogamous system, is enumeration necessary to determine that the number of husbands and wives is equal?

44. Easternmost Greenland is farther east than western Africa; western Africa is farther east than South America. Complete and classify this argument.

45. "Men differ more in stature than in body temperature, more in weight than in stature, more in intelligence than in weight, more in musical ability than in intelligence." Complete and classify this argument. (Thorn-dike *loc. cit.*)

CHAPTER. XII. FORMAL RELATIONS

1. Define: relation, referent, relatum; concrete relative terms, abstract-universal relational terms.

2. The behavior of two persons depends not only on the nature of the one and of the other, but also on the relation between them. Illustrate.

3. How are horses, men, and bullets related with respect to speed of traveling?

4. Underline words denoting relations: This view is more beautiful than the other. Those boys are normal children. The diagonal of a square is incommensurable with its side. The children watched the monkeys.

5. State the number of terms connected by the relations: between; playing chess; playing bridge; give; a pair of equals; a legal debt; $a : b :: c : d$; separated by.

6. Determine various meanings of the verb "to be": Whatever is, is right. Snow is white. Whales are mammals.

Two and two are four. A square is an equilateral rectangle. *Tout comprendre est tout pardonner*. Rome is the Eternal City. There is no Santa Claus. This car is a Pullman. Who was the culprit? That *was* an experience.

7. Classify as transitive, intransitive, or nontransitive:

ancestor of	friend of
greater than	hates
more probable than	acquaintance of
above	voted for
father of	mutually imply
twin of	differs from by one
married to	other than
contradictory	not coextensive

8. Judge valid or invalid: A seems less than B; B than C; therefore A than C. Hen A pecks hen B; B pecks C; therefore A pecks C. Team A beats team B; B beats C; C beats D; therefore A will beat D. Mary and Jane resemble Dorothy and therefore each other.

9. If A is to the right of B, B of C, and C of D, can they be so arranged that D is not to the left of A?

10. Classify as symmetrical, asymmetrical, and non-symmetrical:

hits	in tune with
greet	three miles from
brother of	married to
friend of	different from
SaP	SiP
master of	not implied by
represents	comprise
author of	alternant of
superaltern to	incompatible with
SeP	SoP

11. Does the relation of equality possess its transitivity because of its symmetry?

12. Characterize as to transitivity and symmetry:

contemporary	uncle of
match in color	characterize
better than	closest neighbor of
subalternant	coextensive
not less than	fastened to
sister of	protector of
mutually opposed	not implied by
wedded to	coincident

13. Classify as one-one, one-many, many-one, many-many relations: eldest son of a father; the centre of a circle; sovereign of; square of; wife of the Sultan; 1° of latitude north of; the set of positive numbers and the set of negative numbers.

14. Classify as connected or not: less than; factor of.

15. Relations like identity, for which propositions of the form aRa are always true, are called reflexive. Classify as reflexive, irreflexive, or nonreflexive:

as long as	longer than	proud of
not greater than	coextensive	hate yourself
not disjunct	unequal	work for

16. What laws for the relation of identity are stated in the expressions: $x = x$; if $x = y$ then $y = x$; if $x = y$, and $y = z$, then $x = z$?

17. Characterize as to reflexiveness, symmetry, and transitivity: congruence of line segments; similarity among planes; simultaneity among physical events.

18. Every series (natural numbers, days, months, years, points on a line, progeny in a line of descent, etc.) requires a serial relation. This requirement is fulfilled by

any relation that is irreflexive, transitive, connected, and (by implication) asymmetrical (e.g., greater than, follows, to the left of, descendant of, etc.). Relations such as "taller than," which are irreflexive, asymmetrical, and transitive, are semiserial relations. Such relations as "equal" or "equivalent to," which are reflexive, symmetrical, and transitive, are called isoid or equivalence relations. Give further illustrations.

19. Define in a general way the relation of precedence among words in lexicographical order.

20. What is meant by $n!$ as the number of permutations; by 3.14159 as a successive summation; by .1111. . . . as a sum to infinity; by 1, 1, 1, etc., as a divergent series; by uniform and nonuniform convergence of a series; by the exponential function? (A. N. Whitehead *An Introduction to Mathematics*, chap. xiv.)

21. Characterize the conjunctions and translate into alternative or disjunctive form:

The officers of the President's cabinet are

Algae and Fungi are Thallophyta.

Algae are relatively simple plants which possess chlorophyll and can manufacture carbohydrates; fungi lack chlorophyll and obtain carbohydrates and other foods from external sources.

Thallophyta, Bryophyta, Pteridophyta, and Spermatophyta are plants.

To be a seed plant is to possess vascular tissues, form pollen tubes, and produce seeds.

Living protoplasm as found in cells possesses irritability, conductivity, metabolism, and reproduction.

Verbs are transitive, intransitive, linking, and auxiliary.

Pittsburgh's rivers are the Allegheny, the Monongahela, and the Ohio.

22. Achievement = Ability \times Motivation. Explain.

23. Is human behavior an addition of the behaviors of cells?

24. Is organic behavior a succession and compounding of independent discrete reflex arc units?

25. Is the relationship of heredity and environment in "development" to be compared with addition or with multiplication?

26. How is one special activity related to another in the endeavor to carry out a plan?

27. How are the problems of drawing tangents to curves, and of determining the rate of increase of a function, related in the differential calculus? (Whitehead *op. cit.*, chap. xv.)

28. Identify the formal relations of propositions: in the enumeration of pathfinders in the history of the differential calculus; in the listing of types of functions considered in the science; in the definitions of a limit, of the rate of increase of the function x^2 minus any value x of its argument, and of the differential coefficient of a function. (*Ibid.*)

29. Show why the notion of the variable, or of any and of some, is important in the differential calculus and in mathematical analysis generally. (*Ibid.*)

30. Does the appeal from Phillip drunk to Phillip sober go counter to the law of identity?

31. May different parts of the same thing be beautiful and ugly?

32. Is the formula: " $4 = 2 \times 2$ " contradicted by the formula: " $4 \neq 2 \times 2$ "?

33. "On Sundays the 10 A.M. train will start at 9:30." In what sense would it be the "same" train?

34. Is it the same problem when we ask whether an individual remains the same and whether two things are the same?

35. In what ways is the law of identity often violated in partisan propaganda and the like?

36. What does one who denies the truth of the canon of identity assume about his denial? Comment.

37. The word "short," being itself short, is autological; the word "long," not being itself long, is heterological. Is the word "heterological" heterological? How would you, in the light of the canons of reasoning, deal with paradoxes generally?

38. Punchinello said: "Let us share as brothers; I'll take everything, you take the rest." What law of thought does this violate?

39. "I can't say breakfast!" What law of thought does this violate?

40. Is there anything which can both be and not be?

41. Interpret the significance of the definition of a liar as one who tells the truth about something that never happened.

42. To the consumer: The law of supply and demand is inexorable. To the government: Increase the tariff! Comment.

43. Does the counter recording the ayes and noes and those not voting at an election violate the law of excluded middle?

44. Is a in algebra neither $+a$ nor $-a$?

45. By enabling higher wages to be paid in particular industries, tariffs maintain the high American standard of living. Discuss in the light of the law of excluded middle.

46. Substitute p for q : p implies q ; not both p true and q false; either p false or q true.

47. Substitute S for P: SaP ; $P'aS'$; SeP' ; everything is either P or S'.

48. If "this" and "that" stand for different things, and P and Q stand for different qualities, in what ways are identity and difference mutually involved in the propositions: This is P; this is Q; that is P; that is Q?

49. If the apostles numbered as many as the months of the year, does it follow that one who denies that the apostles numbered twelve necessarily denies that the number of the months is twelve?

50. If faith works hope, which is opposed to fear, can faith work fear?

51. Show that the rules of the categorical syllogism apply the laws of contradiction and of the distribution of terms.

52. What ambiguity is contained in the alleged contradiction that 111,777 is "the least integer not nameable in fewer than nineteen syllables" (which is itself a name consisting of eighteen syllables)?

53. Either what I am saying is false, or I'll eat my hat. Is this statement self-applicable?

54. Can there be "important nonsense"?

55. What is meant by "axioms of quantity"? (Whitehead *op. cit.*, chap. xvii.)

56. Discuss: All dragons breathe flame. All brakeless trains are dangerous. An integer between 3 and 4 is prime; an integer between 3 and 4 is composite. William Tell submitted to the Austrians; William Tell defied the Austrians.

57. Contradict the conjunctive proposition: There are just men, and all of them are happy.

58. If S is nothing at all, what would it mean to say that "S is not"? What about the remark: "I don't believe in ghosts; but I hope they never find it out"?

59. Would a notice to the effect that "trespassers will be prosecuted" imply that "some prosecuted person is a trespasser"?

60. How would the interpretation of particular propositions as existential and of universal propositions as non-existential affect the interpretation of: relations of truth-value among propositions; the square of opposition; obversion, conversion, contraposition, and inversion; valid categorical syllogisms?

61. Geometry has been called "the science of dimensional order." Explain. (Whitehead *op. cit.*, chap. xvi.)

62. Would you prefer a clock that does not go at all or one which loses a minute every day? How often is the former right; how often the latter? (Lewis Carroll.)

63. Give examples of chemical and other structures.

64. Illustrate conditions of logical ordering from the formal relations of terms; from the formal relations of propositions; from the canons of reasoning.

CHAPTER XIII. INQUIRY

1. Trace the way in which you solved a practical problem in your recent experience. What was the situation? How did you state your problem? What suggestions towards a solution did you entertain? How did you reason them out? What was the outcome of your inquiry?

2. Analyze similarly one of your recent attempts to solve a theoretical problem.

3. Delineate the salient phases of reflective inquiry in the story of Archimedes testing Hiero's crown. (O. Blackwood *Introductory College Physics*, p. 135.)

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4. Sketch the main stages of reflective inquiry in a physician's treatment of disease or in some other vocational activity.

5. Compare and contrast modern applications of scientific method with the procedure of the primitive medicine man.

6. Compare the democratic or constitutional process with the procedure of reflective or scientific inquiry. (Charles Beard *The Republic*, p. 327.)

7. Drawing upon your own experience, give a brief account of daydreaming; of the contemplation of beauty; of the acquisition of information; of unreflective and reflective response to a situation; of attention to themes which promise to elicit lifelong interest.

8. Why is it important, in trials at law, that a confession be voluntary? Give additional examples to which the distinction between the voluntary and the involuntary is relevant. Consider, e.g., the laws of error, duress, and fraud in contract.

9. Recount some failure or misunderstanding due to misrepresentation or inadequate sizing up of a situation.

10. State with care how you may explore a definite problematic situation, noting in particular the role of observation and description, explanation and clarification, classification and contextual interpretation.

11. Read: John Dewey *How We Think* [1933 ed.], chap. i. What do you learn about reflective thought as surveyed in terms of its occasion?

12. How does a familiar task, like adding figures, differ from a difficult problem?

13. Why is it a first rule of cartography that a projection must be appropriate to its use?

14. Illustrate the importance of framing significant questions.

15. Estimate the questions: What is the biggest number? Is x an integer? How far is the moon from the earth? How can the United States acquire more credit abroad? What is nature for? Why did this happen to me?

16. What different types of problems do different kinds of scientists see in the atmosphere?

17. How could Marie Antoinette, told the people were clamoring because they could not get bread, ask why they did not eat cake?

18. List specific questions frequently arising in some vocation.

19. Show how specific questions may suggest general ones.

20. What would be the effect of waiting to learn until a problem arises in which a given item of knowledge is needed?

21. What are some of the values of reflective thought and some of the hindrances and aids to its effective development? (Dewey *op. cit.*, chap. ii.)

22. Does the nature of a problem remain fixed in every inquiry?

23. Discuss a problematic situation as a source of clues for suggested solutions.

24. How do facts and principles serve as evidence and explanations?

25. Can you name typical sources of varied suggestions concerning problems of special interest to you?

26. How does explicit formulation of hypotheses bear upon ascertainment of relations between them?

27. Do happy suggestions come to a thinker by pure inspiration?

HOW MAN THINKS

28. Why do only a few, like Charles Darwin, take special pains to note facts, observations, or thoughts opposed to their own ideas?

29. In what sense are ideas plans of action?

30. Point out the barrenness of the notion that actions due to desire are involuntary.

31. Point out the lack of predictive significance or of a specifiable difference in the assertion that pleasure and nobility coerce us.

32. Point out the want, or excess, of simplicity in the doctrine that all virtue is knowledge and that all vice is ignorance.

33. Can thought be trained? Indicate some native resources in the development of habits of reflection. (Dewey *op. cit.*, chaps. iii, iv.)

34. Why does an inquirer apply hypotheses to fresh instances?

35. Discuss the logical significance of detailed acts in large action patterns: turning a knob, opening a door, going to the garage, taking friends to the station, etc.

36. Examine critically the hypotheses: The universe is shrinking in such fashion that all lengths are contracting in the same ratio. If a woman is intelligent, she is in that respect masculine. If I cannot prove the appearance of disembodied spirits, neither can you disprove it. All human behavior may be explained by the impulse to self-preservation. A German invasion of the United States was prevented by thought waves from leaders of the "I am" sect located in California.

37. What different possibilities did extreme advocates and opponents see in prohibition, and why? (I. Edman *Human Traits and Their Social Significance*, p. 57.)

38. Exhibit the relation of reasoning to every other phase of inquiry.

39. When is the judgment that a given piece of cloth may fade, a reasoned judgment? What is the advantage of conceiving heat as motion; of noting that a stove in a railroad car will stop smoking when the car begins to move so as to allow of rapid draught of air over the stovepipe's mouth? (William James *The Principles of Psychology* II.340-43.)

40. How are the psychological and the formal aspects of thought related? (Dewey *op. cit.*, chap. v.)

41. Do facts speak for themselves?

42. State some conditions of good observation. (E. A. Burt *Principles and Problems of Right Thinking*, [1931 rev. ed.], chap. v; H. A. Larrabee *Reliable Knowledge*, pp. 156-64.)

43. "The ability to think depends upon the power of seeing connections." (L. S. Stebbing *A Modern Introduction to Logic* [1933 ed.], p. 4.) Explain.

44. Suppose men had never eaten strawberries, in the belief that eating them would cause pain. Would it be adequate to say that the system of the strawberry abstainers fitted all the facts known to them? (F. P. Ramsey *The Foundations of Mathematics*, p. 253.)

45. In what sense are facts tested by hypotheses?

46. Comment on the refusal of one of Galileo's contemporaries to look through Galileo's telescope.

47. If laboratory experimentation destroyed confidence in observation as source of information, what effect would such experimentation have upon itself?

48. Why does laboratory training include use of a laboratory manual and like apparatus?

49. Why is a definition or theory of truth controversial? Consider the many sorts of ideas to which a test of truth would have to be applied.

50. How are inferences tested, and with what import for the chief function of reflective thought? (Dewey *op. cit.*, chap. vi.)

51. How is reflection related to learning; to impulse and habit; to bodily action; to imagination; to social conduct and traits; to individuality and selfhood; to language; to cultural continuity; to the formulation of ideals? (Edman *op. cit.*)

52. Analyze, in a typical instance, the method pursued by Thomas Aquinas in his *Summa Theologica*.

53. Separate 124 into two parts such that when the larger is divided by the smaller, the quotient is 4, and the remainder is 9. Read the problem over and over until it is clearly understood. Describe what a letter selected for an unknown quantity represents: x , the larger part; y , the smaller part. Write expressions for all quantities involving the unknown: $x + y = 124$; $x = 4y + 9$. Set equal to each other two expressions representing the same number: $x + y = 124$; $(4y + 9) + y = 124$. Solve the resulting equation, and check each solution to see if it satisfies the conditions of the problem.

54. If a grocer has two kinds of coffee worth 30 and 40 cents per pound, respectively, how many pounds of each must be taken to make a mixture of 70 pounds, worth 36 cents per pound? Give an account of the steps in the solution of this problem.

55. In what ways do you, in your inquiries in the present, utilize inquiries you have made in the past? Why ought we, in our inquiries, to have regard to the need of future

inquiries? How would insight into principles of inquiry further the formation of sound habits of inquiry?

CHAPTER XIV. SCIENTIFIC PRINCIPLES

1. In what sense do we "explain" the block and tackle?
2. Can we reasonably rely upon accidental discovery of order?

3. Discuss the role of order in scientific inquiry; e.g., if the movements of the planets are due to the sort of space they move in, a ray of light passing near the sun and moving through this distorted space should deviate from the straight line by a calculable amount, as verified by photographs of stars whose light passed near the sun when the sun was eclipsed. (J. A. Thomson *Outline of Science*, pp. 1041 f.)

4. Why have investigators stressed the maxim that "nature does nothing in vain"?

5. In what respects does the circulation of the blood resemble the water supply system of a city? (J. F. Williams *A Textbook of Anatomy and Physiology* [6th ed.], p. 280.)

6. Why do we speak of the circulatory "system"? Of what other "systems" do we speak in anatomy and also in physiology?

7. What organizing principles of science are involved in the statements: "the chromosomes are the essential mechanism of inheritance"; "chromosomal and cytoplasmic substances endow the organism with certain possibilities of development"?

8. Do the readjustments an airplane pilot has to make disprove "natural law" or "mechanism"? (*Harper's Magazine* [April, 1941].)

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9. Illustrate the relation between mechanism and purpose from the actions of a baseball player.

10. Discuss the significance of the statement that "mechanism is the means by which the ends of nature are reached." (F. J. E. Woodbridge *Nature and Mind*, p. 104.)

11. Compare and contrast the scientific, speculative, artistic, religious, and practical moods. (J. A. Thomson *Introduction to Science*.)

12. Summarize the functions of reflective thought in the solution of a problem. (John Dewey *How We Think* [1933 ed.], chap. vii.)

13. Describe the nature of intuitive induction: Red traffic lights differ, in all instances alike, from the yellow and the green. In Beethoven's *C Minor Symphony*, the four opening notes form a single figure traceable, by the composer's rich use of it, throughout the composition. Perceiving that if some Mongols are Europeans then some Europeans are Mongols, and that if some Russians are Asiatics then some Asiatics are Russians, leads to the recognition that if some S is P then some P is S. As we note that twice 2 inches plus twice 3 inches equal twice the sum of 2 and 3 inches, we realize not only that thrice 3 days plus thrice 4 days equal thrice 7 days, but also that, in general, $ab + ac = a(b + c)$.

14. Neglecting air friction, let us give the kinetic energy of a baseball thrown vertically upward as 200, 150, 100, 50, 0, and the potential energy as 0, 50, 100, 150, 200. Of what law is this a case, and how is the law recognized? (O. Blackwood *Introductory College Physics*, p. 76.)

15. A traveler in the Arctic, pleased with the flavor of a fish frozen rapidly in weather below zero, introduced quick freezing methods of refrigeration. (*Ibid.*, p. 109.) What intellectual activity does this illustrate?

16. Give some historical examples of apparently sudden insights.

17. What is the pedagogical significance of intuitive induction?

18. What points seem evident in the names of compounds containing two elements: sodium chloride, potassium chloride, ferrous chloride, ferric chloride, mercurous oxide, mercuric oxide, carbon monoxide, carbon dioxide?

19. Describe the logic of complete or perfect induction: Mercury, Venus, etc., move round the sun; Mercury, Venus, etc., are the known planets; the known planets move round the sun. The parabola, the ellipse, and the hyperbola meet a straight line in less than three points; parabolas, ellipses, and hyperbolas are the conic sections; hence every conic section meets a straight line in less than three points. Washington, Adams, etc., are the Presidents of the United States; hence all the Presidents of the United States to date have been Protestants.

20. Is it possible to make a complete enumeration of our rights; of eternal verities; of current trends?

21. Cite instances in which perfect enumeration is significant and important.

22. With what warrant did Harvey apply his discovery to all living organisms?

23. List the numbers by which 60 is divisible. If we argued that 60 is therefore divisible by all its predecessors and itself, to what feature of scientific method would this false generalization point?

24. What is lacking when, from patent medicine testimonials, inferences are made to a reader's own case?

25. Is size paramount in the sampling of mass opinion?

26. Distinguish primary, statistical, and functional generalizations: Eighteen per cent of deaths in New York

City result from automobile accidents. All who have a superiority complex suffer from an inferiority complex. Most dances are presented with music. The need for social control varies with population density. Benzoate of soda is deleterious.

27. Show the interrelation of induction and deduction: Galileo, having formed the hypothesis that a body starting from rest falls with a uniform acceleration and that its velocity varies with the time of its fall, concluded that a body falling according to this law would fall through a distance proportionate to the time of its fall. How is Newton's law of celestial gravitation related to Kepler's laws? From a series of comprehensive studies, Herbert Spencer inferred that predominantly industrial states are democratic, whereas predominantly military states are undemocratic; he argued that the trading relations, characteristic of the former, train men to good humor and consideration of others. What is the "case history" method?

28. Complete and classify the arguments: This specimen of argon has atomic weight 39.9. A star visible at night becomes invisible by day; a voice becomes inaudible in the noise of a machine shop. The thirteen colonies, Massachusetts, Virginia, etc., had a property qualification for voting. Against the contention that the states can tax a national bank, Marshall declared: "If the States may tax one instrument employed by the government in the execution of its powers, they may tax any and every other instrument." Plants and beasts and men breathe. Every L is P, every M is P, every N is P, and every S is L or M or N.

29. If nations throughout history have prepared for or engaged in war, will this necessarily be the case in the future?

30. Why are facts and ideas correlative in reflective thought? (Dewey *loc. cit.*)

31. How do we know that a force of 1 pound with a lever arm of 2 feet is as effective as a force of 2 pounds with a lever arm of 1 foot? (Blackwood *op. cit.*, p. 20.)

32. What principle was violated in the contention directed against Galileo that the apparent valleys of the moon were covered with transparent glassy material so that, as tradition had maintained, the moon's surface was really smooth?

33. What distinguishes experiment from observation?

34. How are logical methods of reasoning related to technical methods of observation and experiment?

35. What principle would you put into operation if you were to convince another that the sun warms the earth?

36. Of what principle did Copernicus avail himself when he insisted that that center of reference in astronomical motions was to be taken in terms of which the facts could be accounted for on the fewest postulates?

37. What are the main lines of converging evidence for evolution? (W. S. Scott *The Theory of Evolution.*)

38. Trace the principle of converging evidence in the constitutional issue of states' rights and nationalism. (F. A. Ogg and P. O. Ray *Essentials of American Government* [7th ed.], pp. 47-50.)

39. Do all the implications of most hypotheses in science become verified in time?

40. Can the meaning of "truth" remain absolutely fixed while methods of inquiry change?

41. Describe significant traits of reflective "judgment." (Dewey *op. cit.*, chap. viii.)

42. Note the main stages of scientific progress from Copernicus to Newton. (E. A. Burt *Principles and Prob-*

lems of Right Thinking [1931 rev. ed.], chap. xxi: "How Science Progresses.")

43. How did Vesalius deal with the notion that men had one less rib on one side; that the body contains an indestructible resurrection bone as the nucleus of the resurrection body; that human thighbones are curved in their natural condition; etc.? (William A. Locy *The Growth of Biology*, p. 174.)

44. What principles of scientific progress stand out in the history of physiology since Harvey? (*Ibid.*, chap. xx.)

45. Has chemistry discarded or extended Dalton's atomic hypothesis?

46. May twentieth-century physics be properly said to supplant the physics of Galileo, Newton, and Faraday? (Blackwood *op. cit.*, p. 447.)

47. Illustrate the role of false hypotheses in the progress of science. (*Ibid.*, p. 462.)

48. How do new tools and techniques contribute to the advance of science?

49. Does scientific progress confirm and refine old conceptions or institute new problems?

50. Explain: "Science is an attempt to obtain complete, precise, generalized, and systematic knowledge of the world in which we live." (C. A. Mace *The Principles of Logic*, p. 9.)

51. Of what elements does intellectual progress consist? (Dewey *op. cit.*, p. 140.)

CHAPTER XV. RIVAL METHODS

1. On what method, if any, will one typically rely who claims to be "practical" and to have absolutely no use for "hypotheses"?

2. Trace some of the ways in which you were conditioned early in life to like or dislike somebody or something.

3. To what extent does some group with which you are acquainted restrict individual opinion to what is conventional for the group?

4. List some problems not solved by common sense.

5. Compare and contrast common sense with science "as a body of systematized and verified knowledge, expressing in general terms the relations of exactly defined phenomena." (I. Edman *Human Traits and Their Social Significance*, p. 368.)

6. Discuss the effectiveness of repetition in the formation of beliefs.

7. Does the need for a faith warrant turning to some given particular faith?

8. What analogy, according to Bede, prompted Edwin, king of Northumberland, and his wise men to embrace Christianity in 627?

9. Ought we to be more certain of what we want to believe or of the reasons for a belief? Is there any evidence that pessimism shortens life whereas optimism lengthens it?

10. John Locke: "I find every sect, so far as reason will help them, make use of it gladly; and, where it fails them, they cry out, 'It is matter of faith and above reason.'" How does this illustrate the importance of method.

11. In what sense is faith in democracy a hypothesis? (S. Hook, in *Naturalism and the Human Spirit* ed. Y. H. Krikorian, pp. 51-61.)

12. May we claim to "know" what we "believe"?

13. Trace some of the ways in which you have come to doubt something you had previously accepted.

14. Is "critical" inquiry necessarily purely negative in its aims?

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15. Must a thoughtful reader of political platforms "believe in" at least one of them? (Charles Beard *The Republic*, p. 195.)

16. Examine arguments for and against various types of skepticism.

17. Evaluate the ability to suspend belief.

18. The ancients said nothing about moons on Jupiter; hence these cannot exist. What method does this argument exemplify?

19. Give examples of positive and negative values of traditions.

20. Would you observe the injunctions of Exodus 22:18; I Thessalonians 5:26; John 13:14; etc.?

21. Was it reasonable for a New England town meeting three generations ago to vote whether and where to build a bridge across a stream; whether the village doctor should use ether as an anesthetic?

22. How would you try to determine, though only tentatively, whether or not a given person is an expert in psychology; in education; in government?

23. Why do speakers and writers quote "authorities": Adam Smith regarded war as man's noblest profession; Thomas Jefferson held all men to have been created free and equal; etc.?

24. Why do experts disagree? In lieu of knowledge, what recourse would you take in weighing expert opinions favoring the "gold dollar," the "commodity dollar," or the "managed dollar"?

25. Are the ablest men of a community on the whole to be trusted or mistrusted?

26. Whose opinion, other things being equal, would you be inclined to favor in problems of labor, of armament, of banking, of education, of retail trade, etc.?

27. "This is why able men have to change their minds and their line of action in middle age, and to begin life again, because they have followed their party, instead of having secured that faculty of true perception as regards intellectual objects which has accrued to them, without their knowing how, as regards the objects of sight." (John Henry Cardinal Newman *The Idea of a University* [1905 ed.], p. 499.) Discuss.

28. If sense experience as a source of information is fallible, is it therefore also incorrigible?

29. Does reason as a function of inquiry take recourse to extrinsic supports and sanctions?

30. Why does the search for pure "immediate experience" end in failure?

31. Show the need of method in the control of evidence, of conceptions, of experience. (John Dewey *How We Think* [1933 ed.], chaps. xii, xiii.)

32. Do we face the alternative of having to choose between "logic" and "experience"?

33. Explain Immanuel Kant's dictum that concepts without percepts are empty, and percepts without concepts are blind.

34. Set forth some of the "hindrances and aids to right thinking." (E. A. Burt *Principles and Problems of Right Thinking* [1931 rev. ed.], chap. iii.)

35. Discuss understanding in relation to ideas, meanings, conceptions, and definitions. (Dewey *op. cit.*, chaps. ix, x.)

36. Can intuition and other alternatives to reason be alternatives to unreason also? What acknowledgment seems implied in the giving of reasons for such alternatives? (James MacKaye *The Logic of Language*, p. 43.)

HOW MAN THINKS

37. Do belief, doubt, experience, reason, and allied factors in the quest for certainty suggest considerations restricted to an isolated organism?

38. If science cannot achieve unqualified certainty, is it therefore futile?

39. If scientific method is not infallible, to what method shall we resort for its correction?

40. How would the belief that science and intelligence cannot help us, affect intellectual integrity and the scientific enterprise?

41. Contrast with other methods the method of science or reflective inquiry as public, as progressive, and as effective. (M. R. Cohen and E. Nagel *An Introduction to Logic and Scientific Method*, chap. x.)

42. Discuss "the self-corrective nature of scientific method." Evaluate other methods in this regard. (*Ibid.*, p. 395.)

43. If one committed to methods of critical examination does not necessarily reject all statements which make appeal through some alternative method, what would he do with such statements?

44. Name outstanding exponents of methods alternative to reflective inquiry.

45. For further study of rival methods, see: J. H. Randall, Jr. and J. Buchler *Philosophy: An Introduction* (especially Part II: "The Analytical Function of Philosophy"). See also: W. P. Montague *The Ways of Things*, chap. ii. Are these methods "ways of knowing"?

CHAPTER XVI. CAUSAL HYPOTHESES

1. If A is the cause of B, and B of C, is B the cause of C because A is the cause of B?

2. Explain the paradox of scientific experiment as interfering with the course of nature for the purpose of discovering more determinately nature's laws.

3. Explain: heterogenesis, biogenesis.

4. Energy is absorbed when a hand is held under the bulb of an incandescent lamp. Air being a poor conductor of heat, the energy is transferred by conduction; heated air moving upward, the energy is not carried by convection. The energy must accordingly be transferred by radiation. Put the argument into the form of syllogisms.

5. Brewster took an impression from mother-of-pearl in cement of resin and beeswax and then in balsam, fusible metal, lead, gum arabic, isinglass, etc., thus accounting for the iridescent colors by the form of the surface, whatever the chemical nature of the substance. What method does this example illustrate?

6. Whatever fluid, air or water, gas or liquid, be forced into a siren, the sound produced is the same. What method does this illustrate?

7. In showing that a person's illness is due to the eating of certain foods, what test would one apply to the consideration that other persons were ill after eating the same food?

8. How do you account for coincident discoveries (Newton and Leibniz, James and Lange) and inventions (the telegraph three times in 1837, the telephone twice in 1876)?

9. How is action, including experiment, related to thought? (See: John Dewey *How We Think* [1933 ed.], chap. xiv.)

10. What method do you apply when you estimate the benefit of a possession by the consequences of its loss?

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11. By what method did Redi show that no maggots appeared upon meat that was screened so that flies could not lay their eggs on it?

12. If friction of solid surfaces is independent of surface area, why is a sled with narrow runners harder to drag over soft snow than one with wide runners? (O. Blackwood *Introductory College Physics*, p. 38.)

13. Why may a passenger in a crashing automobile bump against the windshield, and a boy sitting at the rear end of a truck which unexpectedly starts forward fall off? (*Ibid.*, p. 55.)

14. What is the effect of pouring oil on troubled waters? (*Ibid.*, p. 113.)

15. What is proved by an increase in the weight of a liter flask when opened to the atmosphere? (*Ibid.*, p. 141.)

16. With what varying effect may a differentially permeable membrane in the form of a bag, filled with a 20 per cent sugar solution, be immersed in a vessel of water, and in a vessel containing a 20 per cent sugar solution? (G. H. Smith and University of Wisconsin Associates *Textbook of General Botany* [4th ed.], p. 24.)

17. What kinds of experiments would you institute to demonstrate that photosynthesis is possible only at a suitable temperature and in the presence of living matter, chlorophyll, water, carbon dioxide, and light? (*Ibid.*, pp. 123 ff.)

18. Rats barely crawl at birth, but walk when the nervous system and muscles have developed. Interpret this observation.

19. Students having remained without sleep for three days took a test in which their performance fell off 25 per cent from their normal. What conclusion would you draw, and by what method would you arrive at it?

20. In what sense does study of foster children constitute a "crucial experiment"?

21. What differences appear when you try to break a green stick from when you try to break a dry one?

22. Why is frost a factor in the splitting of rocks and the formation of soils?

23. Mr. Walter Lippmann has thought it significant that the founders of our liberties had been educated in the arts of free men and that we, not having been so educated, have mismanaged our liberties to the point of being in danger of losing them. Discuss from the standpoint of the logic of causal hypotheses.

24. A frog's ears are in the legs: rap on wood, and he will jump—except when his legs have been removed. Comment on this reasoning.

25. Explain the joint method of agreement and of difference in inquiring into the causes of the decline of the rural church; into the connection between liquor and crime; etc.

26. Is the same home the same environment to all its occupants?

27. What is meant by the phrase "control group," in an experiment?

28. Drop a feather and a coin at the same instant. Which reaches bottom first? Why? What if the two are released in a long, glass tube which has been evacuated? (Blackwood *op. cit.*, p. 51.)

29. Translate into German, Dutch, Danish, Swedish, Icelandic, Latin, French, Spanish, Italian, and Portuguese: "Give us this day our daily bread." What conclusion would you draw, and by what method, from the vocabulary, grammar, and pronunciation? (Frederick Bodmer *The Loom of Language*, p. 180.)

30. Explain the meaning of the term "function" as applicable to the following examples: the volume of a gas and its temperature and pressure; $y = x^2$; the length of the circumference of a circle and its radius; expansion and temperature of mercury; income tax and income.

31. Why is plague less frequent in Indian villages abounding in cats?

32. What correlations are recorded by Keeler's "Polygraph"? (*Scientific American* [June, 1937].)

33. Why is starting-friction greater than sliding-friction? (Blackwood *op. cit.*, p. 38.)

34. Suppose an automobile coasting down a slope increases its velocity in each successive second by 3.2 feet; it travels in the first five seconds a total distance of 1.6, 6.4, 14.4, 25.6, and 40.0 feet, respectively. Formulate the manner in which the total distance travelled varies with the time. (*Ibid.*, p. 49.)

35. How does pressure vary with velocity? (*Ibid.*, p. 154.)

36. When I speak loudly, I get attention; I must therefore make my wishes known in a loud voice.

37. When the price of cotton is low, lynchings increase. Comment.

38. Aesop tells of a woman who expected her hen, with a double measure of barley, to lay two eggs a day instead of one; but it grew fat and ceased to lay any eggs. Comment.

39. Does agreement among traits of a phenomenon suffice to establish a conclusion about them? Can differences be determined in utter disregard of agreements? What bearing has the varying of conditions on negative traits?

40. If it took you ten minutes to memorize a stanza and eight to relearn it, how do you account for the time saved?

41. Given a force X capable of carrying particle P from A to C , and a force Y capable of carrying P from A to B , compute the joint effect of X and Y upon P . Given the conjoint effect AD , and given AB as the effect of Y , what is the cause of the residual effect?

42. In a tug of war, the total north pull is 400 pounds; the total south pull, 360 pounds. What is the resultant force? (Blackwood *op. cit.*, p: 11.)

43. If a river carries a boat southward with a velocity of 4 miles per hour, and the boat is headed eastward with a resultant velocity of 5 miles per hour, with what velocity does the man row his boat? (*Ibid.*, p. 63.)

44. What led to the discovery of argon?

45. "If then the removal of the causes of this Spirit of American Liberty be, for the greater part, or rather entirely, impracticable; if the ideas of Criminal Process be inapplicable, or if applicable, are in the highest degree inexpedient; what way yet remains? No way is open, but the third and last—to comply with the American Spirit as necessary; or, if you please, to submit to it as a necessary Evil." Discuss as an example of method. (Edmund Burke "Conciliation with America" *Select Works* I.195, quoted in J. F. Genung *The Working Principles of Rhetoric*, p. 626.)

46. How would you proceed to test the hypotheses of a causal relation between study and grades; advertising and large sales; living in small towns and the prevalence of gossip; neurotic personality and repressed anxiety; an explosion and a rapid expansion of gases; application of a specific serum and immunity to a particular disease to which the "control" group remains susceptible; tension of a violin

string and pitch; criminality and the sum total of our institutions?

47. Does each distinct letter stand for a distinct number in this addition problem? SEND + MORE = MONEY.

48. What, in a causal inquiry, is the status of a circumstance neither invariably nor differentially nor functionally related to the phenomenon?

49. State some of the difficulties in the application of Mill's methods. (H. A. Larrabee *Reliable Knowledge*, pp. 335-40.)

50. Name some of the causes of misperceiving.

51. Give an example of an inference from a prior cause; from a subsequent effect; from concomitant events.

52. Where does the search for an origin begin?

53. Are men true causes?

54. Is a seed a cause of its own growth?

55. What is the logical significance of variety in the causes of plant and animal diseases?

56. Discuss the distinctions between pure and applied science; between science and other studies; between science and the sciences. (N. Campbell *What is Science*, chap. i.)

57. Do you consider it appropriate to speak of "belief" in the existence of an external world, of other people, etc.? (*Ibid.*, chap. ii.)

58. Distinguish invariable conjunctions of properties, sequences of events, and numerical relations. (*Ibid.*, chap. iii.)

59. In what sense are scientific laws predictive? (*Ibid.*, chap. iv.)

60. State carefully what is meant by scientific explanation. (*Ibid.*, chap. v.)

CHAPTER XVII.

MEASUREMENT AND STATISTICS

1. How do measurable and nonmeasurable properties differ? (N. Campbell *What Is Science*, chap. vi.)
2. State three rules for counting. (*Ibid.*)
3. Illustrate the laws of measurement. (*Ibid.*)
4. Discuss with care the distinction between fundamental and derived measurement. (*Ibid.*)
5. Wherein, basically, consists the value of mathematics for experimental science? (*Ibid.*, chap. vii.)
6. Show how a numerical law may give rise to a system of derived measurement. (*Ibid.*)
7. What is meant by "calculation"? (*Ibid.*)
8. Indicate the part played by the sense of form in mathematical theories. (*Ibid.*)
9. Point out some of the limitations and values of "science." (*Ibid.*, chap. viii; M. R. Cohen and E. Nagel *An Introduction to Logic and Scientific Method*, pp. 399-405.)
10. Why must a unit of measurement, taken as such, be discrete? May what is taken as discrete in one use be treated as continuous in other respects?
11. A: "Give me one of your sheep, and I shall have twice as many as you will." B: "Rather give me one of your sheep, and each of us will have the same number." How many sheep did each have? What is the value of suitable symbols in the solution of problems?
12. How do statistical techniques differ from simple enumeration?
13. If a teacher has taught 11 years, does this mean 11, or from 10.5 to 11.5, or from 11.0 to 11.99, or from 10.001 to 11.00? Determine similarly the unit interval for: age, 35; brothers and sisters, 6; average hours of prepara-

tion per week, 15; years which father attended school, 7; average number of pupils in classes, 26.

14. Why do we no longer keep statistical records of solar and lunar eclipses?

15. A class of 30 pupils scored, respectively, 48, 49, 60, 56, 48, 42, 50, 53, 54, 42, 45, 50, 42, 43, 36, 52, 26, 44, 35, 50, 40, 36, 45, 40, 47, 38, 36, 41, 34, 45. Construct a grouped frequency distribution of the scores. Compute the mean.

16. Give the arithmetic mean of a runner's record in the 100-yard dash: 10.2, 10.1, 10.0, 10.0, 10.1, 10.0, 9.9, 10.1, 11.4, 10.2 seconds.

17. A group of 25 students earned a mean score of 30; a second group of 75 averaged 40. What was the mean score of the 100 students?

18. What is the significance of a median length of life of 15,000 miles for tire A and 17,000 for tire B as compared with a mean length of life of 17,000 miles for A and 15,000 miles for B?

19. If a teacher desires to raise the median score of her pupils, on which pupils would she concentrate?

20. In a given school group, 1 boy is 16 years old; 2 boys and 1 girl, 15; 1 boy and 2 girls, 14; 4 boys and 3 girls, 13; 9 boys and 2 girls, 12; 25 boys and 10 girls, 11; 5 boys and 17 girls, 10; 2 boys and 2 girls, 9; 1 girl, 8; 2 girls, 2; 1 boy, 6. Compute the mean and the median age of the boys, of the girls, of the total group; compute the mode of each distribution.

21. Is the value of a piece of cultivated farm land best indicated by the mean, median, or modal yield per acre?

22. Which average would you use in determining the most common age of college graduation; an average wage; the size of ready-to-wear clothing most frequently de-

manded; the batting strength of a baseball team; the "average" reader, customer, community, school; etc.?

23. Are climatic conditions in Boise, Idaho, and in Seattle, Washington, sufficiently characterized by the mean annual temperatures of 50.9 and 51.0 degrees, respectively?

24. On different tests on which the respective average marks received were 85, 90, and 25, with standard deviations of 7, 2, and 7, respectively, A made a score of 72; B, of 85; C, of 17. Arrange A, B, and C in order of excellence as judged by these results.

25. Of 80 pupils, 1 scored between 60 and 63; 0, 56-69; 0, 52-55; 1, 48-51; 2, 44-47; 2, 40-43; 5, 36-39; 8, 32-35; 16, 28-31; 32, 24-27; 13, 20-23. Estimate the range of variation. If the third quartile is 31.3, and the first quartile is 24.4, what is the semi-interquartile range? If the mean is 28.9, estimate the average deviation; the standard deviation.

26. Given 2 men 61 inches tall; 10, 62; 11, 63; 38, 64; 57, 65; 73, 66; 106, 67; 126, 68; 109, 69; 87, 70; 75, 71; 23, 72; 9, 73; 4, 74; draw several inferences. State approximately the average height; the median; the lower and upper quartile; the range; the range for the middle half.

27. If the mean height of 1,017 freshman women is 63.86 inches, with a standard deviation of 2.09 inches, and the mean weight is 115.65 pounds, with a standard deviation of 15.78 pounds, are these women more variable in height or in weight?

28. Given ten individuals with scores in one test of 4, 6, 7, 8, 10, 10, 11, 12, 14, 18; and in another, 2, 5, 6, 3, 7, 5, 4, 8, 11, 9; find the coefficient of correlation by Pearson's formula.

29. Let C represent the circumference of a circle; D , its diameter; then $C = 3.1416 D$. Distinguish the independ-

ent and the dependent variable. Give several examples of correlation.

30. Spingarn's rank correlation coefficient is expressed in the formula: $\rho = 1 - \frac{6 \sum d^2}{n(n^2 - 1)}$; where d refers to the difference in rank between paired items in the two series. Apply to the rankings of 8 tennis players in 2 years: A, 1, 1; B, 2, 3; C, 3, 4; D, 4, 2; E, 5, 8; F, 6, 5; G, 7, 6; H, 8, 7. Compare with a rank correlation coefficient of $-.143$ in 8 baseball batting averages.

31. Sketch some typical social conditions accounting for the extensive use of statistics.

32. Account for the confusion when a 32 per cent increase of employment from May, 1933, to May, 1934, was interpreted as meaning that one extra man had been added to every two; when percentages of white, foreign-born population in New England States in 1930 (12.6, 17.8, 12.0, 25.2, 25.2, 24.3) were averaged to 19.5 per cent of the total, although the total white population numbered 8,065,113, of whom 1,834,310 were foreign-born; when .34 per cent is read as 34 per cent.

33. Why did the attempt, in the United States census of 1890, to discover the number of persons of pure and mixed African blood, by distinguishing between blacks, mulattoes, quadroons, and octoroons, end in failure? Describe some of the difficulties in taking a census; in the use of questionnaires; in estimating unemployment; in calendar variation; in the unification of world statistics.

34. Is the statistical method appropriate in compiling a word list in foreign language textbooks?

35. How are IQ averages arranged according to occupational groups, to be interpreted?

36. If an individual's IQ remains unchanged, is his mental development arrested?

37. According to results of intelligence tests applied to different national and racial groups, what can be concluded from a given individual's nationality or race?

38. If the mean income in one community is larger than in another, can you conclude that the first community has a higher standard of living?

39. Before World War II, Europe had about 17,357 statute miles of boundaries, or 4.0 per 1,000 square miles of area; Europe west of Russia, 14,846, or 7.3; Asia, 26,113, or 1.6; Asia, except Russia, 17,507, or 1.7; Africa, 28,670, or 2.5; North America, 11,433, or 1.3; South America, 18,961, or 2.7. Interpret the methodological significance of these figures.

40. A coin has fallen "heads" nine times in succession. What is the probability that it will fall tails on the tenth throw?

41. Suppose you desire information about the amount of profit in farming in a given locality; to this end, you make a statistical study of a sample of the most accessible farms operated by farmers able and willing to give you the desired information. What kind of conclusions can you safely draw?

42. Does random sampling call for knowledge?

43. Is an "error in random sampling" an "error" in the sense of an avoidable mistake?

44. Can a liar be described as "reliable"?

45. Is there a general answer to the question how large a sample must be employed in statistical inquiry?

46. How would you view concomitant variations in the curves of industrial production and the length of women's skirts; in industrial activity and the use of soap or

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the consumption of ice cream; in the smoke from industrial plants and the smoking of cigars and cigarettes?

47. Is a highly trained musician likely to be a poor mechanic?

48. If women of unusual beauty are more noted for their beauty than for their intelligence, does this justify the expression "beautiful but dumb"?

49. List some bigotries refuted by the correlation of superiority in desirable traits. (E. L. Thorndike *Human Nature and the Social Order*, p. 276.)

50. Express in the language of probability changes in the relation between a given stimulus and a given response, including the strengthening, acquiring, weakening, and abolition of tendencies. (*Ibid.*, p. 6.)

51. What is the significance of the information that 80 per cent of the working population were employed in agriculture in 1820, but in 1930 only about 10 per cent; that in 1790 about 3.3 per cent of the population lived in towns of 8,000 and over, but in 1930 about 49.1 per cent?

52. Among primitive peoples what does the participation of men in the care of little children imply for statistical interpretation?

53. What advantage over an observational approach is gained when the United States Department of Agriculture predicts the size of crops from the number of acres planted and from multiple correlations of inches of rainfall, hours of sunlight, and average temperature?

54. By what type of reasoning do educators, from kindergarten tests (in vocabulary, discrimination of letter forms and combinations, selection and matching of words, discrimination and memory of forms, and following of directions), predict reading achievements in the first grade?

55. What do you make of the fact that the mean income of college graduates is much higher than that of high school graduates?

56. According to Sorokin, our science is bankrupt; and statistical findings point to the coming collapse of our culture. Evaluate this judgment.

CHAPTER XVIII. HISTORICAL INFERENCE

1. Illustrate attempts to verify memory-affirmations; narrations and descriptions; historical inferences.

2. What did *historia* mean in its original Greek usage?

3. Is history the past? Is history about the entire past? In what sense, if any, does the past "live again" in history?

4. Is a chronicle a history? Is history past politics? Is history past biography?

5. In what respects is "history" a "double-barrelled" word?

6. Illustrate as phases of historical investigation: selection, sources, auxiliary sciences.

7. How does memory become history? (George Santayana *Reason in Science*, in *Works* [Triton ed.; Charles Scribner's Sons], chap. ii: "History.")

8. "The principle that elicits histories out of records is the same that breeds legend out of remembered events." (*Ibid.*) Explain.

9. How does historical investigation involve reconstruction, verification, and survey of alleged past events? (*Ibid.*)

10. Compare Mark 14:47; Matthew 26:51-54; Luke 22:50, 51.

11. Distinguish "higher" and "lower" criticism.

12. What test did David Hume apply to the alleged occurrence of miracles?

13. How was it possible for Giesebrecht to reconstruct, from chronicles of the eleventh century, a missing chronicle which was found some twenty-six years after its reconstruction? (Allen Johnson *The Historian and Historical Evidence*, p. 17.)

14. Trace the role of evidence in textual criticism. ("Textual Criticism" *Encyclopedia Britannica* [14th ed.])

15. Note the methods by which Lane Cooper, in his *Aristotle, Galileo, and the Tower of Pisa*, refuted one of the many legends in the history of science.

16. Are historical events "in our minds"?

17. Select several quotations from those given in Professor Randall's *The Making of the Modern Mind*. Indicate some of the factors upon which adequate interpretation depends.

18. Why do historians sometimes write as if the events chronicled were inevitable?

19. What would be implied by the denial of any continuity in history?

20. Can the history of philosophy be properly presented as if each thinker continued where his predecessors left off?

21. Comment on the phrase: "the wave of the future."

22. Point out procedures involved in asking the question: "Why was Lincoln murdered?" (H. A. Larrabee *Reliable Knowledge*, pp. 15-19.)

23. Can historians dispense with generalizations?

24. May a multiplication of historical illustrations prove one theory of history to the exclusion of others? (Charles Beard *The Republic*, p. 338.)

25. Show the need, and some of the difficulties, of recovering complex historical events in their fullness.

26. May written histories be properly discredited because of their incompleteness?

27. Would human life be meaningless without an absolute historical purpose?

28. How may we avoid both overconfidence and underconfidence in historical knowledge?

29. "English literature will ever *have been* Protestant." Account for this remark. (John Henry Cardinal Newman *The Idea of a University* [1905 ed.], p. 314.)

30. How does historical theory involve rhetorical unities, efficacious and ideal relations, and moral critique? (George Santayana *loc. cit.*)

31. How does historical romance involve moral affinities, dramatic imagination, and fidelity to past fact? (*Ibid.*)

32. "The function of history is to lend material to politics and to poetry." Illustrate. (*Ibid.*)

33. Why would no one today follow the advice of an old Roman treatise on agriculture that more grain can be produced at lower cost by increasing the number of slaves and using iron ploughs drawn by oxen? May we interpret the second World War according to the pattern of the first? May we interpret it apart from that pattern?

34. How do you interpret Charles Beard's use of epigrams concerning the lessons of history: the mills of the gods grind slowly, but they grind exceedingly fine; the bee fertilizes the flower it robs; whom the gods destroy, they first make mad; sometimes on a dark night men can see the stars?

35. Select, from a standard history, several passages embodying moral judgments of the people discussed or of the historian. Account for such moral judgments.

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36. Is it possible for the historian to give a pure or final description of past events?

37. May history be justly characterized as mythology?

38. Discuss self-knowledge as a test of theories about the past.

39. Is there an all-inclusive process of which particular histories constitute a part? Does meaningful history depend upon discovery of a single future goal?

40. In what sense, if any, may the world be said to be "returning" to the dark ages or to some other period of the past?

41. Does science destroy human freedom, origination, and progress? (E. L. Thorndike *Human Nature and the Social Order*, chap. xv.)

42. "With all your talk of progress you do not believe in progress, for you do not believe in the changed heart and changed mind which alone are creative and which make progress possible." Discuss. (Paul Wolfe "Conversations with the Old Man," *The American Scholar* XIV, No. 1 [Winter, 1944-45], p. 43.)

43. Estimate the value of historical knowledge.

CHAPTER XIX. JUDICIAL PROOF

1. What is required for a balanced approach to the analytical study of legal conceptions? (John Chipman Gray *The Nature and Sources of the Law* [2nd ed. 1921] Introduction.)

2. How may legal rights and obligations be defined? (*Ibid.*, chap. i.)

3. In what sense does the law recognize different kinds of "persons"? (*Ibid.*, chap. ii.)

4. State some fundamental issues posed by the concept of the "state." (*Ibid.*, chap. iii.)

5. Compare and contrast several functions of the law, and note the value of such comparison and contrast. (*Ibid.*, chap. iv.)

6. In what ways is judicial power limited? (*Ibid.*, chap. v.)

7. Distinguish particular, comparative, and general jurisprudence. (*Ibid.*, chap. vii.)

8. Why does Gray consider important the distinction between law and sources of the law?

9. Read: M. R. Cohen *Law and the Social Order* (1933), pp. 165-83: "The Place of Logic in the Law"; pp. 184-97: "Law and Scientific Method."

10. Having assigned to an argument a probability of 9/10 and to the trustworthiness of a witness against the conclusion a weight of one thousand to one, may we depend upon the argument alone? (A. De Morgan *Formal Logic*, p. 224.)

11. Four witnesses, whose accuracy I estimate at two, three, four, and five to one, favor an assertion to which I assign a weight of 1/100. What is the value of the united testimony? (*Ibid.*, p. 225.)

12. Given four assertions, of which one, and one only, must be true, evidence weighted 4/7 is offered for the first; none, for the others. What is the proportion of the probabilities of the four assertions? What is the probability of the first? (*Ibid.*, p. 231.)

13. Given four assertions, not more than one of which can be true; given the probabilities for the assertions 2/7, 3/11, 1/8, 4/5; and given an assertion true if either of the first three be true. What are the probabilities of the several assertions? What is the probability of the fifth assertion? (*Ibid.*)

14. If a man is alive, would a court of law content itself with witnesses to his having been recently alive? (*Ibid.*, p. 233.)

15. Given a conclusion and its contradiction. What effect does an argument for, or an argument against, either side have on the other? What would be the effect if the two sides of the question are not contradictories but contraries? (*Ibid.*, p. 235.)

16. Given both testimony and argument to both sides of a contradiction. Is it essential for the truth of either side that the argument for it be valid or that the argument against it be invalid? (*Ibid.*, p. 236.)

17. What are the odds for and against the testimony of the Cretan Epimenides that all the Cretans are liars? (*Ibid.*, p. 242.)

18. Classify the evidence according to which, if the thing to be found has certain marks and "this" has those marks, "this" is therefore the thing we want to find. Under what circumstances would the evidence be demonstrative? (*Ibid.*, p. 262.)

19. Account for the contention that strong evidence which has been presented, may be false. (*Ibid.*, p. 321.)

20. On what conditions would a court of law accept, in lieu of an original document, copies of it or testimony concerning its contents?

21. Distinguish "evidential" fact from "ultimate," "main," or "principal" fact. (John Jay McKelvey *Handbook of the Law of Evidence* [5th ed. 1944], p. 5.)

22. What would be the function of a court if all facts of a case were admittedly presented with utmost accuracy? Do rules of evidence alone suffice to determine the main facts of a given case? (*Ibid.*, p. 15.)

23. On whom rests the burden of proof when the facts alleged by a plaintiff are denied by the defendant; when the defendant admits those facts, but alleges others in defense? How does the "burden of proof" differ from the "burden of proceeding with the evidence"? (*Ibid.*, pp. 95, 101.)

24. Does "corroborative evidence" have reference to quantity or quality of evidence? (*Ibid.*, p. 110.)

25. Why is it important to distinguish between an admission and proof of admission? (*Ibid.*, p. 219.)

26. How does "hearsay evidence" differ from "evidence of hearsay"? (*Ibid.*, p. 384.)

27. State some circumstances which disqualify a witness. (*Ibid.*, 526.)

28. What is a "leading question"? (*Ibid.*, p. 573.)

29. What is meant by "authentication of documents"? (*Ibid.*, p. 620.)

30. In what sense is a writing "the best evidence" of its own contents? (*Ibid.*, p. 604.)

31. On what assumption concerning intent is a signer bound by the terms of a written contract whether he has read it or not? (*Ibid.*, p. 152.)

32. Is silence in the face of a charge an "implied admission"? (*Ibid.*, p. 183.)

33. Does possession of weapons by the accused have the characteristics of a confession? (*Ibid.*, p. 245.)

34. Ought confession of guilt by a third party to be treated as hearsay or as circumstantial evidence? (*Ibid.*, pp. 246-49.)

35. Is the fact that a witness has not observed a "Private Way" sign admissible as tending to prove the sign was not there? If one witness swears that a thing occurred in the presence of another, and the other swears it did not, is the latter "negative" testimony? (*Ibid.*, p. 256.)

36. Do precautions taken after the occurrence of injuries show previous negligence? (*Ibid.*, p. 273.)

37. How does inference from "quarrelsome character" to "assault" compare in probative value with inference from "peaceable character" to "innocence of committing an assault"? (*Ibid.*, p. 299.)

38. Does failure of an accused person to testify furnish ground for an inference against him? (*Ibid.*, p. 534.)

39. Why do rules of evidence change? (*Ibid.*, p. 11.)

40. Discuss the "power of courts" over "statutes." (*Gray op. cit.*, chap. viii.)

41. To what questions does consideration of judicial precedents give rise? (*Ibid.*, chaps. ix, x.)

42. What is to be said about formation and influence of expert legal opinion? (*Ibid.*, chap. xi.)

43. How does custom operate as a source of law? (*Ibid.*, chap. xii.)

44. How does Gray interpret the concepts of morality, equity, and law? (*Ibid.*, chap. xiii.)

CHAPTER XX. DEMONSTRATION

1. It is impossible for a rational number (i.e., a number which can be expressed as a quotient of two integers) to denote exactly $\sqrt{2}$. Were there such a number, it could be written as a fraction s/t reduced to lowest terms, with s and t having no common factor. Then $s^2/t^2 = 2$, and $s^2 = 2t$; so that s is even (say $s = 2x$), s^2 is divisible by 4 (or $s^2 = 4x^2$), $2x^2 = t^2$, and t is even. But now s and t contain 2 as a common factor—a result which contradicts our assumption. In what respects is this knowledge reasoned, determinate, and necessary?

2. What undefined terms are presupposed in Euclid's definitions that a point is that which has no parts; that a line is breadthless length; that a surface is that which has length and breadth only? Is the indefinable in a deductive system that which is not understood?

3. What purpose does a list of undefined terms and relations serve in a deductive system?

4. What does it mean for postulates of a deductive system to be consistent and independent?

5. When are the postulates of geometry sufficient for demonstration of theorems?

6. By what means are theorems deduced in a deductive system?

7. Is it an axiom or a postulate in arithmetic that everything is either A or not-A?

8. What is the status of an alleged postulate that can be demonstrated?

9. What would be the consequence of postulating in arithmetic that $2 + 2 = 4$ and that $2 + 2 = 5$?

10. In what sense must the postulates of a deductive system be taken as true?

11. Is a thinker free to make whatever presuppositions he chooses?

12. In supposing what things are, what difference does it make if we stress the "supposing" or the "what"?

13. Show the futility of attempting to demonstrate everything.

14. Is it possible, from unity, to deduce plurality; from plurality, unity?

15. Give an example of what W. E. Johnson calls a subsumptive syllogism; a functional syllogism; functional deduction.

16. Explain: $1 \times 1 = 1$; $2 \times 2 = 1 + 3$; $6 \times 6 = 1 + 3 + 5 + 7 + 9 + 11$; etc.

17. Criticize: The angles of a brass isosceles triangle are equal to two right angles. (Aristotle *Prior Analytics* i.5.)

18. Are perpendiculars to the same line parallel because the angles thus formed are right angles?

19. Would you prove separately for numbers, lines, solids, etc., that if $A : B :: C : D$ then $A : C :: B : D$?

20. Note, as an example of invalid demonstration, the "proof" that a "right angle is sometimes equal to an obtuse angle." (Lewis Carroll, quoted in H. W. B. Joseph *An Introduction to Logic* [2nd ed.], p. 571.)

21. Given three propositions p, q, r , of which one, and one only, is true; three propositions, x, y, z , of which one, and one only, is true; if p then x ; if q then y ; if r then z . How would you demonstrate that if x then p ; if y then q ; if z then r ?

22. Are proverbs comparable to first principles? Give reasons for your answer.

23. Discuss the significance of the fact that any system is governed by the initial principles of the system.

24. In what different ways are scientific principles and the foundation of a building considered "fundamental" to architecture?

25. What respective sciences introduce such basic terms and relations as: implies, and, or, not, false; thing, all-is-, non-; number, is less than, is equal to quantitatively, is greater than; point, line, plane, intersects; time, motion; mass or particles of matter; groups of particles (taken collectively), electricity, atom; valence; life, animal, plant; mind, intelligence; groups of biological individuals or groups of men?

26. A science may use terms of sciences which it presupposes. Illustrate.

27. Why cannot a cube be defined in plane geometry?

28. What difference would it make for physics if it were assumed, with Euclid, that through a point outside a line, one and only one line can be drawn parallel to the given line; with Lobachevsky, more than one; with Riemann, not even one?

29. Generalize in an algebraic formula: $4 \times 3 = 4 \times 4 - 4$; $8 \times 9 = 9 \times 9 - 9$.

30. According to a Chinese proverb, the man who loves his wife is a good man; he who loves wife and children, a better man; he who loves wife, children, and friends, a still better man; and he who loves his wife, children, friends, and all the rest of mankind, the best of men. What difference would it make if we said instead: "A man's goodness is proportionate to his loves"?

31. How would you go about applying the distinction between matter and form to what is known in geometrical demonstration?

32. Distinguish and relate the parts that may be played by definitions in deductive systems.

33. If $\text{Work} = \text{Force} \times \text{Displacement}$, and a 100-pound cask is to be lifted onto a truck 4 feet above the pavement, the work required is 400 foot-pounds. Show the role of the middle term.

34. A 150-pound man runs up a stairway, raising himself 16 feet in 10 seconds. What is the work done? If one horsepower = 550 foot-pounds per second, what is his power?

35. Define efficiency; e.g., if 1,000 foot-pounds of work are done in pushing a 200-pound safe up an inclined plane

4 feet high, the useful output work is 800 foot-pounds, and the efficiency is 80 per cent.

36. How is the principle of work (output work equals input work when frictional effect is negligible) related to the principle of the conservation of energy (energy input equals energy output plus energy in overcoming friction)?

37. Formulate the syllogism condensed in the proposition that the object of taste, being something tangible, is not perceived through the medium of a foreign body.

38. May desire for prestige be treated as a form of acquisitiveness?

39. Distinguish homologous and analogous animal structures. Are the arm of a man and the wing of a bird homologous or analogous?

40. Define: solar eclipse.

41. What part does Spinoza's "General Definition of the Affects" play in Book III of his *Ethics*?

42. Determine the common element in the following terms: Englishman, Frenchman, German, Paul, Italian; whale, horse, man, dog, ape; kangaroo, opossum, wombat, nototreme; dolphin, frog, eagle, snake, man; lion, tiger, vulture, condor, wildcat. (Avey.)

43. How do you recognize a face; see a motion; read a newspaper?

44. What insight did Binet use as the criterion of validity for measurements of mental age?

45. Show how recognition of the one in the many, or of identity in difference, is involved in the classification of a plant; in the tracing of an evolutionary series; in the reference of sound and light to conditions mediating them; in the formulation of laws; in the discernment of purpose.

46. What sort of intellectual activity takes place when I call a bird a "thing-a-bob" and then a "robin"?

47. How did Kepler arrive at his laws of astronomical phenomena? How is Newton's law of gravitation deduced from Kepler's laws?

48. By what respective processes do we become aware of things not ourselves; fix attention on their general or essential features; pass from some to all possible cases of a type; trace correspondences in analogous considerations; affirm a universal with composite certainty?

CHAPTER XXI. SYMBOLIC LOGIC

Identify principles of symbolic logic as they occur in the following exercises. Express them in symbolic form. Symbolic logic likewise provides opportunity for practice in demonstration.

1. Why do we not write out in ordinary language:

$$x^3 - y^3 = (x - y)(x^2 + xy + y^2)?$$

2. In what ways is chess, with its symbols and rules of formation and transformation, analogous to a calculus?

3. What would be meant by taking as indefinable in a system of classes: class, multiplication, negation, equality?

4. Define by multiplication and negation: addition, inclusion, zero. Define: 1.

5. Postulate in terms of multiplication: commutation, association, inclusion, $aa' = bb'$.

6. Whence would you obtain as axioms: the mutual implication of two equivalent propositions; the truth of a proposition implied by a postulate or theorem; the continued validity of an expression in which a term has been replaced throughout by another term or expression; the substitution of equal terms for each other; the transitivity of the implication relation; the suppression of a true premise in an argument? How would you judge an attempt to

prove or derive the laws of identity, contradiction, and excluded middle from other propositions?

7. Explain the properties of equivalence indicated in the expressions: $a = a$; if $a = b$, then $b = a$; if $a = b$, and $b = c$, then $a = c$; if $a = b$, then $ac = bc$, and $a + c = b + c$; if $a = b$, then $a' = b'$; if $a = b$, then $(a \text{ in } b)$ ($b \text{ in } a$).

8. Days when it hails; days when it thunders; days when it hails and thunders. The only member of the class of even primes is 2. A circular sector is the intersection of a circle and an angle whose vertex lies in the center of the circle.

9. The straight angle is the union of two supplementary angles. Houses are combustible objects. Greeks; barbarians. Each of two athletes can repeat all the performances of the other.

10. The largest of two classes including both. The smallest including both. The class remaining unaltered no matter what class is selected from it. The class in which all individuals in any class are contained.

11. What algebraic laws are expressed in the equations: $x + y = y + x$; $(x + y) + z = x + (y + z)$; $xy = yx$; $(xy)z = x(yz)$; $x(y + z) = xy + xz$?

12. French Catholics are Catholic Frenchmen.

13. American or British soldiers or sailors are American soldiers, American sailors, British soldiers, or British sailors.

14. Those who are intelligent or both intelligent and pleasant are those who are intelligent.

15. Artists who are artistic or who are temperamental are artists.

16. One who has been tried and convicted has been tried.

17. Young clergymen are clergymen.

18. Boys will be boys.

19. If Americans are nonmonarchists, then monarchists are non-Americans.

20. If Americans are democratic, and bankers are capitalists, then American bankers are democratic capitalists.

21. Everyone (i.e., no one) having robbed this bank has been sent to jail.

22. No momentum is lost in a collision, since the momentum of one body has increased and that of the other has diminished.

23. Fact is fact.

24. One who sings and dances, sings.

25. All centaurs are elephants.

26. If silver prints fade in the sun, and the photographs I have bought are silver prints, then the photographs I have bought will fade in the sun.

27. If the competent are hard workers, and business executives are well-paid, then competent business executives are well-paid hard workers.

28. Pardoned transgressions are transgressions.

29. The negative of: clear and warm, clear or warm, male guardians, those either killed or wounded.

30. The dual of: *a* or *bc*.

31. Students are ingenious and industrious, ingenious and idle, dull and diligent, or dull and negligent.

32. Some have convictions but no principles; others, principles but no convictions; others, neither convictions nor principles; still others, both convictions and principles.

33. One party fails to favor, whereas the other favors, high tariff and prohibition; but some I know favor only the high tariff, others only prohibition.

34. Who are those other than patriots or executed traitors?

35. Into how many classes may employees fall as they are or are not: hard workers, in good standing, and justly rewarded?

36. Arrange in an order of increasing extension: mathematicians or logicians, mathematical logicians, philosophically-minded mathematicians or not-philosophically-minded logicians.

37. If there are no stars other than self-luminous bodies, or no self-luminous planets, then there are no stars which are planets. If men are tender-minded Platonists or tough-minded Aristotelians, they are either Platonists or Aristotelians.

38. State this proposition in several other ways: There are none who are either unlearned scholars or unscholarly men of learning.

39. State in several other ways: There are unambitious barbers.

40. If $b = ac'$ or $a'c$, how is the class a constituted?

41. Reduce to lowest terms: All the members of the program committee are on the executive committee; no one not a member of the program committee shall be a member of both the executive and the membership committees; none of the membership committee are on the program committee.

42. Everything that is S is P ; nothing that is S is P' ; everything that is S is $(P')'$; everything that is S is P . Nothing that is S is P ; nothing is SP ; nothing is PS ; nothing that is P is S . Not everything that is S is P ; something that is S is P' ; something that is P' is S .

43. If x is a positive number, then $2x$ is a positive number; and vice versa. This triangle has equal angles; this triangle has equal sides.

44. What I have written, I have written.

45. My assets and liabilities just balance.

46. The inhabitants of this town are natives, or they are adventitious.

47. The content of what is asserted is not affected by reassertion; by a different order among the assertions; by a different grouping of the assertions.

48. We must live and learn, or we shall be ne'er-do-wells; so we live and learn, or we live and are become ne'er-do-wells.

49. Hobson's choice.

50. Contradict: Some red houses are not red.

51. You told the truth and hurt his feelings; so—you hurt his feelings.

52. If I am leaving today, then I am leaving today or tomorrow.

53. Inasmuch as, if the world is getting smaller, we shall have to learn to live together, it would seem that to assert that the world is getting smaller, amounts to asserting both that the world is getting smaller and that we shall have to learn to live together.

54. If you become wise, you will be admired; and if you cultivate a friendly disposition, you will be liked. So, if you become wise and cultivate a friendly disposition, you will be admired and liked.

55. If, on the assumption that man is the measure of all things, astronomers can create planets, then, if astronomers cannot create planets, man is not the measure of all things.

56. If a student does not get an early start, he will fall behind in his studies; and if he falls behind in his studies, his education will be sadly neglected; consequently, if a student does not get an early start, his education will be sadly neglected.

57. If he who says A must say B, and he who says B must say C, and he who says C must say D, then he who says A must say D.

58. If it is false that God is either unwilling or unable to prevent evil, then God is both willing and able to prevent evil.

59. If human life can be improved, then either human life can be improved, and your gospel is to be believed; or human life can be improved, and your gospel is not to be believed.

60. Ice is lighter than water; it is true that ice is lighter than water.

61. Translate into ordinary language: If p' imp p , then p .

62. If $2 \times 2 = 5$, then any given number is equal to any other given number.

63. If $3 = 7$, then $3/3 = 7/7$, or $1 = 1$.

64. State in two equivalent symbolic forms: German aggressions imply either that the German people wanted war or that they were betrayed by their leaders. If Germans either wanted war or were misled, does it follow that either all Germans wanted war or all Germans were misled?

65. Since the fact that heat can be transformed into mechanical work implies that if mechanical work is not a substance then heat is not a substance, we may say that, if heat can be transformed into mechanical work, and if mechanical work is not a substance, therefore heat is not a substance.

66. If the assumption that all the good are wise and that all the happy are good implies that all the happy are wise, then, if none but the wise are good, it would follow that, if none but the good are happy, none but the wise are happy.

67. Construct truth tables for principles of the calculus of propositions.

68. What is the paradox of material implication? (On Exercises 68-75, see: W. E. Johnson *Logic* I. 39-47.)

69. Draw the appropriate conclusions: q' would imply p' , and q' would imply p ; q would imply p' , and q would imply p ; p' would imply q' , and p' would imply q ; p would imply q' , and p would imply q .

70. Draw the appropriate conclusions: p would imply q , and p' would imply q ; p would imply q' , and p' would imply q' ; q would imply p , and q' would imply p ; q would imply p' , and q' would imply p' .

71. In a major premise, p would imply q , are p and q entertained or asserted?

72. "If the millenium were to come tomorrow, human nature would be different from what it is." Complete the presumably intended argument. Translate into an alternative syllogism. Examine critically: p false; therefore either p false or q true; therefore p would imply " q "; therefore, if p would imply " q ", and p true, then q true.

73. "If Woodrow Wilson was a visionary, he did make valuable suggestions for international relations." Complete the presumably intended argument. Translate into a disjunctive syllogism. Examine critically: q true; therefore either p false or q true; therefore " p " would imply q ; therefore, if " p " would imply q , and p true, then q true.

74. Does the paradox of material implication warrant an *inference* from a false proposition to any proposition; from any proposition to a true proposition?

75. If, from: q true, we may infer: either p false or q true, may we further infer: p , therefore q ; or: q false, therefore p false? From a similar inference: p or q , may we further infer: p false; therefore q true; or: q false, therefore p true? If, from: q false, we may infer: p true or q false, may we further infer: p false, therefore q false; or: q , therefore p ? From a similar inference: p false or q false, may we further infer: p true, therefore q false; or: q true, therefore p false?

76. Express in the notation of propositional functions: Everybody has relatives. Nobody is everybody's friend. Nothing can come out of nothing. Some people are twins. Some mushrooms are not poisonous.

77. How is the relation of parenthood related to the relations of fatherhood and motherhood?

78. Whenever x is smaller than y , then x and y are diverse.

79. How are the relations related in the equivalent formulas $x < y$ and $y > x$?

80. Show that the statement: Polygons P and Q are similar, is equivalent to the statement: The shapes of P and Q are identical.

81. Do all people belong to the domain and the counterdomain of the relation expressed by: x is father of y ?

82. What is the domain and the converse domain of the relation of identity?

83. x is brother or sister of y . x is friend and benefactor of y . x is the friend of the father of y .

84. Give the relative product of the relations: husband and daughter, brother and father, father and brother, brother and son, father and father, ancestor and ancestor.

85. Obtain, by various operations, new relations from the relations of being: father, mother, child, brother, sister, husband, wife.

86. Characterize the negation of a relation when a relation R is reflexive; is symmetrical; is asymmetrical; is transitive and connected.

87. What are the properties of the converse of a relation when a relation R is reflexive; irreflexive; symmetrical; asymmetrical; transitive; connected?

88. Can an asymmetrical relation R be reflexive?

89. What about the symmetry of a relation R which is irreflexive and transitive?

90. Given that "if there is an x having the relation R to every y , then for every y , there is some x having the relation R to that y ," may we infer the converse (in which x and y are interchanged)?

91. The logical product of a set of propositions is a propositional function that is always true (i.e., for all values of the variable). The logical sum of a set of propositions is a propositional function that is sometimes true (i.e., for one or more values of the variable). Explain.

92. What is meant by the uniqueness of the converse of a given relation? What is the converse of the converse of a given relation? Are these properties of the conversion of a relation analogous to those of negation?

93. Give the square of the relation: father, of, greater by one.

94. Compare "ancestor" and "father" from the standpoint of the transitivity and the square of the respective relations.

95. How are the "fields" for x and y restricted when $y^2 = x$, where x and y are integral numbers? When $x + y = 1$, where x and y are positive numbers and either is greater than 1?

96. How are a and b related when either a proposition is true of a or that proposition is false of b ? (C. S. Peirce *Collected Papers* IV.57 f.)

97. Take x as small as y . With what relation must this relation be conjoint if $x = y$? Contrast with: x less than y . Contrast the last with: x greater than y . (*Ibid.* III.28 f.)

98. Given a school in which every teacher teaches every pupil, construct and fill out a table giving at the side the multipliers: colleague (c), teacher (t), pupil (p), and schoolmate (s); and at the top the multiplicands c, t, p, s . (*Ibid.* III.77.)

CHAPTER XXII. EVALUATION

1. Distinguish statements of fact and of value: The earth is round. State Socialism would benefit the country. The American Indians were beasts. Sulphuric acid and zinc, if left together, will become zinc sulphate and hydrogen. Giving free seed to farmers is Socialism. Individualism is compatible with high protective tariffs. You mean everything to me.

2. Distinguish factual, honorific, and vituperative elements in the meanings of words: culture, reality, practical, order, propaganda, evolution, progress, a vocation, a "racket," habit, dirt, a beautiful wound, a good spanking, an excellent forgery, subsidies for shipowners, minimum wage legislation.

3. In what sense is the eating of mice by owls a case of co-operation in nature?

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4. List some words you tend to regard favorably; to regard unfavorably.

5. Classify objects of value judgments: sunshine is in general better than inky darkness; laughing, than wailing; Jane Addams, than the average woman; much intelligence, than little; craving for food, than for morphine; an ounce of gold, than of sand; health, than illness.

6. Does telling how we like a person adequately describe him?

7. May the same statement express both a judgment of fact and of value?

8. Give examples of impressiveness of size, speed, novelty, and power in the life of children; in contemporary life.

9. Pope: "Whatever is, is right." Discuss.

10. Does the knowledge a man secures determine what he does with it?

11. Generalize the distinction between a theory of radio transmission and what is said over the radio.

12. Show the need of discriminating the greater and the less; good and evil; etc.

13. Are pictures of the world in which we live, pictures of our living in it?

14. Is scientific psychology inimical to human values?

15. Show how the same thing (e.g., a wedding ring) may be evaluated from various points of view.

16. How may morphine do good and harm?

17. What grounds appear reasonable to you for preferring (or not preferring) happiness to wealth; happiness to pleasure; prudence to courage; health to exercise; recovery of health to a surgical operation; spiritual goods to bodily goods; good character to good reputation; classical music to jazz; justice in friends to justice in enemies;

achievement to good luck; friendship to wealth; competence to mediocrity; literary classics to the best sellers of the day?

18. Cite instances to which you may apply as standards of evaluation: consequences, means and ends, degrees of comprehensiveness, permanence, absolute and relative good, essential and accidental good, quantity and quality, difficulty of attainment, reality and appearance, shared experience, the example of others.

19. Show how comparison and contrast may enter into evaluation.

20. Does the exercise of a preference have about it something tentative or something final?

21. Can anything be done about unintelligent or unhappy participation in what is going on?

22. Why is it that, as Gilbert Murray has said: "A good Moslem believes in Mohammed far more passionately than anyone believes in the multiplication table"?

23. Define: absolute optimism, absolute pessimism, meliorism.

24. What appears to you a reasonable attitude toward schools of art, of psychology, etc.?

25. Distinguish and relate the practical and the moral.

26. How does the distinction between the moral and the nonmoral differ from that between the moral and the immoral?

27. "Political science involves human beings, and human beings are moral beings. Note that I do not say righteous beings." (Charles Beard *The Republic*, p. 39.) Explain.

28. Can a moral problem be solved by an antecedent conviction that no solution is possible?

29. Analyze the procedure of deliberating; e.g., deliberating what to do with a sum of money.

30. When you are in doubt how to estimate someone or something, do the facts of the case in conjunction with accepted standards of judgment always suffice to decide your evaluation?

31. Illustrate the function of the Golden Rule in moral deliberation.

32. How would you test the statement that Americans look upon divorce as an evil?

33. Are the purposes for which men live subject to continuous testing?

34. What is the lesson of "the preface to morals"? (F. J. E. Woodbridge *Nature and Mind*, pp. 471-83; *Yale Review* XX [1931] 691-704.)

35. Discuss the uses of practical reason in moral issues. (Arthur E. Murphy *The Uses of Reason*, Part II.)

36. Show from a variety of considerations how intelligent social action may be illuminated by a reasoned social philosophy. (*Ibid.*, Part III.)

37. Compare and contrast physical sciences and social studies. (H. A. Larrabee *Reliable Knowledge*, chap. xiii.)

38. Why did the radio dramatization of an invasion of the earth from Mars on October 30, 1938, result in a near panic?

39. Are human motivations to be studied in terms of reflexes, stimulus-response patterns, instincts, organic needs, or vital interests?

40. List some common fallacious or dubious notions about social matters. (Charles Beard *The Discussion of Human Affairs*.)

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41. Is "Socratic dialectic" still applicable to the clarification and organization of social concepts?

42. Can lack of precision in the results of social inquiries be offset by an admission of their tentativeness?

43. Discuss "factors in successful prediction." (Larabee *op. cit.*, chap. xvi.)

44. Suppose you were favorably inclined to the theory that a co-operative store system which would eliminate middlemen would be an economic benefit to society. Would you proceed at once to act or to advise another to act on the theory?

45. What lesson may be derived from the part played by compromises in the formulation and adoption of the national Constitution?

46. Does the fact that the forces of nature have been discovered and applied by a few exceptional individuals shed some light on the frustration of efforts on behalf of social progress?

47. Do you think of the "golden age" as lying in the past or in the future?

48. Discuss Clemenceau's saying that: "War is much too important a business to be left to the soldiers."

49. If constitutional government implies at least some democracy, will democracy necessarily always guarantee constitutional rights?

50. Show the relation between economic and cultural reconstruction. (J. H. Randall, Jr. *The Making of the Modern Mind* [rev. ed.], chap. xxii.)

51. Discuss, as a method, "the experimental application of social intelligence to specific demands, under democratic control and with democratic responsibility." (*Ibid.*, p. 641.)

52. Distinguish and relate the natural and the ideal.

53. Discuss as neglective fictions, abstractions, or ideal limits: Kepler's first law of motion; Newton's law of gravitation; Boyle's law of gases; frictionless pulleys; etc. (Read: M. R. Cohen and E. Nagel *An Introduction to Logic and Scientific Method*, pp. 367-75.)

54. With what limitations do we take the statement that a given stimulus evokes a given response?

55. What types of "personality" are distinguished in psychology?

56. What is the point about the phrase "other things being equal"?

57. Is normal color vision a "limiting case" of color blindness?

58. What is the logical significance that the pattern of heredity is subject to modification?

59. Does pure water exist in nature?

60. Can ideals be validly set up in separation from available means and without reference to obstacles?

61. Is Isaiah's picture of universal peace a description of a future scene or a vision of an ideal?

62. "Where there is no vision, the people perish." Illustrate.

63. Do any of the arts achieve their ideal aims in all cases?

64. Distinguish several approaches to the study of religion. (E. A. Burt *Types of Religious Philosophy*, chap. i.)

65. What strains of thought were combined in the theological synthesis of Augustine? (*Ibid.*, chap. ii.)

66. Examine "the major disputed assumptions" of Roman Catholicism; Protestant fundamentalism; the religion of science; agnosticism; ethical idealism; modernism; humanism. (*Ibid.*, chaps. iii-ix.)

67. Name some philosophies and trends in recent discussions of religious philosophy. (*Ibid.*, chap. x.)

68. State some major problems underlying conflicting religious points of view. (*Ibid.*, chap. xi.)

CHAPTER XXIII. ART AND CRITICISM

1. Is all art good art? What bearing does your answer have on the analytical, in contrast with the eulogistic, use of the word "art"?

2. Trace, in your own or in another's description of a natural or artificial process, the Aristotelian distinction of matter, form, efficacy, and end.

3. What is meant by the passage from potentiality to actuality? How does this differ in nature and in art?

4. What general significance do you find in the description of green plants as "the original carbon chemists"?

5. The chemist is said to consider, in the study of an element or a compound, its occurrence, preparation, properties, and uses. Comment.

6. Government is a matter, not only of machinery, but also of reactions, relationships, and controls. Comment.

7. Why is the statement important that "the nature of man is not what he is born as, but what he is born for"? (Aristotle, quoted in J. F. Dashiell *Fundamentals of General Psychology*, p. 69.)

8. Of the final, formal, material, and efficient factors, note which is stressed and in what way: This is a silver bowl. A friend advised me what to do. This statue is a sculptor's work in bronze. In the pilot's absence, the ship was wrecked. The conclusion follows from the premises. This painting is a Titian. Our architect must now be chosen. Octave, 1:2. I exercise for my health. He is fit because he works hard, and vice versa. A whole consists of

parts. At present, doctors are scarce. John Doe, M.D. Man builds because he is a builder, and a builder builds because he has mastered the art of building. (Consult: Aristototele *Physics* ii.3.)

9. Why must "chance" be recognized in processes of nature and of art?

10. Examine the adequacy of the notion that an artist is given to "self-expression."

11. How do the universal and the particular enter into artistic content?

12. Illustrate the possibilities and the limitations of special arts.

13. Give examples of positive and negative values in aesthetic effects.

14. Show the need of fusing the intellectual and the emotional in the educative process. (John Dewey *How We Think* [1933 ed.], chap. xviii.)

15. Discuss the factor of personality in the creation of art of significance. (Sam A. Lewisohn *Painters and Personality*.)

16. Bring out, by means of the contrasts suggested, the significance of the phrase adopted for the title: *The Intent of the Artist* ed. Augusto Centeno, pp. 3-35.

17. How does Sherwood Anderson (*ibid.*, pp. 37-39) interpret imagination in relation to observation and narration, and especially to everyday life and the social order?

18. Comment on the factors of collaboration, the audience, convention, and imaginative narration in the art of the playwright. (Thornton Wilder *ibid.*, pp. 83-98.)

19. Is appreciation of music enhanced by analysis in such terms as movement, harmony, expression; style, form, content; association, progression, contrast? (Roger Sessions *ibid.*, pp. 110-34.)

20. What general principles does the architect employ in the solution of his problems? (William Lescaze *ibid.*, pp. 137-62.)

21. Explain and illustrate as types of unity in variety: multiplicity in uniformity, indeterminate organization, determinate organization. (George Santayana *The Sense of Beauty*, in *Works* [Triton ed.; Charles Scribner's Sons], chap. ii: "Form.")

22. What is the demand for integration in art? Can it be satisfied to perfection?

23. Give examples of rhythm and symmetry in their dynamic interrelations and in their qualifications by their opposites.

24. Why must an artist discipline his originality?

25. Distinguish typical forms in literature or in some other art.

26. Set forth some problems of the art historian. (W. H. Constable *Art History and Connoisseurship*.)

27. It is not the case "that the form in which the aesthetic element expresses itself is independent of the purpose the work is to serve." (*Ibid.*, p. 2, note.) Comment.

28. How is the word "art" used in the assertion that "it is through the emotions that the art historian has to judge whether he is in the presence of a work of art"? (*Ibid.*, p. 7.)

29. According to Aristotle, an artist initiates change in something other than himself. Illustrate.

30. Distinguish the raw material for the carpenter and for the lumberman; for the ironmaster and for the smelter.

31. Distinguish subject matter, content, and title.

32. Are the "fine arts" identical with "art"? How do the arts vivify, clarify, and unify experience? (I. Edman *Arts and the Man*, chap. i. [Reprinted by permission of

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33. What is the place of the arts in civilization, with special reference to industry, morality, politics, efficiency, and the pursuit of happiness and of truth? (*Ibid.*, chap. ii.)

34. Account for the effect of poetry and of fiction. (*Ibid.*, chap. iii.)

35. "At its freest and most inspired triumphs, it [architecture] has all the color and line of painting, the decorative and monumental character of sculpture, the imaginative suggestiveness and suasion of poetry." (*Ibid.*, chap. iv.) Explain.

36. On what different levels does music afford enjoyment of tone, rhythm, and melody? (*Ibid.*, chap. v.)

37. What philosophic issues of morals and of truth and knowledge are posed by the arts? (*Ibid.*, chap. vi.)

38. How are means and ends related in the attitude of an artist? (John Dewey *How We Think* [1933 ed.], chap. xix.)

39. Read: Robert Henri *The Art Spirit*. What do you learn from his comments on painting, technique, self-education, etc.?

40. Discuss the table of the species of art (as skilled activity) given by C. J. Ducasse in *The Philosophy of Art*, chap. viii: skilled play; skilled work; skilled objectification of meaning, of will, and of feeling.

41. Does the analysis of the way a problem is solved apply to inventive production; to aesthetic perception?

42. Give an account of the general principles applied in the invention of the steel frame structure for tall office buildings, as described in *The Autobiography of an Idea* by Louis H. Sullivan.

43. Read: C. J. Ducasse *Art, the Critics, and You*.

44. What have we the right to expect from a book review? (Joseph Wood Krutch *The Nation* [April 17, 1937].)

45. Is there a correlation between achievement in plastic or other arts and success in telling about such achievement? (*The Painter's Object* ed. Myfanwy Evans.)

46. Is immediate liking or disliking sufficient for grounded aesthetic judgment?

47. Name one of your favorite products of art. Specify what you would miss if it had been eliminated from your experience.

CHAPTER XXIV. PHILOSOPHY AND EDUCATION

1. What is the etymological meaning of "philosophy"?

2. Contrast the popular and the historical meanings of "metaphysics."

3. Discuss, from a philosophic point of view, the Greek maxims: "Know thyself"; "Nothing in excess." (F. J. E. Woodbridge *Nature and Mind* pp. 457-70: "The Practice of Philosophy.")

4. Show the interrelation between "philosophy as logical faith"; as "social criticism"; as "mystical insight"; as "nature understood." (See: I. Edman *Four Ways of Philosophy*.)

5. Contrast the status of values in religion as a faith and in philosophy as a vision. (W. P. Montague *The Ways of Things*, pp. 171 f.: "The Vision of Philosophy.")

6. Why do philosophies diverge? Consider, in this connection, the social scene and the philosophic enterprise of expressing a culture.

7. Is democracy ultimately a form of administration or a way of life? Can you suggest other examples of phil-

osophy as comprehensive thinking? In what sense does philosophy make for unity?

8. What part is played, in "the discipline of philosophy," by contextual analysis; by reason as inclusive; by personal commitment? (Arthur E. Murphy *The Uses of Reason*, Part IV.)

9. Show the bearing of language and of observation on logic and education. (John Dewey *How We Think* [1933 ed.], chaps. xvi, xvii.)

10. Does the value of logic as an art consist chiefly in rules or in intellectual habits?

11. Does the value of logic as a science depend chiefly upon the questions or the answers suggested?

12. Illustrate the importance of logic as an instrument of exploration. (M. R. Cohen *A Preface to Logic*.)

13. Does mechanism justify disbelief in intelligence? (Charles Beard *The Republic*, p. 342.)

14. With what expressed motivations did John Stuart Mill compose his *Autobiography*?

15. Note the subjects, methods, advantages, and disadvantages of Mill's early education.

16. To what reflections was Mill led by his father's attitude toward Butler's *Analogy*; toward moral educational standards; toward personal intercourse?

17. What influences crystallized Mill's early thinking in law, psychology, and religion?

18. Note the traits and ideas stressed in Mill's accounts of his social and professional contacts, of his youthful propagandism, and of his participation in group discussions and debates.

19. Trace carefully the stages in the chief crisis of Mill's mental history and discuss their significance.

20. What light does Mill shed on the beginnings of his *System of Logic*; on the reactions of the nineteenth century against the eighteenth; on liberty and necessity; on English and French politics in his day?

21. Note the formation of Mill's altered views in practical politics; of his views on induction in logic.

22. What conceptions dominated Mill's matured reflection on: logic, political economy and social philosophy, liberty, representative government, suffrage, utilitarianism, philosophy, public affairs, liberal education, religion?

23. Summarize the lessons impressed upon you by Mill's *Autobiography*.

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